



# Centre for Teaching Mathematics News



Issue 8  
www.tech.plym.ac.uk/maths/CTMHOME/CTM.HTML

## Welcome

Welcome to the Spring edition of the CTM News. We publish this newsletter every term and distribute it to schools, colleges and interested people. If you are reading somebody else's copy please contact the Centre secretary to be added to the mailing list. The newsletter will contain information on the staff and activities of the CTM. Each issue will contain a teaching resource which might be a graphic calculator activity, a problem solving activity or a practical mechanics problem. This term we have a photocopiable resource for all ages – the 24 challenge – can you make the number 24 out of the four numbers given, and a problem solving task for statistics.

## The Centre for Teaching Mathematics

The CTM is an inter-faculty group of mathematics educators based at the University of Plymouth within the Mathematics Department and the Education Faculty at Exmouth plus associate members.

The aims of the Centre are:

**C**reative Resources and Research  
**T**raining for Teachers  
**M**athematics Enrichment for Pupils

## Contacting Us

Members of the CTM can be contacted via the Secretary:

Julie Tombs  
Centre for Teaching Mathematics  
University of Plymouth  
Plymouth  
Devon  
PL4 8AA  
Tel/fax 01752 232772  
Email [jtombs@plymouth.ac.uk](mailto:jtombs@plymouth.ac.uk)

## Inservice Course News

We are running a number of courses at the University this summer:

### Enabling you to teach AS/A2 Mathematics

These courses are designed for teachers who are new to teaching AS/A level mathematics or need to update their subject knowledge. The courses are applicable for all examination boards.

Pure 1: Wed 25<sup>th</sup> June – Fri 27<sup>th</sup> June  
Pure 2: Mon 30<sup>th</sup> June – Wed 2<sup>nd</sup> July  
Pure 3: Wed 2<sup>nd</sup> July – Fri 4<sup>th</sup> July  
Mechanics 1: Mon 30<sup>th</sup> June – Wed 2<sup>nd</sup> July  
Mechanics 2: Wed 2<sup>nd</sup> July – Fri 4<sup>th</sup> July  
Statistics 1: Wed 2<sup>nd</sup> July – Fri 4<sup>th</sup> July  
Decision 1: Mon 30<sup>th</sup> June – Thurs 3<sup>rd</sup> July  
Decision 2: Mon 30<sup>th</sup> June – Wed 2<sup>nd</sup> July

### Enabling you to teach Maths with Technology

Just how can you integrate the use of technology in your mathematics teaching? This week of one day courses, which can be joined together to form a comprehensive course, will show you how.

### Key stage 3 and 4

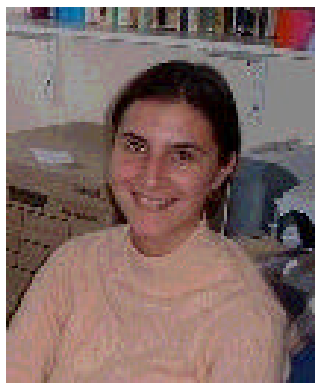
Dynamic Geometry - Mon 30<sup>th</sup> June  
Graphic Calculators in the NNS - Tues 1<sup>st</sup> July  
Using an Electronic Whiteboard - Wed 2<sup>nd</sup> July  
Spreadsheets in learning Maths - Wed 2<sup>nd</sup> July  
**A level**

Using an Electronic Whiteboard - Wed 2<sup>nd</sup> July  
Spreadsheets in learning Maths - Wed 2<sup>nd</sup> July  
Graphic Calculators at A level - Thurs 3<sup>rd</sup> July  
Software for supporting Mathematics - Fri 4<sup>th</sup> July  
For details of these courses please contact the secretary at the address opposite.

### Professional bursaries scheme

The Professional Bursaries scheme offers teachers in their fourth and fifth year of teaching a bursary of £500 to help develop their professional skills. The scheme runs in England until 31<sup>st</sup> March 2004 and eligible teachers can claim a bursary in both their fourth and fifth year. For full information about this scheme visit <http://www.teachernet.gov.uk> and look at professional development.

## Visitor Profile – Eva Dimitrova



We often have visitors to the Centre, and in January, Eva Dimitrov from Bulgaria joined us for 4 months. Eva is Head Assistant Professor in mathematics at the Food Technology University at Plovdiv. She is interested in learning how she can

incorporate technology into her teaching as well as learning about research methods. She will start her PhD when she returns to Bulgaria. We asked Eva about the education system in Bulgaria:

### 1. Describe the type of schools you have.

On the whole, students in Bulgaria begin school at the age of seven, although there are some who start at the age of six. Compulsory education covers the elementary and the primary stage. It follows the unified curriculum approved by the Ministry of Education and Science. Their studies progress as follows:

*Elementary school* – from the age of 7 to 10 (4 years): The subjects studied are: Music, Mathematics, Arts, Sports, Writing and Reading, Bulgarian History and Geography, Nature.

*Primary school* – from the age of 11 to 14 years old (4 years): The subjects studied are: Mathematics, Literature, Bulgarian, a foreign language, Physics, Chemistry, Biology, Geography, History, Sports, Music, and Arts.

Usually school starts at 7.30 and finishes at 13.30 or it starts at 13.30 and finishes at 19.30. In most schools students go to school in shifts, that is, for one month in the mornings and then for one month in the afternoons. Some schools only have classes in the mornings. They do not wear uniforms except for some new private schools opened recently. Some schools offer additional courses to students with specific interests. There are some primary schools with a mathematics, language or natural sciences focus where students have to take an admission test in the respective subject in order to be admitted to the school.

Some students decide to leave school at the age of 15, i.e. after they have completed the compulsory stage. The majority, however, cover the secondary

education stage. According to new regulations, it ends on completion of the 12<sup>th</sup> grade. There is a great variety of secondary schools:

*I. Comprehensive Secondary Schools / High Schools* - from the age of 15 to 18 (4 years): The subjects studied are: Mathematics, Literature, Bulgarian, two foreign languages, Physics, Chemistry, Biology, Geography, History, Sports, Music, and Arts. Students do not get any professional qualification upon graduation. They graduate with a diploma of secondary education and can continue their education at university.

*II. Special Secondary Schools*- These are secondary schools with a mathematics or foreign language focus – on completion of the seventh grade, that is, before the end of their primary education, students sit a national admission exam in Bulgarian and Mathematics for these schools. In their first year students only study a foreign language, Mathematics and Sports. In fact, they complete their primary education at the end of their first year in this type of school. After that they do the same subjects as in ordinary secondary schools, but they have more hours for mathematics or for the main foreign language respectively. They graduate with a diploma of secondary education and can continue their education at university.

*III. Secondary Technical Schools*– from the age of 15 to 19 (5 years). There is also an entrance exam for these schools, which is usually different for different schools, but students sit it after they have completed the primary level. These schools offer training in different fields of engineering, technology or economics. Apart from the subjects common to all secondary schools, students also do some special courses. During their last year at school, they mostly study special subjects and do practical training. They graduate with a diploma of secondary education and a professional qualification education and can continue their education at university.

*IV. Secondary Vocational Schools*– from the age of 15 to 18 (4 years). There is no entrance exam for these schools and students get different kinds of specialised training. They study most subjects common to all secondary schools but for fewer hours per week. They do a lot of special subjects and have many hours of practical training. They graduate with a

certificate of secondary education and a professional qualification and normally do not go to university.

## **2. Describe a typical mathematical lesson.**

The main types of Mathematics classes in Bulgarian schools can be classified according to their purpose as follows:

- for providing new information,
- for practice and exercises,
- for general overview of a topic,
- for revision.

For example:

A typical class aimed at providing new information consists of several stages as follows:

- review and discussion of the last homework,
- updating previously acquired information,
- teaching the new concepts,
- exercises on the new concepts, (consolidation of the new knowledge)
- giving an assignment for homework.

A typical class for practice and exercises consists of the following stages;

- review and discussion of the last homework,
- doing exercises graded according their degree of difficulty,
- generalization and consolidation of the material learnt,
- giving an assignment for homework.

Normally the first 10 or 15 minutes of all types of lessons are used for evaluation of the students' knowledge.

## **3. Can you explain about the examination system for Bulgarian Schools?**

Evaluation on all levels from elementary through secondary is on the basis of current assessment grades given after an oral examination of the individual student in class or after a written test. Their averages make up the end-of-term grades and the annual grades, which stand in the student's certificate or diploma.

Usually at the end of their last year in secondary school students sit two national school-leaving exams: one in mathematics and one in Bulgarian Language and Literature. Some students are exempt from these examinations if they have attended a special secondary school or if they have achieved excellent results in the corresponding subject in the course of their studies. Now some changes have been made in our educational system. Every

student has to sit two exams – one in Bulgarian Language and Literature and another in a subject selected by the student. They could be used as entrance exams for some universities. However, the universities with the largest number of applicants, such as Sofia University, insist on preparing their own examinations.

After a lot of arguments about the introduction of the compulsory school-leaving examinations and several controversial decisions announced by the Minister of Education, the issue was brought to the court. According to the court's decision, students who began their secondary education according to the old curriculum, which did not prepare them for this type of examination, would not have to sit these exams.

## **4. Tell us about University in Bulgaria.**

We have a large variety of Universities. For example, in Plovdiv, my home town, we have Plovdiv University, the University of Food Technology, the Agricultural University, the Technical University, and the Medical University, as well as many other colleges and branches of other higher education institutions.

Most of the Universities and the Institutes accepts students after one or more successful entrance examinations. The exams are different for each higher education establishment and students take them on the premises of the respective establishment. The entrance test in our University is in chemistry for the Technological Faculty and in Mathematics for the Technical Faculty and the Faculty of Economics. There is an additional foreign language test for the Tourism applicants. The grade scored in the entrance examination is calculated together with the grade point average from the student's diploma.

All our universities offer programs towards a Bachelor's, a Master's and a PhD degree. Most of them participate in different European projects in education. The curricula and syllabi, and the most essential criteria for study and graduation have already been developed in conformity with the European higher education systems, and some universities have introduced the credit system.

If you are interested in talking to Eva she will be at the Centre until the end of April and after that you can contact her by email on: [eva\\_di@hotmail.com](mailto:eva_di@hotmail.com)

## Vlth Form Activities

Staff from the Centre are available to visit your school or college to work with your A level Mathematics students.

- **Revision Days:** If Plymouth is too far for your students to travel to, then we can come to your school or college to run the revision days. A possible scenario would be for a school or college to volunteer to host a Revision Day for one or more of the AS or A2 level Mathematics modules, and then we would be prepared to advertise the event to other schools and colleges in the locality.
- **Vlth Form Lectures:** we are always pleased to contribute a sixth form lecture which we hope would be entertaining and enlightening!
- **Masterclasses:** A masterclass is an opportunity for students to immerse themselves for two to three hours in one topic and really get an insight on it.  
For example, Proof is a topic that works well in a masterclass environment. At present the various methods of 'proof' are spread throughout the modules and the students rarely get to see the whole picture. The Masterclass that we have developed talks about the language of mathematics, the need for proof, the rigour of proof and takes the students through a variety of methods from direct proof to proof by contradiction. For those studying Further Maths, we can include a short additional session on Proof by Induction.
- **Visits to Plymouth:** The Centre is always pleased to welcome sixth form students who are studying Mathematics for a day visit. They may join in a lecture or tutorial, meet with undergraduate students and get a flavour of life as a student at University.

If you would like to discuss your Vlth form needs then please do not hesitate to contact us.

Jenny Sharp [jsharp@plymouth.ac.uk](mailto:jsharp@plymouth.ac.uk)

## Continuing Professional Development for Teachers

The Centre offers mathematics teachers opportunities for continuing professional development in several ways. Our programme of summer courses focus on curriculum developments such as the key stage 3 mathematics initiative and Decision Mathematics at A level. Research leading to Masters and Doctorate higher degrees are available in part-time and full-time format.

The DfES funds several initiatives as part of its continuing professional development for teachers.

### The **Best Practice Research Scholarships**

have been set up "to enable teachers to undertake classroom-based and sharply focused small-scale studies in priority areas, and to apply and disseminate their findings. Using research processes to investigate classroom practice is a good way of increasing understanding about how to raise standards of teaching and learning." (DfES, 2001)

### The **Sabbatical Scheme for Experienced Teachers in Challenging Schools**

is designed to create opportunities for experienced teachers working in challenging schools to undertake a significant period of development to enhance their own learning and effectiveness, and bring subsequent benefits to their pupils and to their school.

If you, or a colleague, are interested in one of these schemes and would like to collaborate with a University then the Centre for Teaching Mathematics would be pleased to provide support in the preparation of a submission and supervision for the research or development project.

For further details of these schemes please visit the DfES web site:

[www.dfes.gov.uk/teachers/cpd](http://www.dfes.gov.uk/teachers/cpd) .

For an informal discussion of how you might collaborate with the Centre please contact John Berry.

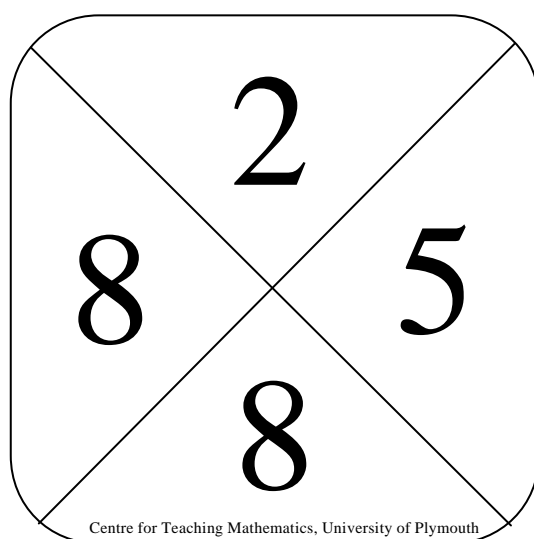
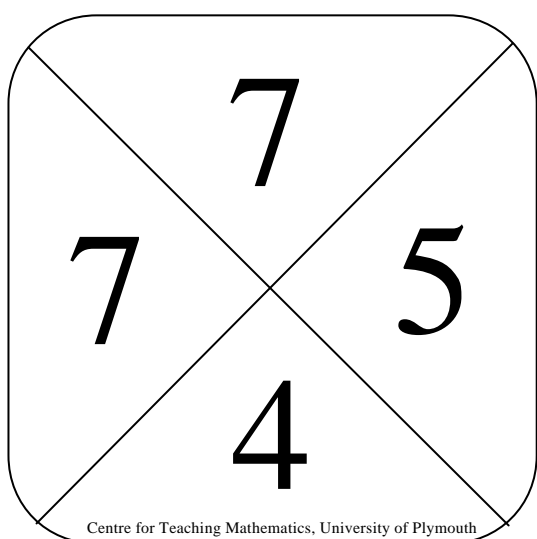
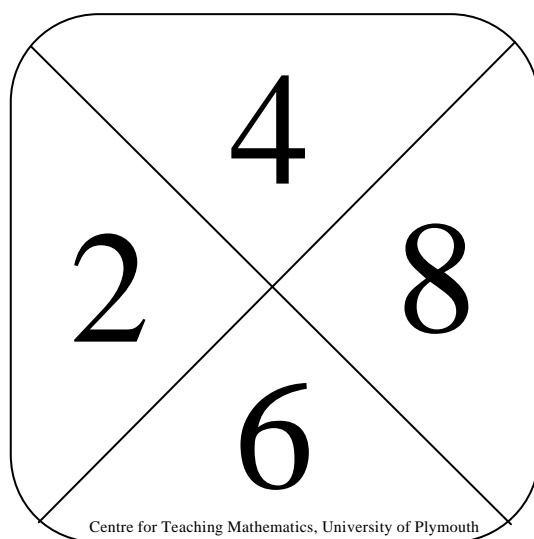
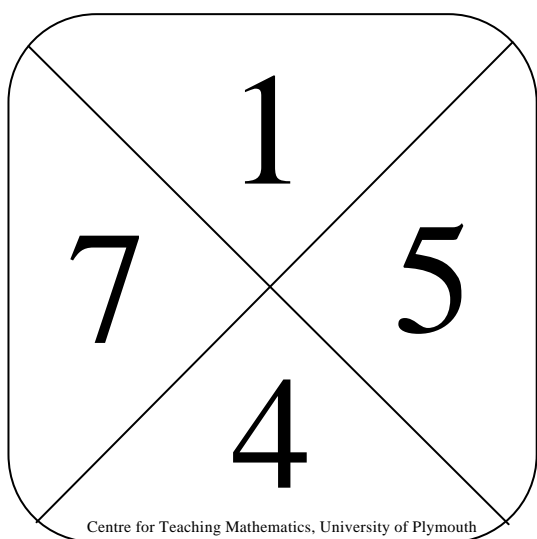
# The 24 Challenge

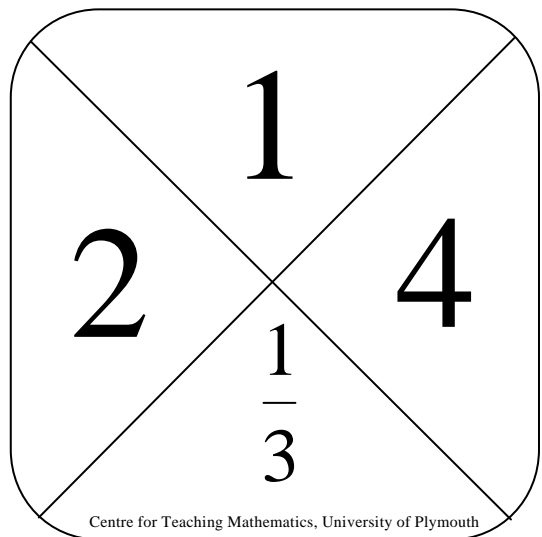
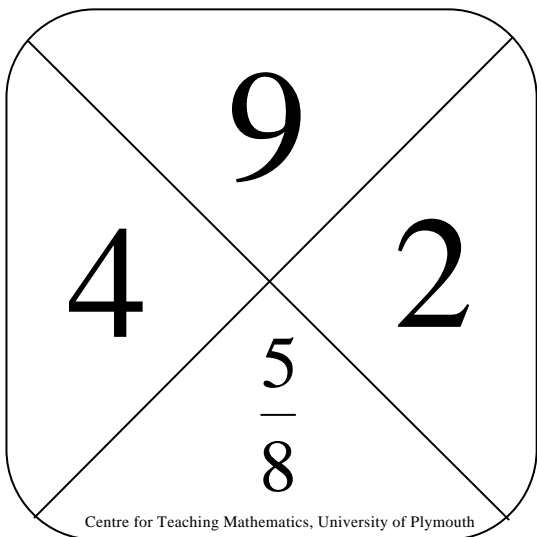
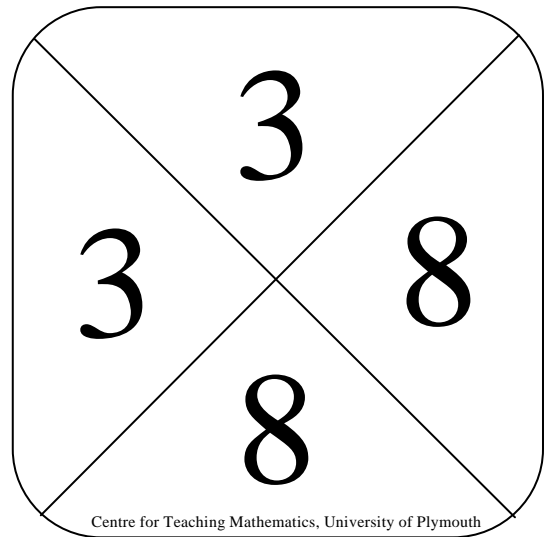
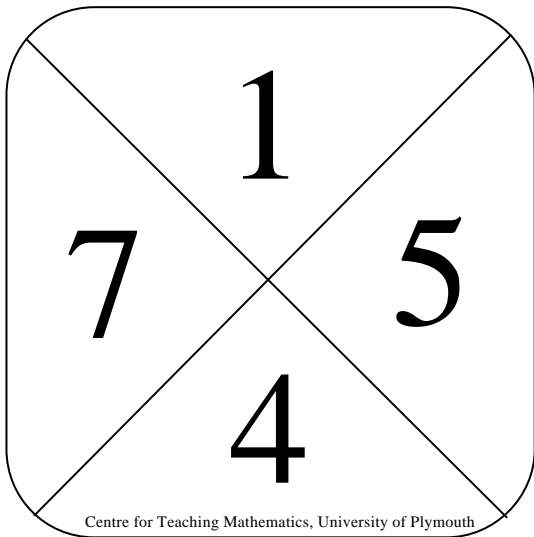
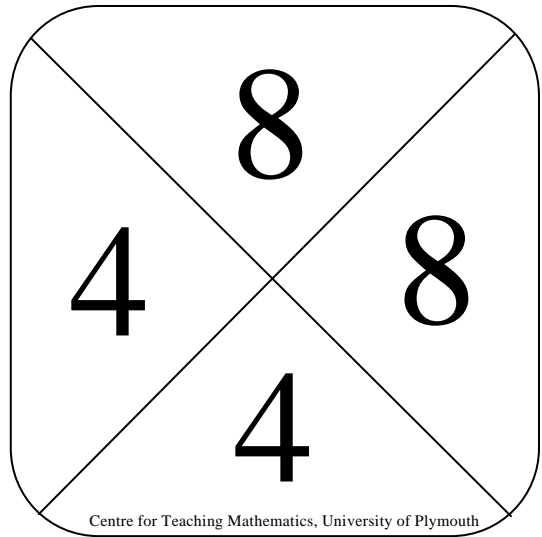
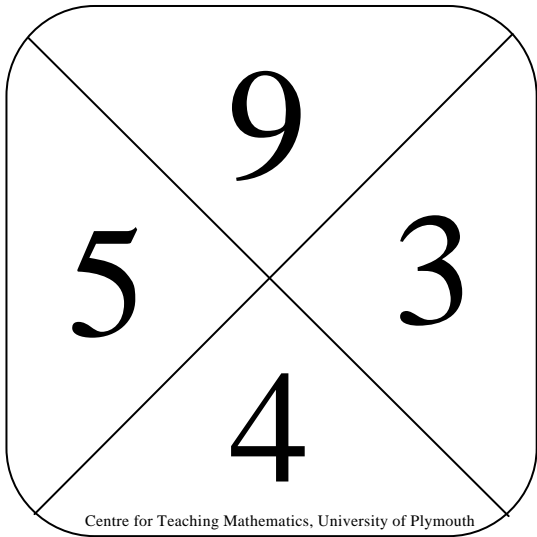
We have used the 24 challenge with students from Primary through to VIth form as a way of getting the brain going – ideal for the mental starter! 24 is a great number to work with because it has a large number of factors. We will be publishing 10 cards an issue, you can photocopy them onto card for students to use individually or in pairs or onto transparency to use with an OHP for a whole class activity.

The rules are simple:

Make the number 24 by:

- Using all four numbers once and only once
- Using the four operations  $+$ ,  $-$ ,  $\times$  and  $\div$

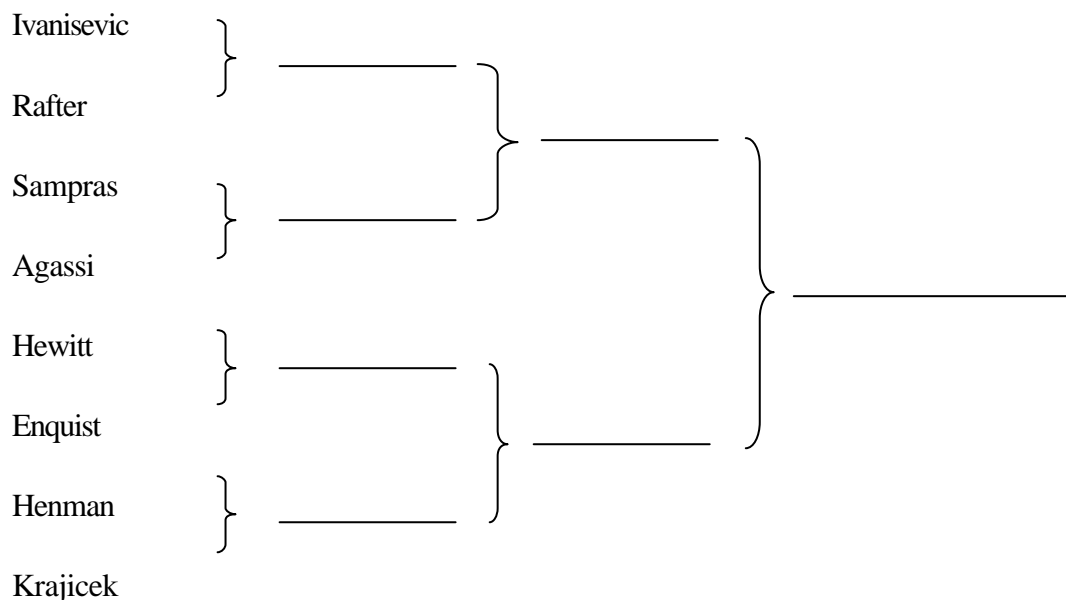




## An Investigation for Statistics: Knockout tournament

Knockout tournaments are used for many events at all levels, for example, the Wimbledon tennis tournament and the FA cup. Bookmakers often take bets on these events and have to calculate odds for a particular team or competitor winning. Investigate how probabilities can be applied to the knockout competition below:

A Wimbledon tournament has reached the quarter final stage with eight players lined up:



Previous results are shown below.

	Quarter Finals	Semi Finals	Finals
<b>2002</b>	Hewitt bt Schalken: 6-2, 6-2, 6-7, 1-6, 7-5	Hewitt bt Henman	Hewitt bt Nalbandian 6-1, 6-3, 6-2
	Henman bt Sa: 6-3, 5-7, 6-4, 6-3	7-5, 6-1, 7-5	
	Nalbandian bt Lapentti: 6-4, 6-4, 4-6, 4-6, 6-4 Malisse bt Krajicek: 6-1, 4-6, 6-2, 3-6, 9-7	Nalbandian bt Malisse 7-6, 6-4, 1-6, 2-6, 6-2	
<b>2001</b>	Agassi bt Escude: 6-7, 6-3, 6-4, 6-2	Rafter bt Agassi	Ivanisevic bt Rafter 6-3, 3-6, 6-3, 2-6, 9-7
	Rafter bt Enqvist: 6-1, 6-3, 7-6	2-6, 6-3, 3-6, 6-2, 8-6	
	Ivanisevic bt Safin: 7-6, 7-5, 3-6, 7-6	Ivanisevic bt Henman	
	Henman bt Federer: 7-5, 7-6, 2-6, 7-6	7-5, 6-7, 0-6, 7-6, 6-3	
<b>2000</b>	Sampras bt Gambill: 6-4, 6-7, 6-4, 6-4	Sampras bt Volchkov	Sampras bt Rafter 6-7, 7-6, 6-4, 6-2
	Black bt Voltchkov: 7-6, 7-6, 6-4	7-6 6-2 6-4	
	Rafter bt Popp: 6-3, 6-2, 7-6	Rafter bt Agassi	
	Agassi bt Phillippoussis: 7-6, 6-3, 6-4	7-5, 4-6, 7-5, 4-6, 6-3	
<b>1999</b>	Sampras bt Phillippoussis: 4-6, 2-1 (ret)	Sampras bt Henman	Sampras bt Agassi 6-3, 6-4, 7-5
	Henman bt Pioline: 6-4, 6-2, 4-6, 6-3	4-6, 6-4, 6-3, 6-4	
	Agassi bt Kuertin: 6-3, 6-4, 6-4	Agassi bt Rafter	
	Rafter bt Martin: 6-3, 6-7, 7-6, 7-6	7-5, 7-6, 6-2	
<b>1998</b>	Sampras bt Phillippoussis: 7-6, 6-4, 6-4	Sampras bt Henman	Sampras bt Ivanisevic 6-7, 7-6, 6-4, 3-6, 6-2
	Henman bt Korda: 6-3, 6-4, 6-2	6-3, 4-6, 7-5, 6-3	
	Ivanisevic bt Siemerink: 7-6, 6-3, 3-6, 7-6	Ivanisevic bt Krajicek	
	Krajicek bt Sanguinetti: 6-2, 6-3, 6-4	6-3, 6-4, 5-7, 6-7, 15-13	

Decide which competitor is most likely to win the final.

## Hints and Nudges

- 1 Consider how the data given could be used to estimate the probabilities that:
  - (a) Ivanisevic beats Rafter.
  - (b) Sampras beats Agassi.
 Decide upon strategies that allow you to make reasonable estimates for these probabilities.
- 1 Use your strategy to construct a table that gives the probabilities of each competitor beating each of the other competitors.
- 2 Find the probabilities that:
  - (a) Ivanisevic beats Rafter and Sampras to reach the final,
  - (b) Ivanisevic beats Rafter and Agassi to reach the final.
  - (c) Ivanisevic reaches the final.
- 1 Find the probability of each competitor reaching the final. Who are the two most likely to reach the final?
- 2 Find the probability of each player winning the tournament.
- 3 Identify any strong contenders or outsiders.

## Teacher Notes

There are many possible ways to let students explore problems such as this. Possible ways of estimating the probabilities is based on games won. Ivanisevic and Rafter have played 51 games, 26 of which have been won by Ivanisevic. We can estimate the probability Ivanisevic beating Rafter as 0.51 and hence the probability of Rafter beating Ivanisevic and 0.49.

There are competitors who have never met, such as Hewitt and Enquist. One possibility here is to assume that both are equally likely to win and assign to each an estimated probability of 0.5.

Based on these assumptions the following table can be drawn up.

	Ivanisevic	Rafter	Sampras	Agassi	Hewitt	Enquist	Henman	Krajicek
Ivanisevic	-	0.51	0.47	0.5	0.5	0.5	0.49	0.53
Rafter	0.49	-	0.43	0.49	0.5	0.66	0.5	0.5
Sampras	0.53	0.57	-	0.61	0.5	0.5	0.63	0.5
Agassi	0.5	0.51	0.39	-	0.5	0.5	0.5	0.5
Hewitt	0.5	0.5	0.5	0.5	-	0.5	0.65	0.5
Enquist	0.5	0.34	0.5	0.5	0.5	-	0.5	0.5
Henman	0.51	0.5	0.37	0.5	0.35	0.5	-	0.5
Krajicek	0.47	0.5	0.5	0.5	0.5	0.5	0.5	-

Based on these figures,

- the probability that Ivanisevic beats Rafter and Sampras to win the final is  $p(\text{Ivanisevic beats Rafter}) \times p(\text{Sampras beats Agassi}) \times p(\text{Ivanisevic beats Sampras}) = 0.51 \times 0.61 \times 0.47 = 0.146$ .
- The probability that Ivanisevic beats Rafter and Agassi to win the final is  $p(\text{Ivanisevic beats Rafter}) \times p(\text{Agassi beats Sampras}) \times p(\text{Ivanisevic beats Agassi}) = 0.51 \times 0.39 \times 0.5 = 0.099$ .
- Hence the probability that Ivanisevic reaches the final is  $0.146 + 0.099 = 0.25$
- Repeat for all competitors to find probability of reaching the final.
- If Ivanisevic is to beat Hewitt in the final, both players must first reach the final and then Ivanisevic must win. This is given by  $p(\text{I reaches final}) \times p(\text{H reaches final}) \times p(\text{I beats H})$ .
- Ivanisevic could also meet Enquist, Henman or Krajicek in the final.
- Calculate the probability of Ivanisevic winning the final against each competitor.
- The overall probability of Ivanisevic winning the final would be the sum of these four.
- Repeat for each competitor to see who has the largest probability of winning.