



Centre for Teaching Mathematics News



Issue 17
Autumn 2006
www.tech.plym.ac.uk/maths/CTMHOME/CTM.HTML

Welcome

Welcome to the Autumn 2006 edition of the CTM News. Apologies for there being no Spring/Summer issues – events at the Centre meant we were too busy to put together the newsletter. We will in future publish two bumper editions, one in the Autumn term and one in the Spring term. It is distributed to schools, colleges and interested people. If you are reading somebody else's copy please contact the Centre secretary to be added to the mailing list. Also if you are moving schools and would like to continue receiving the newsletter please send us details of your new school. The newsletter will contain information on the staff and activities of the Centre for Teaching Mathematics.

Editor – Jenny Sharp jsharp@plymouth.ac.uk

The Centre for Teaching Mathematics

The CTM is an inter-faculty group of mathematics educators based at the University of Plymouth within the School of Mathematics and Statistics and the Education Faculty at Exmouth plus associate members. The aims of the Centre are:

Creative Resources and Research

Training for Teachers

Mathematics Enrichment for Pupils

Contacting Us

Members of the CTM can be contacted via the Secretary:

Julie Tombs

Centre for Teaching Mathematics
School of Mathematics and Statistics
University of Plymouth

Plymouth

Devon

PL4 8AA

Tel/fax 01752 232772

Email jtombs@plymouth.ac.uk

Changes at the CTM

Ted Graham has recently been seconded to work for the National Centre for Excellence in the Teaching of Mathematics. (See page 2 for further details of the NCETM). Ted's appointment is as an assistant director and he has responsibility for research, the web portal, HE liaison as well as the south west and south east regions. Ted is very excited about working on this exciting new project, but is still doing some research work at Plymouth.
egraham@plymouth.ac.uk



Due to Ted's secondment we have an addition to the Centre's staff, **Carrie Headlam**.

She obtained her degree in Mathematics and Physics at the University of Exeter and went on to complete a PGCE also at Exeter University. Her teaching career has been in Plymouth schools, her last position being at Devonport High School for Girls where she was Assistant Head of Maths and the Schools Numeracy Coordinator. Carrie has also been with us for the past few years as a part time research student, gaining her MPhil in 2004 with "An investigation into the use of graphics calculators in mathematics at KS3". Carrie will be continuing her research as well as contributing to the Centre's activities with school pupils and teachers and teaching on the undergraduate and PGCE mathematics courses.

cheadlam@plymouth.ac.uk



NCETM Update

The National Centre for Excellence in Mathematics Teaching is now moving forward rapidly. The directors and assistant directors have been appointed and took up their posts on 1st April. The appointments are:

- **Director** – David Burghes
- **Assistant Directors**
 - Ted Graham
 - Jane Imrie
 - Rob Smith
 - Colin Matthews

Regional Coordinators are now being appointed.

The website for the NCETM is now up and running. The site includes some background to the NCETM, contact information, and also a discussion area to which teachers are encouraged to contribute. The site can be found at www.ncetm.org.uk



All teachers of mathematics are encouraged to register in order to maximise their use of the web portal.

Cornwall and West Devon Further Mathematics Centre

The CTM is playing a key role in setting up the Cornwall and West Devon Further Mathematics Centre. We have reached the stage where we have just appointed a Centre Manager for the region – she is Sharon Tripconey. It is exciting to see Further Mathematics Centres developing all over the country. Part of the brief of the Further Mathematics Centres is to engage more

youngsters in the subject. We should see an increase in the numbers taking A-level mathematics as well as further mathematics. There are a number of events planned for the Cornwall and West Devon area:

Launch of the Centre

Due to the size of the region there are two official launchings of the Centre, one in the west and one in the East:

25th September 4.30-6.00pm at Camborne School

Tuesday 26th September 4.30-6.00pm at the University of Plymouth

Events for students

There are a number of events planned for students:

- **Year 11 days** to promote mathematics post 16 and beyond.

Wednesday 18th October at Combined Universities of Cornwall's Tremough Campus, Penryn

Thursday 19th October at the University of Plymouth

- **Further Maths taster days** to promote AS Further maths

Thursday 1st February at Combined Universities of Cornwall's Tremough Campus, Penryn

Friday 2nd February at the University of Plymouth

More details about these events for the Cornwall and West Devon Further Maths Centre can be obtained from Sharon Tripconey:

sharontripconey@fmnetwork.org.uk

For more details about centres and events in your region please visit the Further Maths Network website

<http://www.fmnetwork.org.uk/>

Furthering Mathematics

The new Further Mathematics Network offers exciting opportunities to make the Further Maths A-level more widely available and to increase the number of pupils choosing to study mathematics after GCSE. A key ingredient for making a success of the Network is cooperation between the regional Centres and Universities. Last term four

members of the School of Mathematics and Statistics at the University of Plymouth presented a series of talks at a day long event for over 300 pupils from 13 schools and colleges in Wiltshire. The very successful day was organised by the managers of the Wiltshire Further Mathematics Centre, Andrea Perks and Ben Griffin. The talks included: "Why study A-level Maths?", "Maths and Bubbles", "Mind-Blowing Statistics" and "Careers in Mathematics and Statistics". Prof. David McMullan from the School of Mathematics and Statistics said: "this sort of event is ideal for exposing a large number of students to the power and variety of mathematics and statistics".

A list of talks on Mathematics and Statistics available from the School can be found at www.plymouth.ac.uk/maths (look under Support for Schools and Colleges).

Revision Days

The Centre runs A level Revision Days throughout the year. We have two planned to coincide with the January Examinations, one in Plymouth and one in London. Both days are suitable for all exam boards.

C1 Revision Day – London

Monday 18th December 2006

C1 Revision Day Plymouth

Wednesday 20th December 2006

The cost of each day is £12. Further details and reservation forms can be found on the website:

http://www.tech.plym.ac.uk/maths/CTMHOME/MEP_revision_days.htm

or contact Julie at the address on page 1.

It is possible for us to come to your school to run a revision day specifically tailored to your students and board. We can do C1, C2, C3, C4, S1 or M1. The cost of such a day would be £360 for groups up to 30. For larger groups we would charge £360 (for 30 students) plus £6 for each additional student. In addition, travelling expenses would also need to be covered by the school. For more information please contact Ted Graham by email: egraham@plymouth.ac.uk

Continuing Professional Development

Courses for 2006/2007 in Plymouth

We plan to offer the following courses in June/July 2007. To register interest please contact Julie Tombs at the address on page 1.

Updating Subject Knowledge at A Level: Core 1, Core 2, Core 3, Core 4, Mechanics , Statistics 1

These one day courses are designed for teachers who are either new to teaching A level mathematics or need to update their subject knowledge. The course is applicable for all examination boards. The aim of the summer course in Plymouth is to provide teachers with the personal competencies, knowledge and skills to teach the syllabus. Each delegate will be provided with a resource pack which contains examples and worksheets to support their teaching.

Technology, Modelling and Investigations in International Baccalaureate Mathematics

This residential course is designed for teachers who are teaching IB mathematics.

The IB curriculum for the Mathematics programmes has an internal assessment component consisting of a Portfolio or Project work based on different areas of the syllabus and representing three activities:

Mathematical investigation; extended closed problem solving; and mathematical modelling.

There are also exciting developments in the use of hand-held technologies including the use of computer technology such as graphics calculators and computer algebra systems (CAS) in the teaching and learning of Mathematics and with the future possibility of the inclusion of CAS in the formal examination assessment. This is a hands-on practical course which will provide you with the skills, resources and classroom teaching ideas for integrating investigations, modelling, computer algebra software and technology of the future such as the TI-84 and TI-89 in the teaching and learning of mathematics.

Graduate Student Programme

The Centre welcomes full-time and part-time graduate students to join its research programme. For research in Mathematics Education we normally recommend that prospective graduate students have at least three years teaching experience.

Research students register for a higher degree at one of two levels: a Master of Philosophy (MPhil) or a Doctor of Philosophy (PhD). The level of entry will depend on previous knowledge and experience.

For all research students, regular contact with the supervising team is essential. Part-time research students are expected to visit the University regularly to meet with their supervisors and other research students; overseas research students are expected to spend at least six weeks of each academic year in Plymouth.

We currently have seven part time students enrolled on the programme working on a variety of different topics:

- Mary Ann Harasymowycz: Integrating mathematics and the arts: Its impact on beliefs regarding mathematics
- Suki Honey: Investigating the relationship between beliefs in mathematics and the use of technology
- Roger Fentem: Multi level modelling in educational research
- Maria Orfanidou: An investigation into the part a spreadsheet can play in learning mathematics
- Mike Peters: An investigation into the learning experience of students being taught mathematics with technology
- Brenda Strassfeld: An investigation into relationships between teachers' beliefs and attitudes and teaching styles
- Annette Taylor: The use of a virtual environment in teaching mathematics to national level engineering students

Successful completions of research degrees include

- Brian Watson - MPhil – 2006: A case study in using electronic presentation media to teach mathematics

- Brian Walker - PhD – 2005: An Evaluation of Desktop Video Conferencing for One-to-One Tutorial Support in Mathematics
- Carrie Headlam - MPhil – 2004: An investigation into the use of graphics calculators in mathematics at KS3
- Peter McWilliam - MPhil – 2002: An examination of the effectiveness of the Socratic dialogue as a tool to promote conceptual change and overcome misconceptions in mechanics.
- Susan Picker - PhD – 2000: An investigation of lower secondary pupils' images of mathematics and mathematicians.

For further details please contact Julie Tombs at the address on page 1.

Mathematics Masterclass: Is Mathematics discovered or invented?

Introduction

For three days every June for the past seven years, talented year ten pupils from Hertfordshire, Devon and Cornwall came together to participate in three-day residential courses. These courses were joint activities between the Hertfordshire and Plymouth Royal Institution masterclass groups and they have been developed to build on the year nine series which each group runs every year.

In the two-hour session "Is Mathematics Discovered or Invented?" pupils are engaged in an introduction to philosophy. The form of the session is Socratic: the teacher asks a question and the pupils respond. The teacher then either draws out the implication of the response or asks another question based on the response. The aim of the discourse is for pupils to reach a position as to whether mathematics is invented or discovered and to argue their position. Fifteen to twenty minutes before the end, pupils are invited to write down their arguments. Below is a sample taken from those arguments. By way of introduction, Pupils are told the story of Socrates and the slave boy in Plato's *Meno*¹. By responding to Socrates' questions, the slave boy is led to Pythagoras'

theorem without information given (although one could say that the questions were 'loaded'). Socrates does this to prove the immortality of the soul: if the slave boy deduces a geometrical theorem and recognises this as a necessary truth, then where does this recognition come from, given that the slave boy has had no education in geometry? It is from this point that the Socratic discussion begins with the class. Pupils are told that they need not fear giving an answer to a question or making a contribution. All are encouraged to partake in the discussion, to listen to other contributions before making their own, not to ridicule others and not to hog the limelight. At certain points explanations have to be made, such as *necessary truth* and *Platonic form*.

Each year there have been a number of different responses in the write-up of their position, such as mathematics is discovered, invented, invention leading to discovery, discovery leading to invention, both invention and discovery, undecided. The following contains 37 extracts from a sample of 34 pupils (perhaps a few more pupils as it is not certain which response was a joint response, which were very few in number) out of a total of 91 responses over the seven years. Unfortunately, there was no opportunity to exploit the issues that they raised in their write-up. However, in a normal classroom setting the opportunity would be there to do just that.

Discovered

There were a variety of reasons given as to why mathematics is discovered rather than invented. There has been no recourse to Platonic forms, but there is a sense in which mathematical forms have existed prior to the realisation that they exist. The parallel with television as a discovery is quite interesting: *Discovered – mathematicians just put it in a sensible way of understanding it. If it's not written down, it doesn't mean that a theorem isn't true.*

Everything has always existed with the purpose of being discovered and used. Patterns, relationships, links and theorems

have been discovered, but have always existed before then.

All inventions are discoveries. The reasons why something like a TV works have always existed but John L. Baird discovered them and invented TV. This means that the TV was discovered not invented just as maths theories have always existed and will be discovered.

Some pupils have given very sophisticated arguments as to why mathematics is discovered. The following two argue that mathematics mirrors our cognitive structures / neural networks in the sense that we are 'wired' with mathematics.

Maths was discovered because it was already in people's brains. They just needed to be told about it or needed to think about it to bring it out of them and make them realise it was there.

I believe that maths is an innate system that we can all relate to, because it has roots in all of us. Maths is something in our minds, and has slowly been discovered as a form we all understand – a universal language.

This may be an answer to the age old problem that, if mathematics exists as Platonic forms, then how does the intuition of mathematicians reveal these forms? The answer given by these two pupils may be expressed as: mathematics mirrors our schematic structures, we only need to reflect and form an appropriate language to reveal the mathematics. In the following excerpt the pupil argues for discovery, but what is discovered may be expressed in different ways:

That the angles of a triangle equalising 180° has always existed but the person who discovered it could have perceived it in a different way and it could have been described completely differently. Just like imperial and metric.

This implies that the pupil would realise the difference between the angles of a triangle equals two right-angles as a necessary truth with 180° as a social convention. It would be almost trivial to state that this pupil would

have no difficulty in understanding $180^\circ = \pi$ radians.

The Platonic realm of Forms may not exist, but mathematics tells us what the Forms are if they did exist:

There are so many shapes in the world, the chance of a perfect triangle occurring is almost impossible. Likewise, we cannot construct a perfect triangle but if we could the rules involved could be proved correct.

We can invent concepts to explain, but what is explained must already exist:

Patterns, formulae and rules exist in nature, in the orbits of planets, the make up of sunflowers etc. Humankind never invented this, if we had invented it then we would have the ability to begin it, to alter it, to end it. If this is what happened, then nothing would have existed before humankind existed – surely things did. We can invent concepts to explain things, to create shortcuts but we aren't inventing things that already exists and maths is explaining truths that exist. We cannot invent something unless it has a basis on something that exists, if we visualise it in our mind then it already exists – we are progressing on concept, a rationalisation. If there were no truths, then things would collapse. We discover these truths by inventing a way to understand them.

After arguing that mathematics is discovered, the following pupil concludes with what first appears to be a contradictory statement to what s/he had written previously:

Truths have always existed, but with no-one to know that they do they can't have existed and every geometric shape is a combination of a discovery and an invention, because it is there but with no-one to know it's there, it isn't there.

If truths have always existed, but they don't exist if no-one knows of their existence, then there has to be at least one knower, *always*. Ergo, truth is in the mind of God. This may not be what the pupil had in mind, but it is an implication and one that s/he may agree with.

The following example is from a pupil who may be described as a philosophical sceptic: he asserts what he thinks is the case but also states that the assertion is one of belief rather than knowledge.

I feel (I say 'I feel' because 'I think' is a far too confident phrase for this answer) that maths was never invented. I believe it exists as a truth. No human intervention needed. But it is human nature to explore the boundary of knowledge.

I am reminded of a quote I once was told "as the island of our knowledge grows so does the shore of our ignorance".

Invented

The following arguments for invention do not mention mathematics as a body of necessary truths, in the sense that, for example, $1 + 1 = 2$ is true and always has been independent of location and how it is expressed. There is, instead, the germ of the idea that mathematics is fallible and subject to change. There is also an implied notion that mathematics is a form of activity. The following excerpt is an exemplar and can serve to represent the position of many pupils over the years:

I think maths is invented:

- *Theories are developed to fit in and work with what's already there.*
- *If something doesn't work it's scrapped.*
- *Theorems are involved to prove/disprove other things (emphasis given).*

Some who opted for invention gave elaborate arguments as to how mathematics was invented. For example, *Mathematics was invented because it is a way of relating one object to another. Before maths existed nothing related to anything else and everything was an individual object. After maths was invented, objects began to be grouped. For example, we now had these trees and not a few individual trees. We choose how to group and relate objects to one another, we did not discover it. For example, three objects did not create a triangle until we invented the lines and the*

lines between them. We could not count until we invented the numbers to count with.

Mathematics is a human activity born out of the ability to group objects, form relations out of doing so and create theoretical objects such as lines, triangles and numbers in the process. The following response is worth quoting in full because it expresses a number of philosophical points that bear a similarity with some of the arguments of the philosophers.

The potential for a series of conventions applicable for a specific numerical system exists, before the system itself, just like the potential for a man jumping off a wall exists before the man does. Therefore, as soon as one element of a potential mathematical system becomes convention, everything peripheral to it automatically becomes convention, waiting to be discovered, as does the basis of the number system.

Also, maths in general does not exist. It is merely a way of simplifying chaos, bringing order to it. Outside of intelligent beings order does not exist and so maths can only be conceptual and no conception is set in stone. From each concept there are an infinite number of divergences, all existing regardless of which becomes convention, meaning they are merely undiscovered.

The first being which stated that $1 + 1 = 2$ was not inventing a way of representing a necessary truth, merely discovering a potential for representing it. Equally, three random objects do not form the pattern of a triangle without someone there to witness it. It is just three random objects. It requires the person there to place the concept of a triangle over it, or to witness it and relate it to a triangle to be a triangle. Meaning that person is discovering their potential to represent a shape. Therefore the natural world merely provides starting points, the potential of the random points to be a triangle is discovered. The whole system is built around mathematical truths, and because we accept they are true there are other things we have to accept are true in turn. Therefore we cannot invent truths, merely recognise or discover them.

Here we have Rubin Hersh's² notion of the *potential* for mathematical existence, Kant's idea of geometry as an example of the imposition of synthetic a priori categories on what is given by sense perception³ with just a hint of Lakatos' notion of fallibility⁴. These three components can be loosely described in terms of Hersh, Kant and Lakatos and have a consistency between them. Ironically, however, the theoretical frameworks of Kant and Lakatos are largely incompatible. One has to bear in mind that this is a 15-20 minute response, yet it contains enough points to discuss at length the philosophy of mathematics for quite a few classroom sessions.

Certain statements by the pupils appear to be muddled yet are pregnant with possible inferences that would have been worth pursuing. For example:

Was maths discovered or invented? Numbers – mental objects. I think that maths was invented because it needs an intelligent species for it to be used. I do not think that it was discovered like for example the planets because that means that maths has been created before that somehow because everything else in the universe has been created and has not existed forever.

Here the pupil distinguishes between discovering a planet and discovery in mathematics, but what does she mean by creation? A discussion with the pupil might have been most fruitful.

Invention leading to discovery

Throughout these sessions pupils were asked questions that prompted the idea that mathematics is both invented (in terms of defining theoretical objects and the operations therein) and discovered (in the sense of coming across the properties of the objects invented) and many responses over the years elaborated different versions of the same view. This was inevitable as the view that mathematics is first invented then discovered was elaborated by the teacher from pupil responses that gave a cue to such an elaboration. The following are snippets taken

from the position that mathematics is first invented then discovered.

I think that maths was invented in the same way that humans created language. . . . This was the main basis for maths and from there rules and theorems were discovered.

Basic properties and basic ideas such as distance, time and numbers must be invented. But theorems surrounding these basic properties are discovered but only based on the invented properties.

The concept of the name and thoughts of our maths was invented, but the universal truth of the facts of maths was discovered – by the concept.

Perhaps most interestingly we have the idea that geometrical shapes are inventions, but their properties are eternal:

Perfect circles, squares, triangles, etc. cannot exist in the physical world; they are concepts which were invented by humans, but the properties of these shapes were discovered – they have always existed, even before the concept of the shapes were invented.

This is similar to the idea that the truth of $1 + 1 = 2$ is both timeless and prior to the first person having the idea that $1 + 1 = 2$ (i.e. the truth of a mathematical proposition is prior to the proposition itself).

Discovery leading to invention

The idea that mathematics is first discovered then invented may at first seem a little odd, until you read the following excerpts. The first does not contradict the idea that mathematics is first invented then discovered; just that prior to the invention is the stimulus from sense perception that gave rise to the theoretical objects of mathematics.

I think that there is a mixture of how maths came to be.

Discovery(1) → Invent(2) → Discover and Invent(3).

(1) I reckon that we discovered the idea of shapes and saw things like spots and

volcanoes for shapes like circles and triangles.

(2) Then we invented a perfected version of our shapes.

(3) Then we realised that our shapes had values and properties and we gave them their values and have been inventing formulas ever since.

The following echoes Popper's notion of the *objective problem situation*⁵ that problems exist independently of the cognising subject, some of which are formalised and a solution sought.

Maths was invented because it is an interpretation of problems/situations that arise, and have arose, since the beginning of time. Those problems were discovered, and as a consequence we have invented maths in order to deal with these problems.

Both invention and discovery

The following three examples of the ambivalence between invention and discovery reveal an attempt to make sense of the notions of invention and discovery:

I believe that mathematical laws are discovered because the rules must always be the same. Although numeracy was invented they all follow the same laws.

There is no answer to this question there is a mixture of both. There are basic laws which have always existed, such as the fact that two right angles make a straight line. It can be argued, however, that before humans existed there was nothing to define these laws. Without definition, something cannot exist. Therefore, when humans 'discovered' the necessary truths, they came into being. Thus, invention. So the answer is, mathematics is invented and discovered at the same time.

We believe that mathematics is both invented and discovered. The concept of ' π ' has always existed, but the name, description, rules and concept of π was invented. After the base 10 number system was discovered and invented, π could be described and therefore was discovered.

Mathematics is infinite and therefore we will never know all of it. Maths is not all recorded anywhere and so future discoveries must come from our brains and so must be invented.

Of course, maths is recorded on printed and digital matter, and π was discovered/invented and recorded prior to the invention of base 10, but the third example above seems to contain the germ of the idea that mathematical objects and the necessary truths concerning them have always existed and hence discovered, but it is through the activities of our brains that we invent these objects and what to do with them. We have a sense that mathematics exists independently of human cognition, but discovery of that mathematics consists of invention that parallels that which already exists. This notion is not uncommon. Consider the following:

The laws of maths were discovered because they were true even before we discovered them. . . . However, our ideas and representations of the laws and principles of mathematics were invented because they are purely made by people (emphasis given).

I think that as these truths exist independently of humans they are discovered, but the tools necessary to discover these truths are invented and lead our thoughts in the right direction to discover them.

The purpose and functions on which mathematics works are discovered but the concept is invented. The functions are always there but unless someone invents the concept it will never be discovered. For example, triangles always existed but unless someone invented the concept of triangles we wouldn't know about them. If someone had divided a circle into 300 instead of 360 we would know it that way.

I think that maths is created and discovered because maths was always there before humans and we had to unwrap a puzzle which lead to mathematics – so we had to discover it to understand it.

A novel approach is to regard necessary truth as prior to cognition but the 'forms' are created so as to reveal that truth:

Mathematics is discovered because the rules that apply are necessary truths. However, the forms to describe mathematics are invented as are the languages and pronunciations which express maths.

Perhaps the most sophisticated response is the idea that we construct (invent) a mathematical concept in a way that it becomes universal

I think mathematics is discovered and invented. Something could have been seen and triggered off an idea. That idea could have then been perfected and made into an invention. One example could be:

Man is out walking. He sees an interesting shape. He could then develop that shape. He could make the lines different or change the angles. He could then call this shape the square.

This shows that man has discovered something, changed it (changed the shape, made it neater). This is in turn creating something new. It is something that hasn't been seen before. It is a new concept. It is then given a name. The name is not just applied to the one shape drawn. It is a name given to a thing with similar properties to the one drawn.

Making it 'neater' is presumably making it abstract or idealised. We have what seems to be an epistemological account of how abstraction is possible, but the point being made is that what triggers an idea could be both a process of invention and discovery. A clinical interview with this pupil would have been most informative.

Undecided

As in any sample you will always have the undecided, which, in this case, is perhaps the wisest position to take, especially since this issue has been unresolved since the inception of philosophy (and abstract mathematics) by Thales. Indecision is not necessarily a form of laziness. In this case on the contrary, it indicates sensitivity to the issues involved

I am not sure whether maths was discovered or invented. I am finding it hard to decide because I can see how some parts of maths can be invented, such as triangles but I can't see how you could invent something like Pythagoras' theorem because it's a rule and you can't just make things like that up.

Before giving arguments for both discovery and invention, one pupil stated what could be described as agnostic:

Is mathematics invented or discovered? I think that the statement can be argued either way, and that there is no real proof for either conviction. Because of this, whatever I write down could have an equal and opposite argument.

Equal and opposite arguments were provided. Of course, indecision may be a principle such as pragmatism. . .

People say things can be proved but nothing can be proved. Everyone sees maths differently and I'm not sure it really matters how it became just like it doesn't matter how we exist. All I know is that it's here and I'm good at it. Nothing else matters.

. . . or a principle that does not admit to any pragmatic (or, more to the point, *utilitarian*) value:

We think that maths was invented by the Devil who wanted to see children suffer in hour long maths lessons because he is an evil sadist. A side effect of this is sad old men with too much time on their hands trying to work out π when they should be eating it.

Teachers are in the pay of the Devil but receive their salary via the LEA! The second sentence is perhaps a little sexist given that there are also elderly women mathematicians (with perhaps very little time on their hands), but the point is taken!

Rather than answer the question, the following pupil decided to place mathematics within a broader context:

*Maths is a section of science which is a theoretical representation of the real world. The human mind wishes to know what is right and will try to explain the world that they live in. When they know everything they will be **in control** of their surroundings. Maybe that **is the meaning of life!** (emphasis given).*

Conclusion

Of course, the pupils of a Maths Masterclass are bright pupils, and perhaps we can expect them to engage in philosophy as well as problems in mathematics. However, we firmly believe that, given the opportunity, most if not all pupils would benefit from a discourse about mathematics which would enhance their understanding of mathematics. An occasional meta-discourse might benefit pupils who may be described as concrete thinkers because it provides an opportunity to think about the very objects that they are asked to perform with, developing their ability to operate with those objects in the abstract. A meta-discourse involves thinking *about* the objects of mathematics in contrast to thinking *with* the objects of mathematics and its success can be gauged by the extent to which the pupils themselves engage the discussion.

We would like to conclude with several excerpts taken from a pupil who expressed scepticism of the slave boy's mathematics related to his immortal soul (e.g. "how did Socrates know that the boy had already learned it"; "if the soul is immortal why would it make the person forget what they had previously learned?"):

I think that the boy may have been quite intelligent and that by asking questions Socrates was actually teaching him about Pythagoras. I have heard a lot of teachers say that the best way to get kids to learn things is by making them think about it and you do that by asking them questions. . . . Socrates gave the boy the idea of the squares so he gave him a starting point. He then asked the boy a lot of questions making him think about the problem. I think it is possible for the boy to have proved Pythagoras without having had an education.

This pupil was arguing against the idea that maths is discovered and had stated previously:

Say there are three stars in the sky, which could look like a triangle. You could not, however, define it as an equilateral triangle or a right-angled triangle etc. because it is merely three dots in the sky. A person had to be there to join up the dots and invent a name for it.

References

1 Plato (1961) *Plato Complete*, Princeton University Press, New Jersey. The *Meno* is of course the classic text concerning mathematics and the Forms. The mature Plato and the Forms can be found in the *Republic*. If you like your Forms a little racy the read the *Symposium*.

2 Hersh, R (1998) *What is mathematics really?*, Vintage, London.

3 Immanuel Kant's *Critique of Pure Reason*, translated by NK Smith, Macmillan, London, 1976. A very readable introduction can be found in Körner S (1972), *Kant*, Penguin, Harmondsworth.

4 Lakatos, I (1976), *Proofs and refutations*, CUP, Cambridge.

5 An excellent discussion can be found in Chalmers, A (1982) *What is this thing called science?*, OUP, Milton Keynes.

Stuart Rowlands* and Alan Davies†

*srowlands@plymouth.ac.uk

† A.J.Davies@herts.ac.uk

This article appeared in the journal *Mathematics in School*, March 2006 and is reprinted with the permission of the Mathematical Association.

The Mathematics Enrichment Programme (MEP) at Plymouth

Since the formation of the Centre for Teaching Mathematics in 1990, it has provided activities for students who are able and interested in mathematics focussing at the start on activities for A level mathematics students – the Mathematics Challenge. These days were problem solving days which

involved group work and presentations based on the mathematics the students were studying for A level. In 1996 Jenny Sharp joined the Centre with responsibility for expanding the work done with schools and the formation of the Mathematics Enrichment Programme. We started with the Year 9 masterclasses which are part of the nationwide masterclass programme launched in January 1981 by the Royal Institution, forming a UK-wide network of around 60 series of classes aimed at Year 8 or Year 9 students organised regionally, mainly by local groups of volunteers, based at universities, schools or LEAs. See www.rigb.org for more details of the RI and to see if there is a masterclass series in your area.

The aim of the Year 9 masterclasses are to provide enrichment for students who are able and interested in mathematics. Each series runs for 5 sessions of two and half hours on a Saturday morning in the Spring term, each session covering a different mathematical topic. Speakers are from Plymouth, other Universities and local school teachers. Over the last 11 series more than 625 students have attended the year 9 masterclasses. The Royal Institution runs a follow – up day in June for masterclass groups at the RI in London. In 1996 we took some of our students to London, departing at 6am in the morning from Plymouth and arriving back after 10pm. While it was a good day and a chance for the students to catch up with each other again, it was simply impractical due to the distance. The following year we decided to do our own follow up and introduced the SouthWest Day. There are two other groups in the south west – Exeter and Truro, and so they were invited to the day, a total of about 100 students came to Plymouth for the day attending a number of different sessions. The MEP quickly expanded due to the demand “what can we do next?” from those who attended the year 9 sessions. In 1997 we ran our first residential course for the year 10's in conjunction with the masterclass group at the University of Hertfordshire. This three day course is always rewarding – both for the students and for the staff involved. The article “Is Mathematics discovered or invented?” in

this issue is based on one of the sessions that is run during the residential course.

The Junior schools then got to hear about our work with the secondary schools and asked “what can you do for us?”. In September 1999 we started our Year 5 days; schools were invited to bring up to 6 students who were able and interested in mathematics. That first year we had no idea of how many to expect – we got applications from 40 schools – a total of over 220 pupils meaning we had to change from the one day we had planned to three! The popularity has continued ever since, this September we have four days with 51 schools and 289 pupils. The days run from 10am to 3pm with introduction and plenary sessions, three one hour workshops and lunch. We usually split the group into smaller groups of about 25 pupils, this means that by the end of the fourth day the staff running the sessions have repeated it 12 times – hopefully the last is as good as the first!

With the year 5’s, 9’s, and 10’s masterclasses and the SouthWest Day up and running and the VIth form Challenges continuing each year we thought that we were providing enough extra curricular activities but no! The secondary schools said “can you do something for the year 8’s – we want to keep the bright ones interested and for some, year 9 is too late”. So 2001 saw the start of our year 8 masterclasses, three after school sessions in the Autumn term. Again the focus is on introducing new applications and ideas that build on the maths they have already met, a two hour session allows real in-depth investigations.

June 2002 saw the introduction of our year 6 masterclasses, following the same format as the year 5’s. Again demand was overwhelming, with just over 200 pupils from 36 school attending in the first year increasing each year to 332 pupils from 58 schools in June 2006.

Unfortunately 2002 also saw the end of the VIth form Challenges. Numbers had been dwindling for the past few years with management in schools being reluctant to let students have a whole day out of school to do something that was not directly curriculum

related! We have continued to work with A level students by running revision days :– useful for the students, acceptable to senior management but not quite as much fun! In those eleven years since 1996 the programme at Plymouth has grown in an amazing way. Over 625 students have attended the year 9 masterclasses, many of these have now finished school, gone through university and are out there hopefully still using their mathematics! A questionnaire has been sent to all students who attended the masterclasses to see what they are doing now – responses will be published in a later edition of the newsletter. Matching up the data bases from all the activities, there are now some students who have attended when they were in year 5, year 6, year 8, year 9 and year 10 – we must be doing something right!

As well as the activities run here in Plymouth for local schools we are also involved in activities around the country. We are invited to run masterclasses for other year 9 groups including Truro, Exeter, Sussex, Glamorgan and many of the London groups. We often work with secondary schools, from running a Maths Day for the whole of year 9 to workshops for the feeder primary schools. A long running event is a number of workshops over three days for all schools on the Channel Island of Guernsey. We also offer talks for schools; a popular one is entitled “Why study A level Mathematics?” and is aimed for year 10 – 11. It lasts 1 hour and gives students an interesting and interactive look at A level, covering pure, statistics, decision and mechanics. In 2006 members of the Centre worked with over 3000 students ranging from year 5 through to VIth form all over the country.

If you would like more details on our activities for schools or advice on setting up your own activities please contact Jenny.

jsharp@plymouth.ac.uk

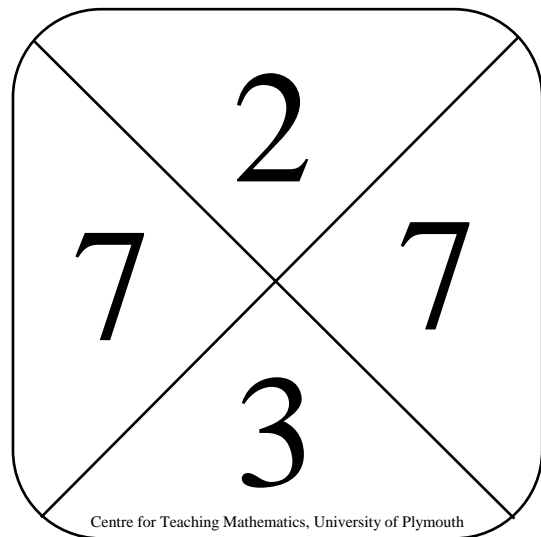
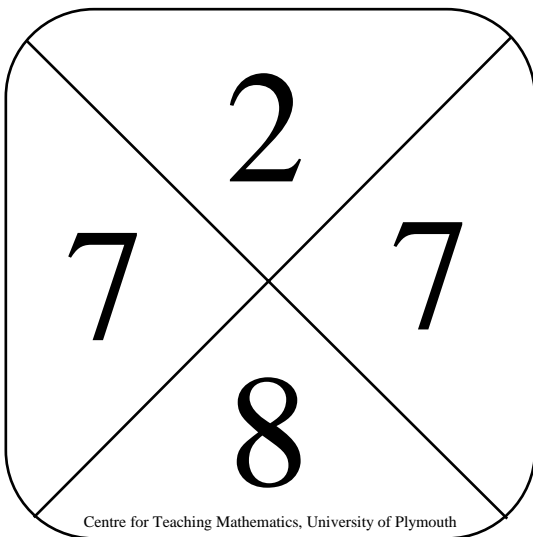
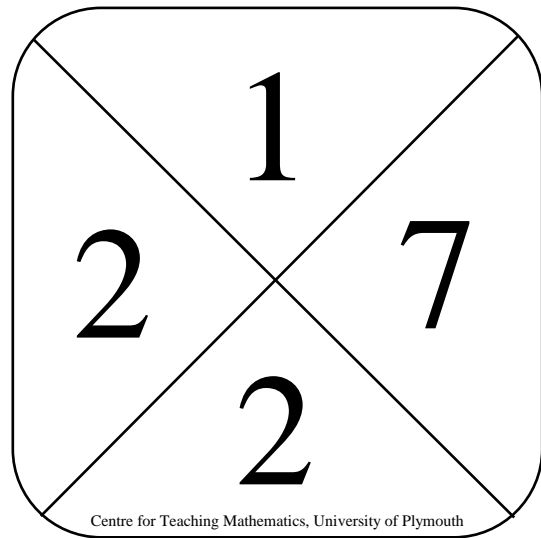
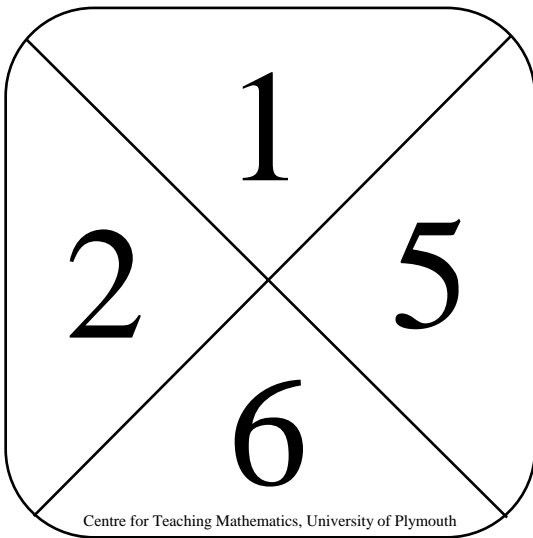
The 24 Challenge

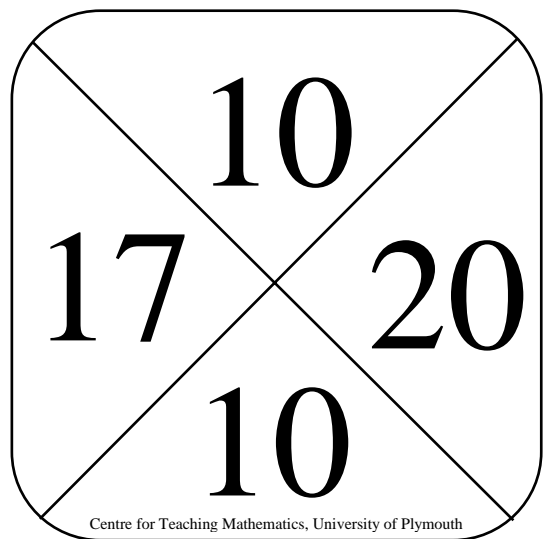
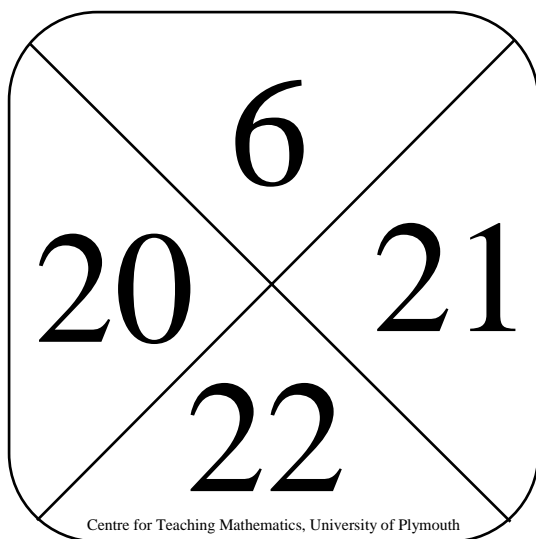
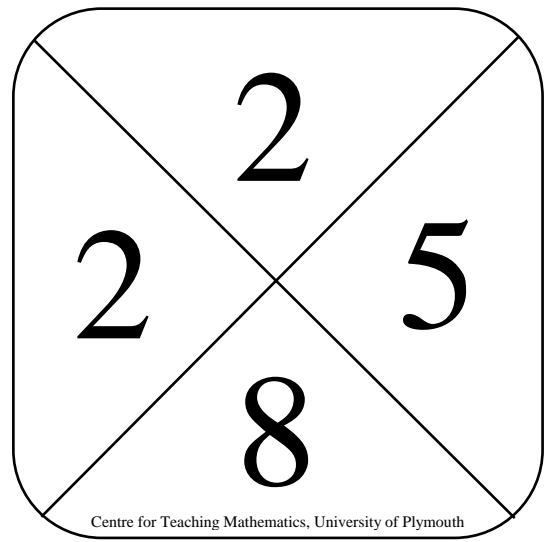
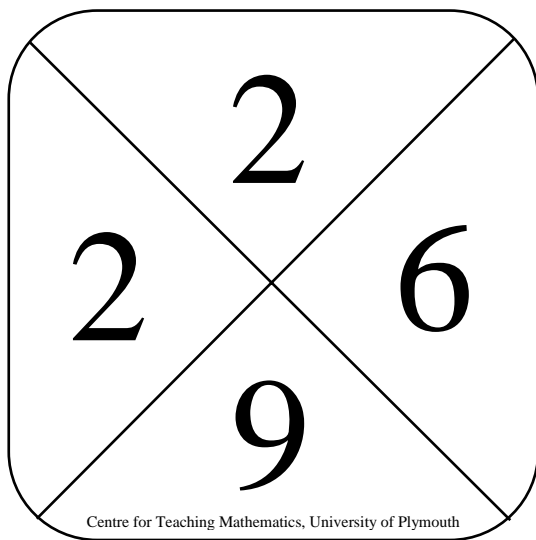
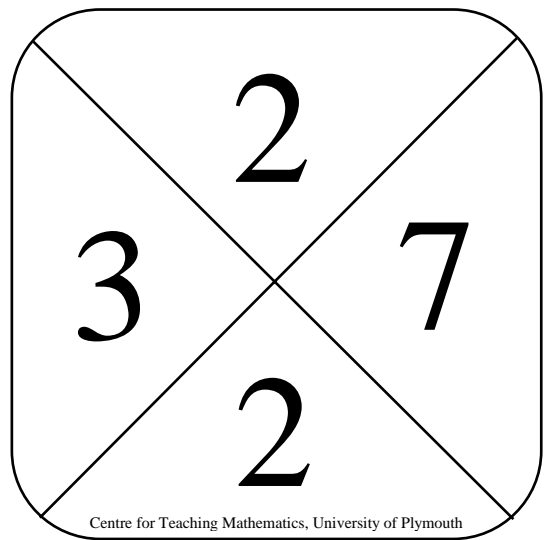
We have used the 24 challenge with students from Primary through to VIth form as a way of getting the brain going – ideal for the mental starter! 24 is a great number to work with because it has a large number of factors. We will be publishing 10 cards an issue, you can photocopy them onto card for students to use individually or in pairs or onto transparency to use with an OHP for a whole class activity.

The rules are simple:

Make the number 24 by:

- Using all four numbers once and only once
- Using the four operations $+$, $-$, \times and \div

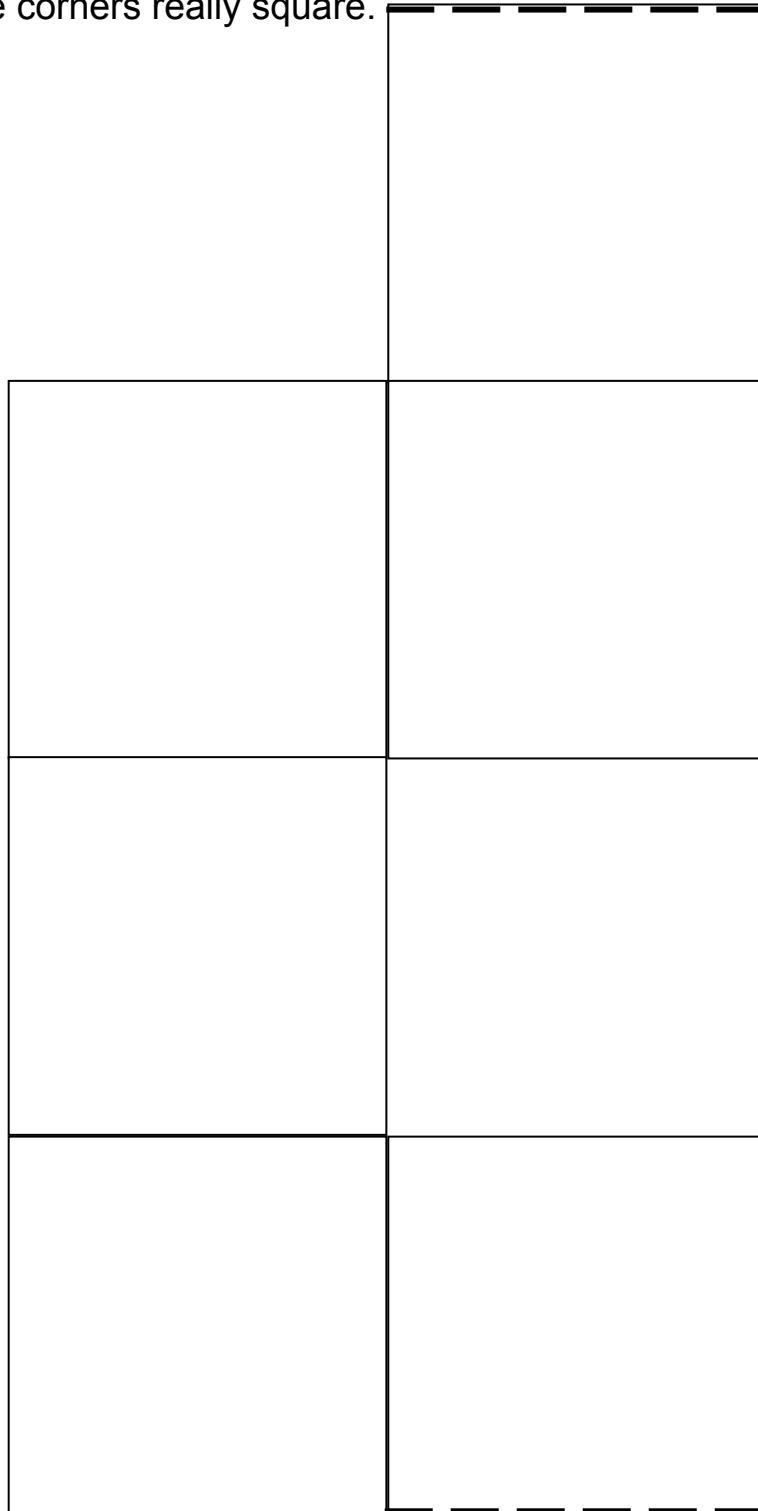




Flexagons

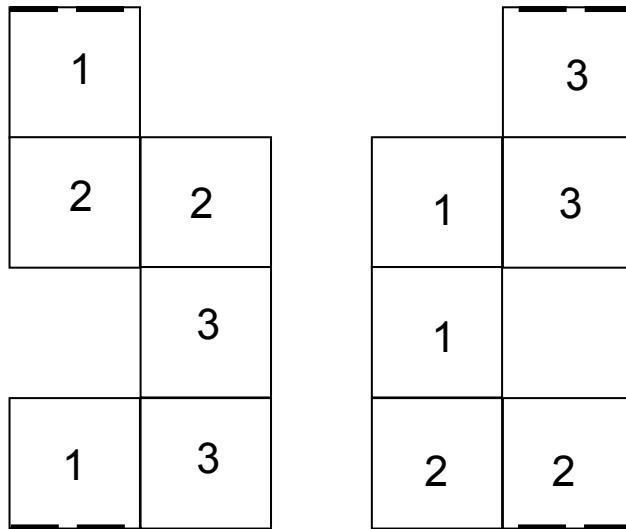
A flexagon is a folded piece of paper which has the property of having more faces than it seems – magic! It fascinates students (and adults) of all ages. Over the next few issues I will be introducing you to a number of flexagons. As you are well aware, reading instructions about how to make things is never easy so I have tried to add some diagrams to make things easier! You will probably want to try it out yourself before trying it with your class.

Photocopy this page and then carefully cut out the shape. You need to stick to the lines and make the corners really square.

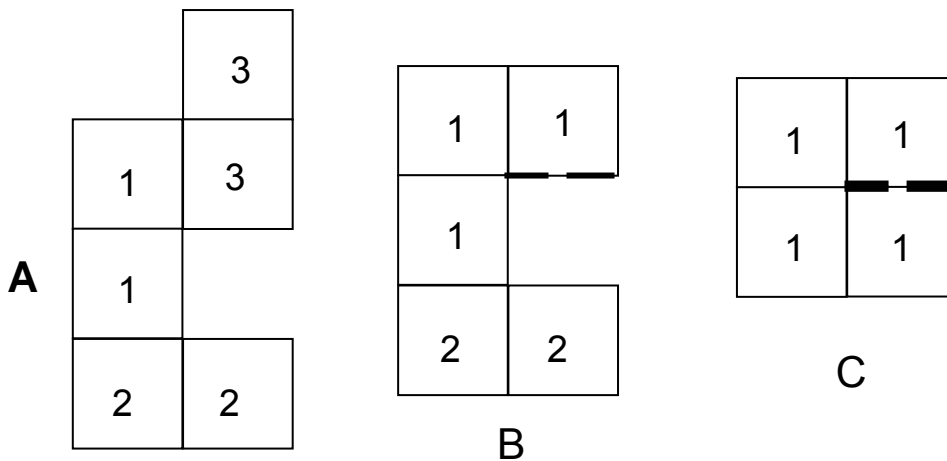


Crease carefully along each of the lines so that it can be folded each way.

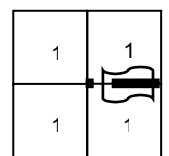
Write these numbers in each square, making sure you have your shape the right way round.



With the 'back' facing (A) you fold the top '3' square down towards you (B) and then fold the bottom two '2' squares away from you (C). You should find that your dotted lines will meet; you should have all 1s on the side facing you and all 2's on the reverse.



Line up the two dotted lines and sellotape carefully along this line making sure you do not overlap or stick it to the underneath.



You are now ready to do the magic. Remember you had 1's on one side and 2's on the other? Well you can fold it up and then unfold it so the 1's disappear and the 3's appear. The best way to do this is:

Have the side with the 2's on facing you (note the orientation of the 2's). Fold in half away from you down the centre line (D). Open up along the centre line and the 3's will appear! With practice you can flip between the faces with ease.

