Centre for Teaching Mathematics News

Issue 9 Summer 2003

www.tech.plym.ac.uk/maths/CTMHOME/CTM.HTML

Welcome

Welcome to the Summer edition of the CTM News. We publish this newsletter every term and distribute it to schools, colleges and interested people. If you are reading somebody else's copy please contact the Centre secretary to be added to the mailing list. Also if you are moving schools and would like to continue receiving the newsletter please send us details of your new school. The newsletter will contain information on the staff and activities of the CTM. Each issue will contain a teaching resource which might be a graphic calculator activity, a problem solving activity or a practical mechanics problem. This term we have a photocopiable resource for all ages – the 24 challenge – can you make the number 24 out of the four numbers given, and a graphic calculator activity for A level

The Centre for Teaching Mathematics

The CTM is an inter-faculty group of mathematics educators based at the University of Plymouth within the Mathematics Department and the Education Faculty at Exmouth plus associate members.

The aims of the Centre are:

Creative Resources and Research

Training for Teachers

Mathematics Enrichment for Pupils

Contacting Us

Members of the CTM can be contacted via the Secretary:
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Devon
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Email jtombs@plymouth.ac.uk

Continuing Professional Development

As this issue goes to print, our two weeks of CPD courses for teachers are underway. We have eleven courses running this year including courses for International Schools -CAS in Teaching and Learning Mathematics and Modelling and Investigations, while for UK teachers we have Decision and Discrete Mathematics, Pure and Mechanics at A Level, and a number of one day courses on technology in mathematics including software, electronic whiteboards, graphic calculator at A level and in the NNS and Dynamic Geometry. The courses for teachers from International Schools have bought together teachers from as far afield as Tanzania, Singapore, Turkey, Russia and many European Countries while the other courses have attracted teachers from all over the UK.

The Centre has been running courses for many years now and have a wide range of expertise. As well as running the residential courses here at the University, members of the Centre are often asked to run a course in a school or college. Courses can last from a few hours to a few days depending on the needs of the school. For example on a non-pupil day, the mathematics department of the school can have a day or half day training session as part of staff development. Alternatively if the school felt that the staff would benefit from a longer training experience, the course can be organised to run over a couple of days (not necessarily consecutive). Often it is beneficial if a number of schools in one location join together for a course. This will provide participants to share experiences with new people and will also reduce the cost.

For more details on the types of courses that we offer please visit our website:
http://www.tech.plym.ac.uk/maths/CTMHOME/CTM.HTML

or contact Jenny Sharp jsharp@plymouth.ac.uk

Staff Profile – Julie Tombs

Julie is one of the most important people in the Centre as she is the Centre Secretary. She has held the position since Autumn 1999 and has become a valued member of the team.

She is responsible for the administration arrangements in connection with the Mathematics Enrichment Programme, so that much of her time is therefore spent liasing with teachers in local schools and colleges either by letter or phone. For example when we run one of our Primary workshops we tend to have over 50 schools each sending up to 6 students and Julie's tasks include making sure each school has details of which day to attend, arrange parking, organising lunch, pestering the schools for names of students so she can make badges and certificates and then sorting out the inevitable phone call that comes the next day – "one of my pupils left a jumper – you haven't found one have you?"!

Julie also takes care of the general administration in connection with the publication of the International Journal of Computer Algebra in Mathematics Education. She is therefore often in communication with authors of papers from all over the world.

This time of year is particularly busy for Julie as we run our CPD courses for teachers. It is her responsibility to process applications and payments, make sure everyone has accommodation, enough meals are ordered, make sure the delegates know how to get to Plymouth and where to go on arrival as well as getting all the teaching staff organised enough to deliver the courses!

On top of all this Julie maintains the well being of the Centre by ensuring that we never run out of tea, coffee and milk and by organising the birthday cards!
jtombs@plymouth.ac.uk

A Rationale for Studying Further Mathematics

Changes to the Mathematics A Level syllabus have made the subject more accessible to a wider range of students (which is good) but with a consequence that the most able students are not fully challenged by A Level Mathematics. Further Mathematics represents an enriching and deepening of the curriculum and thus develops the students' thinking skills. It really is *further* mathematics (not just harder or more of the same) and can provide the academic challenge and fulfilment for the more able students as well as being intrinsically interesting and an excellent preparation for a wide range of Higher Education opportunities. There is also evidence that exposure to Further Maths serves to consolidate the work of the Mathematics A Level.

In order to illustrate some of the benefits associated with Further Maths three general examples are presented below.

- Proof : this is taken beyond that covered in A Level Mathematics. The students are thus immersed in the "thinking" which underpins a proof. They thus develop an appreciation of the need for precise language and an ability to lucidly and rigorously develop an argument.

- Mechanics: the Further Maths modules offer the opportunity for an introductory study of differential equations, sufficient to develop models and understanding associated with a wide range of dynamic processes. This ability could be developed to enable the student to read the research literature with greater understanding and with a firm base from which to critically appraise such work. The modelling approach needed develops the students' problem solving skills thereby giving them valuable transferable skills.

- Statistics: in addition to studying further statistical techniques the ability to critically appraise and reflect on results is developed. Science based subjects are increasingly using larger and larger databases. If the scientist does not know sufficient

mathematics/statistics then their understanding will be compromised.

Reference to the appropriate examination papers shows that beyond the expected technical expertise considerable powers of reflection and thought are also expected of candidates in order that they can deal with the "what if....?" aspects of questions. Far from being "more of the same", Further Mathematics has much to offer the more able student. Precise language, problem solving skills and the ability to handle data with understanding are essential tools for all engineers and scientists.

Several VI form students of my acquaintance are studying Further Mathematics at A Level and have applied to various universities to read medicine. They have been told that Further Maths "won't count" for their A Level points score as it is just "more of the same". In some cases they have been advised to change their fourth A Level subject to something from the humanities as an example of broadening their field of study.

It seems strange indeed that the Medical Schools "lobby" against Further Maths when it develops some of the very attributes one looks for in a doctor.

The Centre would be interested to hear of any other instances of a similar stance being adopted with respect to other potential Further Maths students.

Stewart Townend
stownend@plymouth.ac.uk

Footnote

If you have pupils who would benefit from studying Further Mathematics but are unable to run such a course due to low numbers or time, resource or management constraints please contact the Centre. We currently run a number of modules for students in the Plymouth area. If sufficient students were interested in a particular module then the Centre could offer the course here at the University as part of its Mathematics Enrichment Programme. Please contact Jenny Sharp in the first instance.
jsharp@plymouth.ac.uk

What are your students doing when working with a Graphic Calculators?

One of our research students, Andy Smith, has recently successfully completed his MPhil research study observing a group of students working with graphic calculators. It was no surprise that their working styles left much to be desired. We would argue that the lack of training of the students in the use of the graphic calculator and possible inefficient use of the technology by their teachers leads to the situation.

When students are working with hand-held technology, such as a graphic calculator, we usually only see the outcomes of their activities in the form of a contribution to a written solution of a mathematical problem. It is more difficult to capture their process of thinking or actions as they use the technology to solve the problem. As part of Andy's research he developed a software application that works secretly in the background of a graphic calculator capturing the keystrokes that are used. In this way the students are able to work naturally without the feeling of 'being observed'. After a student has used the graphic calculator we are able to playback the sequence of keystrokes to explore how the students actually used the technology, whether they used 'trial and error' mode and how their working related to the training they had received. There have only been a few empirical investigations on student working styles. The small research study described in Andy's thesis should be seen in terms of new research opportunities to capture and investigate student's actions as they work with hand-held technologies.

The software includes:

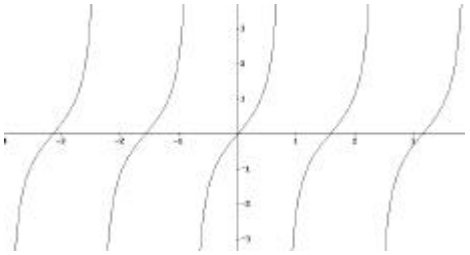
- a module which saves the keystrokes to the calculators internal memory.
- a module which allows the data to be replayed so that the researcher can view exactly what the student has done.
- a module which allows the individual key strokes to be viewed as a list.
- a provision to take the data off the calculator and store it on the hard disk of a computer via the Graphlink.

The software works entirely in the background and the student is unaware of its presence.

A small scale study was undertaken to test out and evaluate the key recorder software. The students who participated had just started the first year of a mathematics degree course. The task the students were set consisted of twelve questions, the first six questions asked students to find the expression that described given graphs and the remaining six involved the students sketching a complete graph of a given function. The students who owned a TI-83 calculator had it swapped for a TI-83 Plus with the Key Recorder software running.

As could be expected for the first six questions the majority of our students used either an analytic construction or graphic trial approach. It was surprising to find that in four of the six questions less than 50% of the students got a correct answer.

For example only 38% of the students gave a correct answer to the question below:
Find an expression for y in terms of x, which describes the graph.

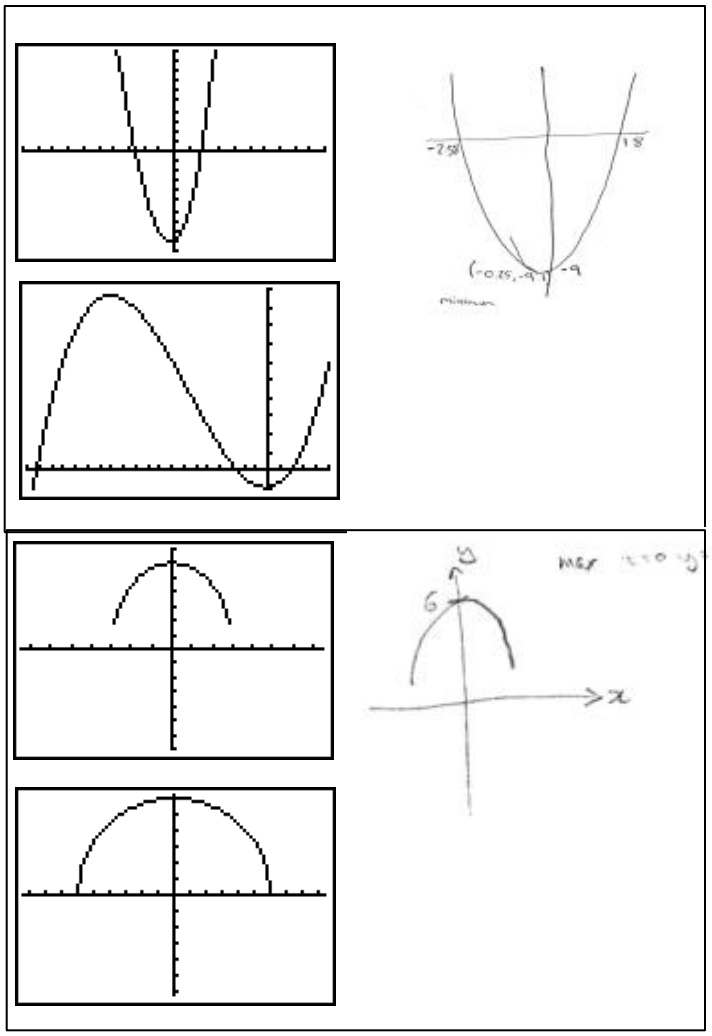


The second part of the exercise involved the students being asked to sketch graphs of six different functions. Two examples are given below showing the calculator screen that the student would have seen having used an unsuitable Window together with the sketch they made. Also shown is the calculator output of the 'complete' graph.

- Sketch a complete graph of the function
  $y = 0.1x^3 + 2x^2 + x - 9$
- Sketch a complete graph of the function
  $y = \sqrt{36 - x^2}$

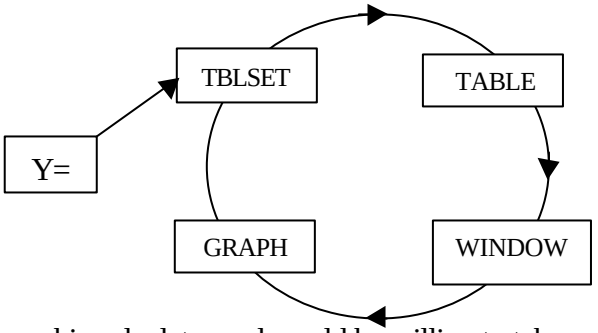
As we see in both these examples the students directly copied the calculator screen. There were many further examples like these.

The apparent failure by many of the students to set an appropriate viewing window when drawing a graph



with a graphic calculator led us to devise the following algorithm to assist students when drawing a graph. It should be stressed that this algorithm is primarily for the inexperienced user of a graphic calculator

We are now interested in exploring how teachers use graphic calculators in their teaching and how students work with their graphic calculators at school and at home, in homework tasks. If you are a user of the TI-



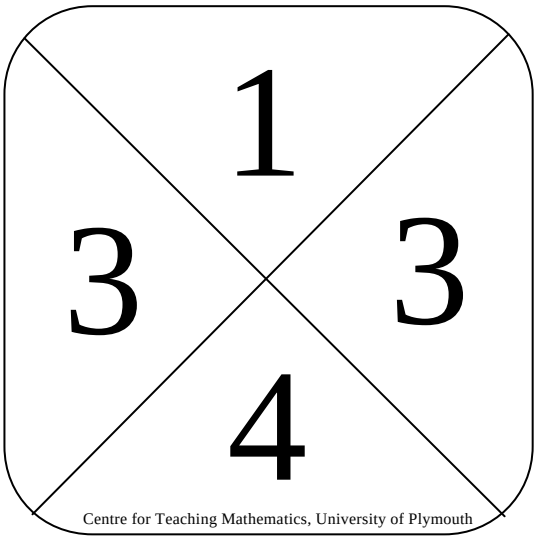
83 graphic calculator and would be willing to take part in a project please write to John Berry (jberry@plymouth.ac.uk).

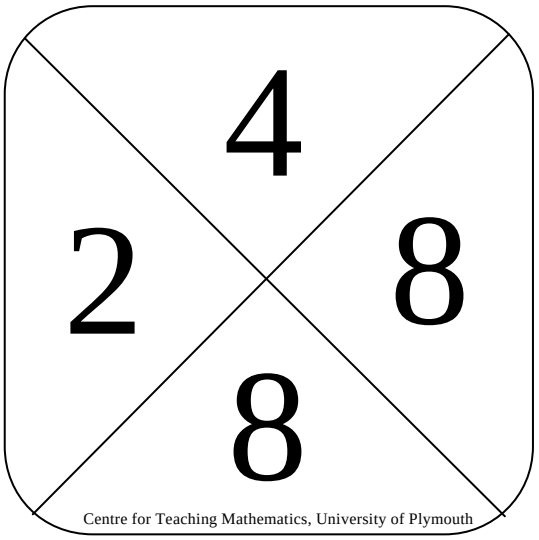
The 24 Challenge

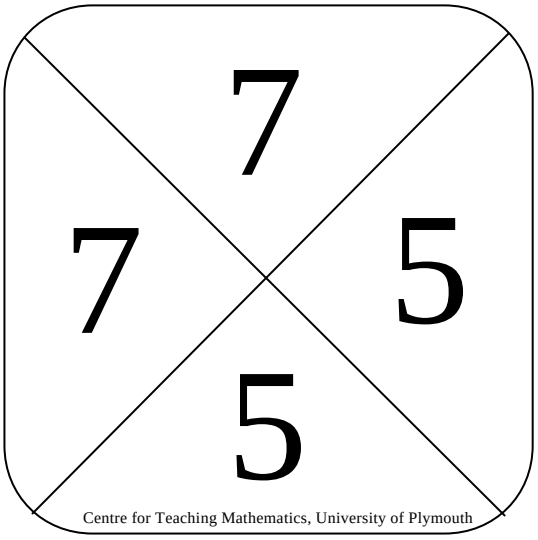
We have used the 24 challenge with students from Primary through to VIth form as a way of getting the brain going – ideal for the mental starter! 24 is a great number to work with because it has a large number of factors. We will be publishing 10 cards an issue, you can photocopy them onto card for students to use individually or in pairs or onto transparency to use with an OHP for a whole class activity.
The rules are simple:

Make the number 24 by:

- Using all four numbers once and only once
- Using the four operations +, –, × and ÷





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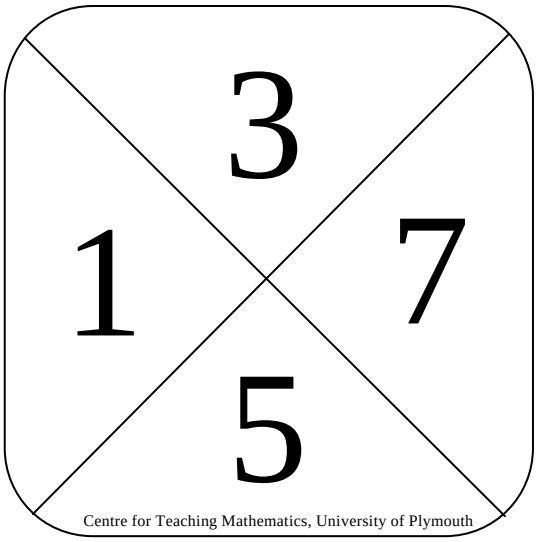
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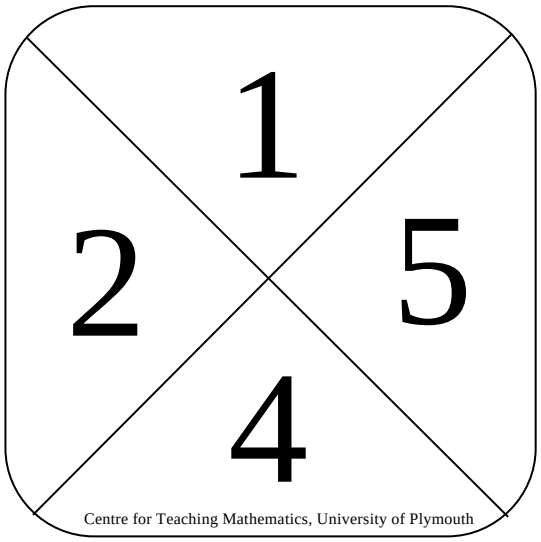
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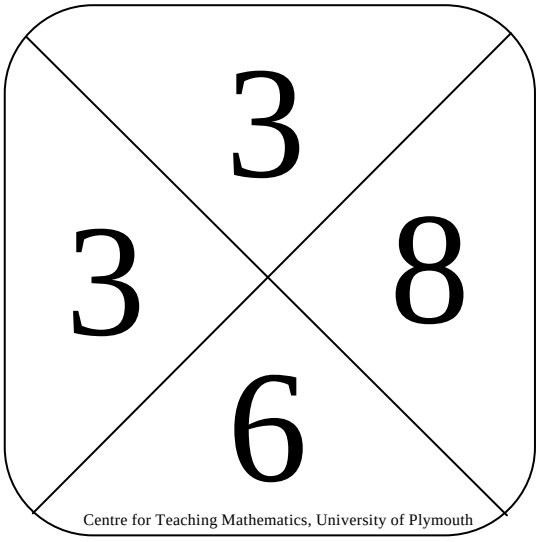
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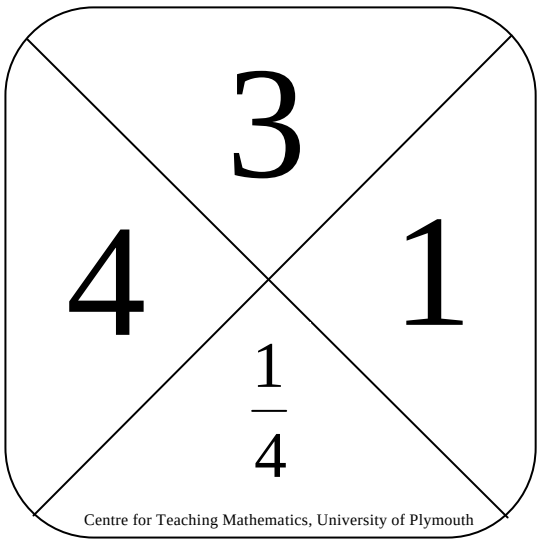
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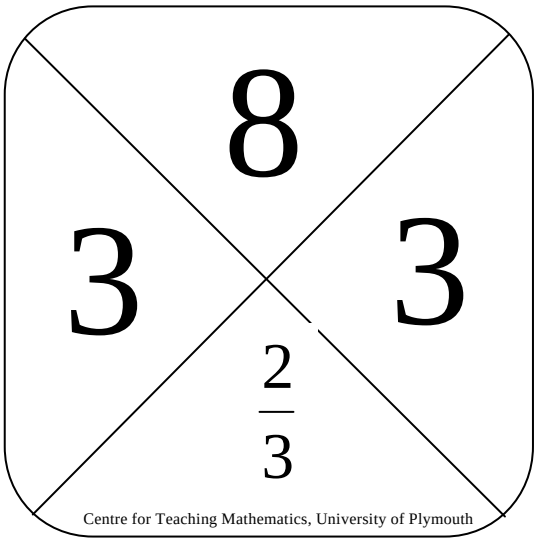
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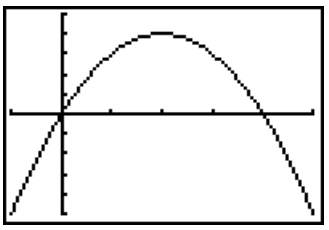
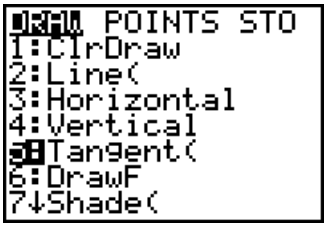
Gradient Functions

This activity can be used to introduce the idea of the gradient function of a function and hence demonstrate (not prove!) the rules for differentiation. It is based around the TI-83 but can be used on any graphic calculator with similar features.

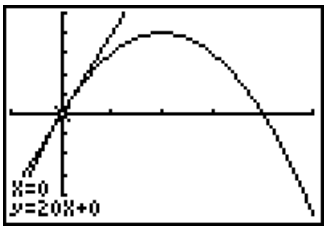
Enter $f(x)=20x-5x^2$ as Y_1 using [Y=] and draw the graph. Change the [WINDOW] options so that you can view the whole graph.



From the [DRAW] menu, select 5:Tangent(and press [ENTER].

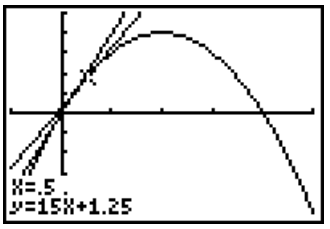


Typing [0] followed by [ENTER] will draw the tangent at the point $x=0$, and give you the equation of the line.

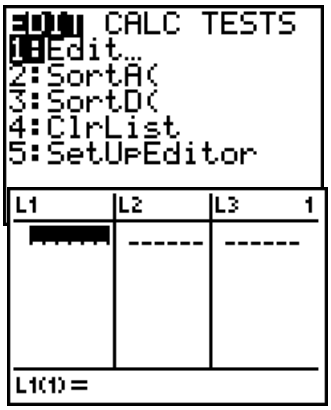


Therefore the gradient of $y=20x-5x^2$ at the point $x=0$ is 20.

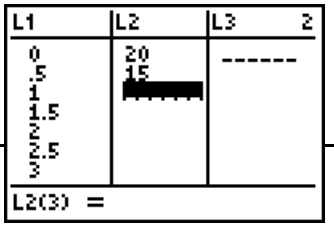
Repeat for $x=0.5, 1, 1.5, ...4$, making a note of the points as you go along. You can delete the tangents if it gets too complicated by [DRAW], 1:ClrDraw



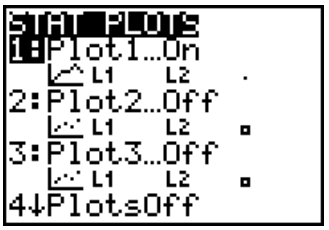
We want to find a relationship between the function $y=20x-5x^2$ and the gradient function, $g(x)$. Let us plot the gradient function.
We can use the list functions of the calculator.
Press [STAT] and choose 1:Edit...



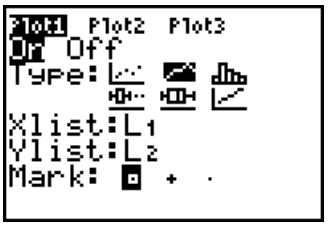
[ENTER] will give you the following screen:
If there is already data in the lists use the up arrow key to highlight L_1 and press [CLEAR]. Repeat for the other lists if necessary.
Enter the x values into list L_1 and the gradient values into L_2



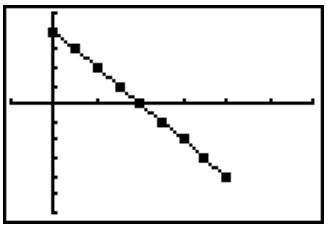
We can plot a line graph of the gradient function using STAT PLOT.



Select the following options for Plot1:

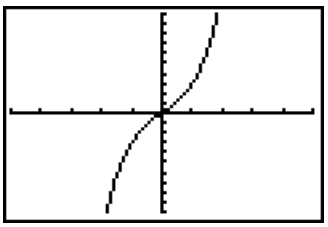


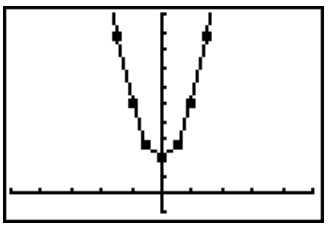
[GRAPH] the plot. You may want to turn off Y1



Can you determine the equation of the gradient function. It is of the form $g(x) = ax + b$. You may need to check your [WINDOW] to see the scales. Enter the function into Y2 to see if you are correct. Can you make a link between the original function and its gradient function?

Try with other functions: $y=x^3 +2x$. Here the gradient function is of the form $ax^2 +b$.





Try with some of the trig functions, remember your calculator needs to be in Radians mode.