Centre for Teaching Mathematics News

Issue 7 Autumn 2002

www.tech.plym.ac.uk/maths/CTMHOME/CTM.HTML

Welcome

Welcome to the Autumn edition of the CTM News. We publish this newsletter every term and distribute it to schools, colleges and interested people. If you are reading somebody else's copy please contact the Centre secretary to be added to the mailing list. The newsletter will contain information on the staff and activities of the CTM. Each issue will contain a teaching resource which might be a graphic calculator activity, a problem solving activity or a practical mechanics problem. This issue again contains a game designed to stimulate logical thought and the teaching resource is an investigation for A level mathematics.

The Centre for Teaching Mathematics

The CTM is an inter-faculty group of mathematics educators based at the University of Plymouth within the Mathematics Department and the Education Faculty at Exmouth plus associate members.

The aims of the Centre are:

Creative Resources and Research

Training for Teachers

Mathematics Enrichment for Pupils

Contacting Us

Members of the CTM can be contacted via the Secretary:
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Devon
PL4 8AA
Tel/fax 01752 232772
Email jtombs@plymouth.ac.uk

Inservice Course News

Although it seems a long way off, we have been making plans for our summer inservice courses here at the University. This year we are running a number of courses under two main headings – teaching A level mathematics and using technology in mathematics teaching:

Enabling you to teach AS/A2 Mathematics

These courses are designed for teachers who are new to teaching AS/A level mathematics or need to update their subject knowledge. Different teaching approaches including the use of technology will be covered. The courses are applicable for all examination boards.

Pure 1: Wed 25th June – Fri 27th June
Pure 2: Mon 30th June – Wed 2nd July
Pure 3: Wed 2nd July – Fri 4th July
Mechanics 1: Mon 30th June – Wed 2nd July
Mechanics 2: Wed 2nd July – Fri 4th July
Statistics 1: Wed 2nd July – Fri 4th July
Decision 1: Mon 30th June – Thurs 3rd July
Decision 2: Mon 30th June – Wed 2nd July

Enabling you to teach Mathematics with Technology

Just how can you integrate the use of technology in your mathematics teaching? This week of one day courses, which can be joined together to form a comprehensive course, will show you how.

Key stage 3 and 4

Dynamic Geometry - Mon 30th June
Graphic Calculators in the NNS - Tues 1st July
Using an Electronic Whiteboard - Wed 2nd July
Spreadsheets in learning Maths - Wed 2nd July

A level

Using an Electronic Whiteboard - Wed 2nd July
Spreadsheets in learning Maths - Wed 2nd July
Graphic Calculators at A level - Thurs 3rd July
Software for supporting Mathematics - Fri 4th July

For details of these courses please contact the secretary at the address opposite.

See page 3 for information on obtaining funding for such training courses.

Staff Profile – John Berry

John joined the Department of Mathematics and Statistics in the University in 1986 - at that time we were called Plymouth Polytechnic! There have been some major changes in higher education since then. Now the University of Plymouth, based on four campuses, has gained a national and international reputation in teaching and research. The Department of Mathematics and Statistics has featured strongly in these developments so that we now have nationally recognised research groups in mathematics education, particle physics, rheology and medical statistics. The Department also has vibrant undergraduate degrees in mathematics, statistics and mathematics with education.
John obtained a PhD in applied mathematics (astrophysics) from the University of Leeds in 1973 after a first degree from the University of Leicester in 1969. He became Professor of Mathematics Education in 1991.

John came to Plymouth after academic jobs in Ulster, Sheffield and with the Open University. Two major influences in the development of the mathematics education at Plymouth were David Burghes at Exeter and John Collins, the mathematics advisor for Devon. Many local teachers will remember John Collins as a mentor and friend to the subject of mathematics and mathematics teaching in Devon. It was John Collins who recommended Ted Graham to the Department as a full time research student in 1988. Subsequently Ted and John (Berry) have developed the mathematics education activities into the Centre for Teaching Mathematics with four main themes: research, mathematics enrichment programme, inservice courses and the development of resources for teaching mathematics. We have welcomed Jenny to lead the schools and mathematics themes, Stewart (Townend) and Andy (Smith) to support the various activities of the Centre and currently we have nine part-time research students from local schools, colleges and two from the USA.

Research Interests
John's research interests are fairly broad covering several aspects of the teaching and learning of mathematics at school and in higher education. For some years in the early 1990's Ted and John worked together investigating student's understanding of mechanics concepts. This formed the subject of Ted's PhD work and was continued by Stuart Rowlands.

Currently John's main research interests are in the use of technology in teaching and learning mathematics and in classroom learning styles. The availability of hand-held technology in the form of graphic calculators and data logging equipment and computer software, in particular computer algebra software such as DERIVE, challenges the curriculum and assessment of mathematics at all levels. Technology not only influences the delivery of the curriculum and its content but has changed the ways students think and do mathematics. We assume that understanding the ways that students work and their 'natural' choice of mathematical representation may contribute to the ongoing discussion of the ways students may use graphical, numerical or algebraic approaches to problem solving. The issues of how students use technology effectively need to be considered at various levels including the curriculum development level, the public examinations level, the school level, the classroom level, the individual teacher level and individual student level. How do students work with advanced calculators when doing mathematics in schoolwork, homework and in examinations? How do students working styles compare with the approach of their teacher and the training the students and teachers have received on the technology? Do students with different levels of ability work 'naturally' in different ways? These questions are at the heart of our work in Plymouth. The Centre has a small research group with two research students Andy Smith and Carrie Headlam and a collaborative research project with three teachers at Coombe Dean School investigating these issues.

Last autumn John spent four months on study leave in Finland, followed by several shorter visits, investigating the role of tasks and the use of co-operative learning in mathematics. The style of teaching and learning mathematics and the type

of tasks used affect the conceptions, attitudes and views of mathematics held by teachers and pupils. The purpose of the research in Finland was to investigate the occurrence of small group learning and the methods of co-operative learning used in school mathematics in Finland and in England. It was also the intention to draw more educators' and researchers' attention to the role of these methods of learning and to the importance of task design in implementing them into school mathematics. We concluded from our research that (i) small group learning is not common or implemented very often in the teaching and learning of mathematics in Finland or England; (ii) teachers believe that the role of pupils working in small groups is to develop social and communication skills and for doing mathematics not for learning new mathematical concepts and skills; (iii) the tasks used in school mathematics determine the teaching and learning styles and these are not conducive to the methods of co-operative learning.
Another group of research students supervised by John and Ted is investigating students and teachers attitudes to mathematics, the curriculum, the use of technology and to the styles of teaching.

Teaching interests
John's enjoyment and enthusiasm for mathematics has remained strong for over half a century! He currently teaches an introductory course in non-linear systems and chaos as well as modules in mathematics education which include *Investigations and Modelling in Mathematics* and *Teaching and Learning Mathematics (How Teachers Teach; How Pupils Learn).*

John is Editor of the International Journal of Computer Algebra in Mathematics Education and associate editor of the Journal of Teaching Mathematics and Its Applications.

Away from academic life John enjoys time with his wife Christine and children Elizabeth and Andrew, travelling (especially to Finland), singing, listening to music and playing golf (not all at the same time!).

Professional bursaries scheme

The Professional Bursaries scheme offers teachers in their fourth and fifth year of teaching a bursary of £500 to help develop their professional skills. The scheme runs in England until 31st March 2004 and eligible teachers can claim a bursary in both their fourth and fifth year.

For full information about this scheme visit
http://www.teachernet.gov.uk
and look at professional development.

A level Pure 1 Revision Day

The Centre is running a revision day for students taking the Pure 1 examination in January. They will be guided through the syllabus topic by topic, provided with a pack of resources to help their revision and given lots of helpful hints for examination success.

Date:	Wednesday 18th December
Time:	10:00 – 3:30
Place:	University of Plymouth
Cost:	£10 to include resources

To reserve a place on this course students need to complete this form and send it **with their fee** (cash or cheque made payable to University of Plymouth) to
Julie Tombs, Centre for Teaching Mathematics, University of Plymouth, Drake Circus, Plymouth, PL4 8AA

Please reserve me a space on the P1 revision Day on Wednesday 18th December
Name.....
School .....
Examination Board.....
Home Address (for correspondence).....
.....
.....
.....

Telephone .....

I enclose my fee of £10 cash/cheque

Work Experience in The Centre for Teaching Mathematics

We recently had two year 11 pupils working with us in the Centre for their work experience placement. The two pupils were from different schools, John Kitto Community College in Plymouth and Tavistock Community College, but were time tabled to be here for the same week and worked together. They followed a programme of varied activities, which left free time to work on a task set by the Centre's staff.

The programme followed by the students included;

- Visiting mathematics and applied mathematics lectures given for students on the University's Foundation Pathways in Technology programme.
- Carrying out experimental work on particle physics and quantum theory, in one of the University's laboratories.
- Talking to a mathematician about the careers that are available for mathematics graduates.
- Talking to a statistician about the career opportunities for graduates from statistics degrees.
- Visiting a primary school, where graphics calculators are being used as part of a research project.

During their "free" time the pupils worked on their task of developing two programmes for the TI-83 graphics calculator. They were given some introductory advice and shown an existing programme. Armed with this, they bravely got stuck into the task. As the week progressed they asked lots of questions, and needed some help debugging their programmes. By the end of the week they were confident and fairly accomplished programmers. The programmes can be downloaded from the resources section of the CTM website (the address is on the front page of this newsletter)

The pupils both seemed to gain a great deal from the experience, and left seeming to have had a good week.

What did the pupils think? Here are a few of the comments that they made during the week.

"This is so cool, I can't believe that I'm here doing all this. My friend is working in a carpet warehouse."

"The University is just made for skateboards!"

"The talk about statistics and what they are used for was really interesting."

"I have to get up so early to catch the bus to get here for nine o'clock."

"Thanks for a really good week."

The Centre does welcome work experience students who are enthusiastic about mathematics and would like to spend time in a mathematical environment. For more information contact Ted Graham on 01752 232773 or via email: egraham@plymouth.ac.uk

Help Needed for my research!

Heads of Department…..
Would you be prepared to take part in a research project into the use of graphics calculators?
Part of my current research project is to develop an understanding of graphics calculators in mathematics classrooms, and I am canvassing data from schools across the country.
I am looking for a wide cross-section of experience and opinion, and would welcome all information. If you think you can help, please email me on s.honey@plymouth.ac.uk,
or phone me on (01752) 232772, or you can download a copy of my questionnaire from the CTM's website on www.tech.plym.ac.uk/maths/CTMHOME/CTM.HTML. You will find it on my page which can be accessed through 'people'.
Whether you are a technowhizz or a technophobe, I look forward to hearing from you.

Suki Honey

Awithlaknannai
A Native American version of Alquerque

Awithlaknannai is played by the Native American Zuni people of the Southwest. The rules are the same as for Alquerque.

Awithlaknannai Rules

One player has 12 white pieces, the other 12 black pieces laid out on the nodes of the board (except the centre).
Decide who will move first.
Each player, in his turn, moves one of his pieces from its current location to another point.

A piece may move along one of the marked lines to an adjacent unoccupied point.

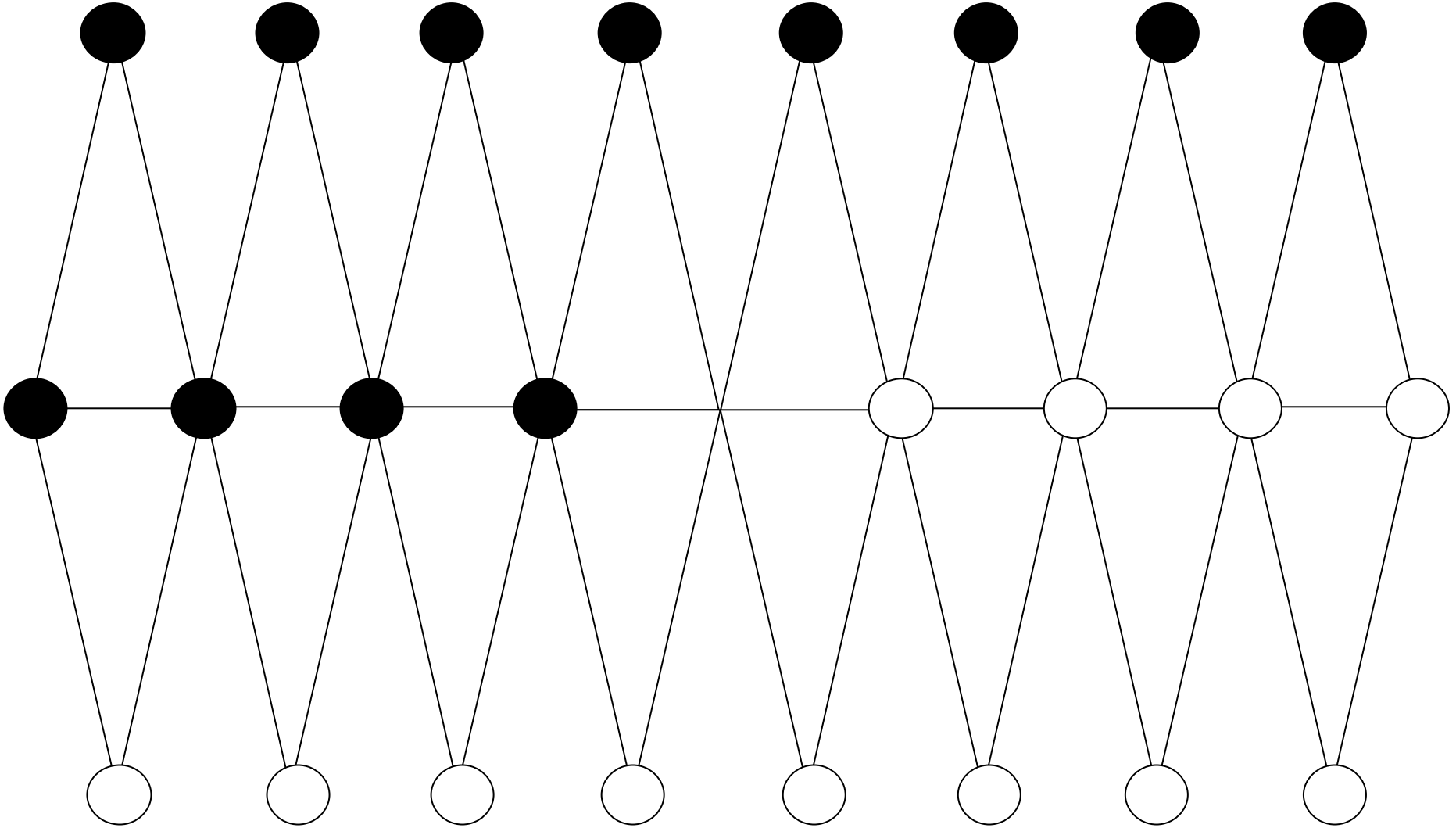
Alternatively, if an adjacent point (along one of the marked lines) is occupied by an opponent's piece but the point beyond that (in a straight line) is empty, the player may capture his opponent's piece by jumping over it to the unoccupied point.

If, after the jump is completed, another of the opponent's pieces is now available for capture, that piece may also be captured even if the second jump is along a different line to the first.

If a player is able to capture an opponent's piece during his move, he must do so. If he does not, his opponent may, at the start of his own turn, take the piece that could have made a capture. (This is in addition to the player's normal move.)
Play continues until one player has lost all his pieces.

Game board overleaf – Enlarge to A3 for ease of play

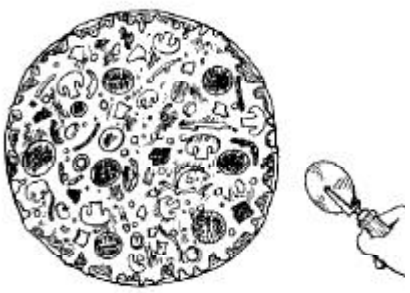
Awithlaknannai



An Investigation for Pure Mathematics

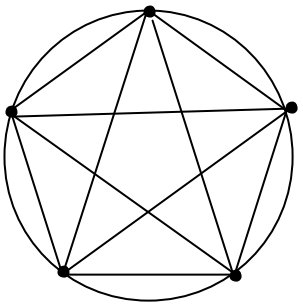
1 The usual method

What is the maximum number of pieces into which a circular pizza can be divided by making *n* cuts directly across the pizza?



2 A mathematician's method!

A mathematician decides to cut the pizza in a strange way. She places points on the edge of the pizza and joins each pair of points with a straight cut. For example, 5 points gives 16 pieces of pizza.



What is the maximum number of pieces into which the pizza can be cut by placing n points on the edge and connecting each pair of points by a cut?

Hints and Nudges

The usual method

1 Investigate, by drawing diagrams, how many pieces you get with 1,2,3 and 4 cuts. Can you find a pattern? Predict what might happen with 5 and 6 cuts.

2 Try to find a simple rule relating the maximum number of regions and the number of cuts.

A mathematician's method

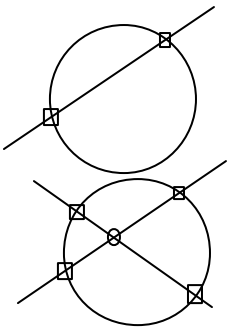
3 Find the maximum number of pieces for 1, 2,3, and 4 points on the circle. Can you identify a pattern? What happens with 5 and 6 points? Does it fit your pattern?

4 Try to find a rule. Consider the number of lines joining the points and the number of intersections of these lines. Does this help?

5 The formula relating the number of intersections, *I*, and the number of points, *n*, is $I = \frac{n(n-1)(n-2)(n-3)}{24}$. Check that this works. Try to explain why the relationship exists.

This investigation comes from Exploring Pure Mathematics, one of three books in the Exploring Series (Mechanics and Statistics being the others). The books are now out of print but many teachers have requested copies after seeing the students work on some of the problems at our VIth form days. We are in the process of adapting these books for our web pages where they will be available free of charge as PDF files. www.tech.plym.ac.uk/maths/CTMHOME/resources.htm

Teacher Material:



Consider a general approach first which looks at the number of regions formed when the interior of a circle is cut by a family of lines. We develop a notation that will provide a simple rule between the number of regions, the number of lines and the number of intersections. First consider one line. This produces two regions. Put a small square where the line cuts the circle, as shown here.

Now if we add a second line, then the number of regions increases by one as the new line cuts the first line and by another one as the line cuts the circle. Put a square where the line cuts the circle and a small circle at the point of intersection of the two lines.

Continue the process, keeping a note of the number of lines and the number of intersection. As each line is added, the number of regions increases by 1 for each intersection. We can now find a simple relationship between the number of regions and the number of lines. We have $R = 1 + \frac{N}{2} + I$ where R is the number of regions. N is the number of squares on the diagram and I is the number of intersections. (Note that N/2 is the number of lines). Clearly, the more intersections there are, the more regions there are.

1 The usual method

Now we can use this formula to solve the first pizza problem. If there are *n* cuts (i.e. lines) then each cut will produce (*n*-1) intersections. The maximum number of pizza pieces occurs when none of these intersections are concurrent. From this table we can deduce that for n cuts there will be $\frac{n(n+1)}{2}$ intersections. So the maximum number of pizza pieces is given by $R = 1 + n + \frac{n(n-1)}{2} + \frac{n^2+n+2}{2}$.

number of cuts	number of intersections
1	0
2	1
3	3
4	6
5	10
6	15

2. A mathematician's method

This problem is interesting because a pattern spotting method looks hopeful for 1 to 5 points on the circle. An unwary student may quickly deduce that $R=2^{n-1}$. However for 6 points the number of regions is 31 and not 32 and for 7 points there are 57 regions not 64. Let us try using the general formula.

number of points	number of regions
1	**1**
2	**2**
3	**4**
4	**8**
5	**16**

$R = 1 + \frac{N}{2} + I$ .The analysis is much harder than for method 1. Consider first the number of lines. Each pair of points produces a line. Take for example 6 points on a circle as shown. For 6 points, there are 5 lines from each point so the total number of lines is ½ (6 x 5). In general with *n* points there will be n(n-1)/2 lines. Now consider the number of intersections. The maximum number of pieces of pizza (i.e. number of regions)will occur when at most two connecting lines intersect (i.e. there are no points of intersection of three or more lines). Each point of intersection is then determined by four points on the edge of the circle, as shown here. Intersection P is determined by the four points, A,C,B,F and joining the 'opposite pairs' of these four points. The number of ways of choosing n things four at a time is given by $^nC_4 = \frac{n(n-1)(n-2)(n-3)}{24}$. If your students have not studied these ideas in their course, then we suggest that they formulate a table showing the numbers of intersections for various numbers of points, and that they are given the formula to validate. (We cannot imagine many students identifying the rule $\frac{n!}{4!(n-4)!}$ without considerable help.) Hence the number of pizza pieces is given by:$R = 1 + \frac{n(n-1)}{2} + \frac{n(n-1)(n-2)(n-3)}{24}$.