

Centre for Teaching Mathematics News

Issue 2

February 2001



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Welcome

Welcome to this 2nd edition of the newsletter of the Centre for Teaching Mathematics. Through the newsletter we hope to inform schools and other interested parties of our activities and services. We are also pleased to hear from you about ways that we can help in enhancing the student experience of mathematics. Even if it's just to say thanks for the CTM News then we know that you are reading it! If you are reading somebody else's copy please contact the Centre Secretary to be added to the mailing list.

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Mischievous Headline Writers!

You may have read in the national or regional newspapers an article about the research of one of our research students on school student images of mathematicians! As reported in the last CTM News the research was about student images of mathematicians and what they do. Although the articles in the newspapers correctly reported the results of the work, the headline writers of the Telegraph and Times misrepresented the content of the articles with headlines about mathematics teachers being nerds! It would of course be a senseless bit of research to ask pupils to draw their teacher so these supposedly respectable newspapers rather missed the point.

We know that several teachers took the opportunity of the reports to discuss with their students the wide career opportunities for mathematicians and some schools had their pupils complete the Draw a Mathematician activity contained in the last CTM News. We are always looking for more data and so if you have the opportunity to explore with your pupils their

images of mathematicians and the jobs that they do we would be pleased to receive them.

Summer Inservice Courses

Each summer, in early July, we run residential inservice course in Plymouth. For July 2001 the following three courses are on offer:

- 2nd–6th July Decision and Discrete Mathematics
- 2nd–4th July Graphic Calculator Course
- 5th–7th July Modelling and Investigations in Teaching and Learning Mathematics

The course in **Decision and Discrete Mathematics** is designed for teachers of GCE AS– and A– level Mathematics who have just started or who wish to teach the first module of Decision and Discrete Mathematics for the first time. The course will assume no previous knowledge of the subject area and by the end of the course we would hope that those attending the course will be able to introduce the module to students in their School or College.

Several teachers have contacted us to say that five days out of school is difficult. If you would like to attend for part of the course then please contact us for a course programme so that you can choose the part of the course that meets your needs.

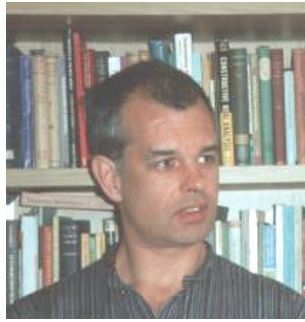
Contacting Us

Members of the CTM can be contacted via the Secretary:

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I received a BSc(Hons) in philosophy from the University of Swansea in 1978 and obtained a Post Graduate Certificate in Education (mathematics) from the same university in 1980. I had taught mathematics to ages 11-18 across the ability range at Culverhay Secondary School, Bath, for thirteen and a half years before joining the Centre in 1994, where I subsequently received a PhD in mechanics education.



I love teaching and I miss the classroom but what I don't miss is the paperwork that has piled up since the introduction of the National Curriculum, something which I regard is fundamentally flawed. Leaving the teaching profession and joining the CTM to complete a PhD has liberated me from the paperwork, but more significantly for me it has satisfied a two decade longing. Ever since my dissertation on the logical structure of Newtonian mechanics in my philosophy degree, I have been fascinated in the way something so abstract (has anybody on Earth seen an object in the absence of force or Galileo's ideal pendulum in perpetual motion) can be used to model, predict and explain the physical world. I have also been interested in how a class of mechanics students can develop the ability to think within something so abstract. My PhD has enabled me to answer many questions in relation to this curiosity and to hopefully contribute to the improvement of mechanics teaching.

I am now a lecturer with most of the timetable given to research. I am now earning what I was earning in the teaching profession when I left in Dec. 1993 but the paperwork is considerably less.

I am currently involved in two research projects. One project is a continuation of the work established in my PhD and involves the supervision of three part-time MPhil research students who are also full-time secondary teachers. The project is the investigation of the

use of concept and parallel questions in the teaching of mechanics.

Over the past decade the work of the CTM in this area has been to use concept questions to challenge student 'misconceptions' of force and motion and to develop a qualitative understanding of the Newtonian system. Central to this work has been the use of parallel questions - questions that refer to different situations that have the same explanation within the Newtonian system as the original concept question. The process of asking concept and the associated parallel questions is both diagnostic, in that it reveals the misconception, and remedial in that it leads to an understanding of the target concept. My research projects tackle this topic and involves three part-time MPhil students who are also full-time maths teachers involved in teaching A-level mechanics. Using the technique of clinical interviews, one MPhil student is researching how concept questions evoke misconceptions and how parallel questions can lead to the target-concept. Another MPhil student is using and applying these results within a classroom context and the third student is investigating the assumptions that have to be made by A-level mechanics students in the understanding of concept questions (with dyslexic students in particular).

The other research project is a collaboration with Robert Carson of Montana State University. The project is the writing of a book that assesses and traces the historical development of the current state of affairs in mathematics education and argues the need for a classical, liberal education. With the exception of utility, many educationalists are at a loss in justifying the teaching of mathematics. We argue that mathematics should be taught for its own intrinsic beauty, and that this can be best accomplished within an historical account of the origins and evolution of our various cultural systems. This way the student can live the experience as it was originally constituted and so experience the insights and the new powers of mind and ways of knowing that attended those cultural developments

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Focus on Research Projects

The Centre has four main Research groups:

Hand-held Technology

Mechanics

Working styles

Student Centred Learning

Each issue will give details about one of the research groups in more detail.

Hand-held Technology Research Group

The Hand-held Technology research group consists of John Berry, Jenny Sharp, Roger Fentem and Stewart Townend with our full-time research student Andy Smith and part time research student Carrie Headlam. The group meets regularly to discuss recent research publications and to discuss the research projects of the group members.

The main aim of our research is to investigate the use of technology to help student understanding of concepts in mathematics as well the use of technology in problem solving.

The acceptance of technology as an integral part of the mathematics curriculum in schools, colleges and universities is a continuing subject for discussion with the level and heat of the debate rising and falling periodically. It's always the same issues and questions that are asked at all levels of the curriculum. "When pupils use calculators they will not be able to do 'such and such'. Should we allow calculators in the classroom before pupils can do mathematics by hand? Should we allow calculators in the examination?"

The negative view adopted by many people results from a sense that technology is there to do mathematics whereas there is good research evidence now that good learning can be achieved through using technology. For three years the Centre for Teaching Mathematics has been involved in leading an programme of continual professional development of teachers called Teachers Teaching with Technology. Out of this programme has come some important research results and resource developments.

A common difficulty that many students have is in interpreting graphs. Imagine that you are walking in a straight line away from and towards a fixed point. A graph can be drawn of your position from the point as a function of time. This is called a displacement-time graph. Ted and Jenny have shown that a graphic calculator connected to a motion detector can be used to develop and test student understanding of displacement-time graphs. Their research was a small study to compare student understanding using traditional 'text-book' type questions with a practical technology based approach. There was evidence that student understanding was enhanced in the practical technological learning environment. (Graham E. and Sharp J., 1999, *J Teaching Mathematics & its Applications*, Vol. 18 No 3, pp 128-135)

Current research with this data logging equipment is on the development of student understanding of the properties of graphs that underpin calculus.

Research in the use of hand-held technology requires an observation of students at work and observing students working with technology can affect their working style. In a current project on the use of hand-held technology in teaching and learning mathematics, software for a graphic calculator is being developed that will capture the user keystrokes while they work on mathematics tasks. The software is hidden from the user who is unaware that her working is being observed. By replaying the sequence of calculator activities we will be able to identify and model student-working styles with technology. In this way we will be able to propose good practice for using technology.

Jenny and Roger are working with four local schools investigating the appropriate training that students require to use hand-held technology successfully in their learning.

For more information about the hand-held technology research group please contact John Berry (jberry@plymouth.ac.uk).

Do you like the idea of research in Mathematics Education?

We at present have six part-time research students who are also full time teachers. If you feel that you like to explore the idea of obtaining a MPhil or PhD in an area of mathematics education then our Graduate Student Programme might be of interest.

Centre for Teaching Mathematics Graduate Student Programme

These notes are designed to provide further information and guidance for prospective applicants. We hope that they will answer most of your initial questions however please contact the Centre if you have any specific or personal questions about making an application.

Research students may register for the degree of:

- Master of Philosophy;
- Master of Philosophy with transfer possibility to Doctor of Philosophy;
- Doctor of Philosophy.

The University of Plymouth regulations defines these qualifications in the following way:

The degree of Master of Philosophy (MPhil) should include the results of an appropriate research programme or a critical analysis of existing knowledge in a defined field. The degree of Doctor of Philosophy (PhD) should include a distinct contribution to the current knowledge of the subject. The thesis should show systematic study and independent, critical and original powers and should be capable of publication in whole or in part.

It is a requirement of the University that research students undertake an integrated programme of related studies in order to gain competence in appropriate research methods. All research students shall normally have at least two supervisors, one of whom is the Director of Studies. The supervising team will ensure that the requirement for the integrated programme of research training is met.

It is one of our aims to encourage our research students in the Centre for Teaching Mathematics to feel part of a research team. To achieve this we propose that for at least two weeks our part-time students should come together for **core study periods** during which we can develop appropriate research skills through a training programme.

Graduate students are welcome to stay on in Plymouth either side of these core weeks.

An outline of the programme for new students starting in 2001 is shown below:

Period 1: June/July first year

- Getting started with research and exploring personal research interests
- Administrative arrangements
- Introduction to research methodology and research methods
- Reading literature in the field
- Research organisational skills
- Personal development plan
- Personal time with proposed supervisory team

Period 2: February

- Further exploration personal research interests
- Research methodology and research methods
- Reading literature in the field
- Research and survey design
- Introduction to Statistical methods in educational research
- Exploring writing styles
- Development of personal programme of research
- Work on personal development plan
- Personal time with proposed supervisory team

Period 3: June/July

- Student presentations/seminars on preliminary research design
- Sharing ideas on personal project
- Further Statistical methods on educational research
- Structure of the thesis: writing styles
- Personal programme of research; ongoing self-assessment, reflection and critical input.
- Work on personal development plan
- Personal time with proposed supervisory team

Period 4: February

- Student presentations/seminars on research progress/goals
- Sharing ideas on personal project
- Start work on a literature review
- Personal programme of research
- Work on personal development plan
- Personal time with proposed supervisory team

For further details please contact John Berry for an informal chat (jberry@plymouth.ac.uk)

Formulas and Rules

In mathematics you can write relationships between quantities in terms of a formula. A formula is just a rule in symbols for calculating something. In this activity we use the TABLE feature of a graph calculator to see the effect of a formula.

Exercise 1

There are two common units of temperature that you meet, °C (Celsius) and °F (Fahrenheit)

water boils at 100°C and 212°F

water freezes at 0°C and 32°F.

This leads to a formula for converting from °C to °F:

$$F = 1.8C + 32$$

Enter $Y1 = 1.8X + 32$

Use TBLSET to enter the settings TblStart = 0, ΔTBL = 5 and use TABLE to find the temperature in °F corresponding to

(a) 10°C (b) 20°C (the temperature of a warm sunny day)

By changing TBLSET find the temperature in °F corresponding to

(c) -3°C (d) 33°C

Use the TABLE to find the temperature in °C corresponding to

(e) 50°F (f) 0°F

(g) 98.4°F (temperature of a healthy person!)

Exercise 2

In the UK, speeds of vehicles are given in miles per hour (mph) whereas in the rest of Europe kilometres per hour is used. If 5 miles is approximately 8 kilometres write down a formula to convert from miles per hour to kilometres per hour. Use TABLE to find the following speeds in kilometres per hour

(a) 30 mph (b) 70 mph (c) 100 mph

Exercise 3

It is important when driving to make sure that you keep a safe distance from the vehicle in front so that in an emergency you can stop safely. In the Highway Code a rule is given for the safe stopping distance in terms of thinking distance and braking distance:

$$\text{stopping distance} = \text{thinking distance} + \text{braking distance}$$

If the speed of a car is v mph then the stopping distance(s), in FEET, is given by the formula

$$s = v + \frac{v^2}{20}$$

Use TABLE to find the safe stopping distance for the following speeds

(a) 30 mph (b) 70 mph (c) 100 mph.

Extension for Example 3

Find a formula for giving the safe stopping distance in METRES assuming that

39 inches \equiv 1 metre. Use the TABLE to find the safe stopping distance in metres for different speeds in mph. Compare your answers with the Highway Code.

Formulas and Rules

Hints and Answers

Exercise 1

X	Y1	
0	32	
5	41	
10	50	
15	59	
20	68	
25	77	
30	86	
X=10		

X	Y1	
0	32	
5	41	
10	50	
15	59	
20	68	
25	77	
30	86	
X=20		

X	Y1	
-3	26.6	
-2	28.4	
-1	30.2	
0	32	
1	33.8	
2	35.6	
3	37.4	
Y1=26.6		

X	Y1	
30	86	
31	87.8	
32	89.6	
33	91.4	
34	93.2	
35	95	
36	96.8	
Y1=91.4		

- (a) $10^{\circ}\text{C} \equiv 50^{\circ}\text{F}$ (b) $20^{\circ}\text{C} \equiv 68^{\circ}\text{F}$ (c) $-3^{\circ}\text{C} \equiv 26.6^{\circ}\text{F}$ (d) $33^{\circ}\text{C} \equiv 91.4^{\circ}\text{F}$

X	Y1	
5	41	
10	50	
15	59	
20	68	
25	77	
30	86	
35	95	
Y1=50		

X	Y1	
-18	-17.8	
-17.9	-17.8	
-17.8	-17.7	
-17.7	-17.6	
-17.6	-17.5	
-17.5	-17.4	
-17.4		
Y1=-.04		

X	Y1	
36.6	97.88	
36.7	98.06	
36.8	98.24	
36.9	98.42	
37	98.6	
37.1	98.78	
37.2	98.96	
Y1=98.42		

- (e) $50^{\circ}\text{F} \equiv 10^{\circ}\text{C}$ (f) $0^{\circ}\text{F} \equiv -17.8^{\circ}\text{C}$ (g) $98.4^{\circ}\text{F} \equiv 36.9^{\circ}\text{C}$

An alternative approach to convert $^{\circ}\text{F}$ into $^{\circ}\text{C}$ is to rearrange the formula and use

$$C = \frac{5}{9}(F - 32).$$

Example 2

To convert V mph to kilometres per hour the formula is $y = \frac{8}{5}V$

- (a) $30 \text{ mph} \equiv 48 \text{ Kmph}$ (b) $70 \text{ mph} \equiv 112 \text{ Kmph}$ (c) $100 \text{ mph} \equiv 160 \text{ Kmph}$.

Example 3

- (a) At 30 mph, $s = 75$ feet (b) At 70 mph, $s = 315$ feet (c) At 100 mph, $s = 600$ feet

Notes for the extension problem

The formula for safe stopping distance is $s = v + \frac{v^2}{20}$ with s measured in feet. To convert

from feet to metres, multiply s by $\frac{12}{39}$.

On the TI-83 if $Y1$ is used as the formula for s then use $Y2 = 12 * Y1/39$. Encourage students to use TABLE to view the stopping distance in FEET and in METRES.

```

Plot1 Plot2 Plot3
Y1=X+X^2/20
Y2=12*Y1(X)/39
Y3=
Y4=
Y5=
Y6=
Y7=
    
```

X	Y1	Y2
0	0	0
10	15	4.6154
20	40	12.308
30	75	23.077
40	120	36.923
50	175	53.846
60	240	73.846
X=0		

X speed in mph, Y_1 stopping distance in feet, Y_2 stopping distance in metres

Parametric Curves

In this activity you will explore families of parametric curves. In each case identify any relationships between the curves and the constant number A. On your calculator choose the PARAMETRIC mode and set up the window for x and y between $-r$ and $+r$ where r is defined in each exercise.

Exercise 1

$$X = \sin T + \sin AT$$

$$Y = \cos T + \cos AT$$

Draw the family of curves for $A = 1, 2, 3, \dots, 10$ using $r = 2$.

```

Plot1 Plot2 Plot3
X1rBsin(T)+sin(
T)
Y1rBcos(T)+cos(
T)
X2T=
Y2T=
X3T=
  
```

```

WINDOW
Tmin=0
Tmax=6.2831853...
Tstep=,1308996...
Xmin=-2
Xmax=2
Xscl=1
Ymin=-2
Ymax=2
Yscl=1

WINDOW
Tstep=,1308996...
Xmin=-2
Xmax=2
Xscl=1
Ymin=-2
Ymax=2
Yscl=1
  
```

What do you find?

Exercise 2

$$X = \cos 3T + \sin AT$$

$$Y = \sin 3T - \cos AT$$

Draw the family of curves for $A = 1, 2, 3, \dots, 10$ using $r = 2$. What do you find?

Exercise 3

$$X = \cos 5T + \sin AT$$

$$Y = \sin 5T - \cos AT$$

Draw the family of curves for $A = 1, 2, 3, \dots, 10$ using $r = 2$. What do you find?

Extension

Explore the families of curves for different values of A and B in the formulas

$$X = \cos T + B \cos AT$$

$$Y = \sin T + B \sin AT$$

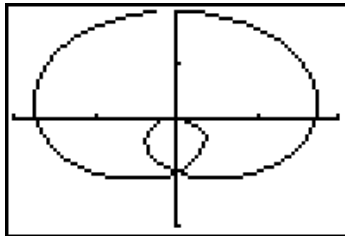
Do you always obtain symmetry? Describe the families of curves that you obtain.

Parametric Curves

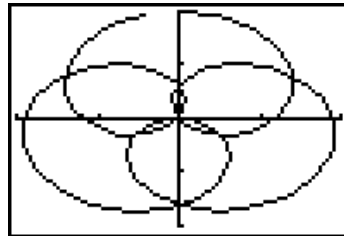
Hints and Answers

This activity is designed to encourage an investigation of the properties of geometrical objects with values of an algebraic constant. Looking for symmetries and describing figures in words are important skills for mathematics pupils/students at all levels.

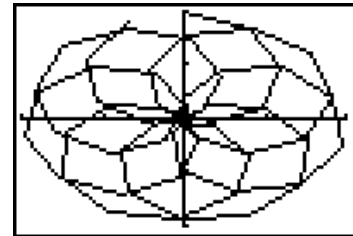
Exercise 1



A = 2



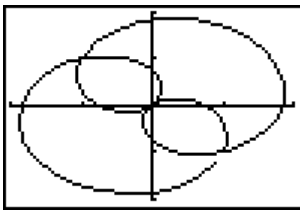
A = 4



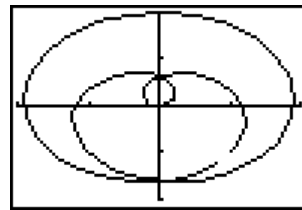
A = 8

The number of loops is always one less than A

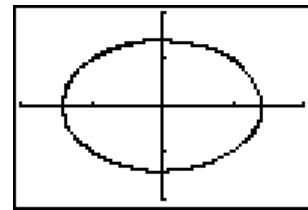
Exercise 2



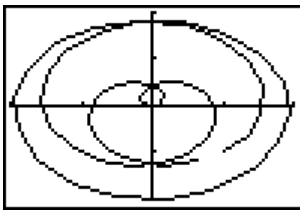
A = 1 and 2 loops



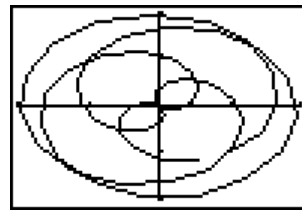
A = 2 and 1 loop



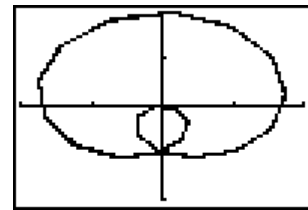
A = 3 and 0 loops



A = 4 and 1 loop



A = 5 and 2 loops



A = 6 and 1 loops

If you watch the drawing of the graph you will see that the shape is drawn out three times (to see this easily choose the cursor leading the graph option under graph styles). So we can deduce that there are 3 loops on top of each other. For $A \geq 4$ the number of loops is $A - 3$.

Exercise 3

This example is similar to Exercise 2; the following table shows how the number of loops changes with the value of A.

Value of A	1	2	3	4	5	6	7	8
Number of loops	4	3	2	1	0	1	2	3