

Emergence of Functional Hierarchy in a Multiple Timescales Recurrent Neural Network Model: A Humanoid Robot Experiment

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Abstract

It is generally thought that skilled behavior in human beings results from functional hierarchy of the motor control system, within which reusable motor primitives are flexibly integrated into various complex sensori-motor sequence patterns. The underlying neural mechanisms governing the way in which continuous sensori-motor flows are segmented into primitives, and the way in which series of primitives are integrated into complex sequential behavior, have however not yet been clarified. In earlier studies, this functional hierarchy has been realized through the use of explicit hierarchical structure, with local modules representing motor primitives in the lower level and a higher module representing sequences of primitives switched via additional mechanisms such as gate-selecting. Complex sensori-motor sequences, however, are not easily handled in such earlier models due to a conflict, induced by this explicitly separated modular structure, between generalization and segmentation. To address this issue, we propose a different type of neural network model. The current model neither makes use of separate local modules to represent primitives, nor introduces explicit hierarchical structure. Rather than forcing architectural hierarchy onto the system, functional hierarchy emerges through a form of self-organization that is based on two distinct types of neurons, each with different time properties ("multiple timescales"). Such multiple timescales lead to complex sensori-motor flows of skilled behavior being segmented into reusable primitives, and the primitives, in turn, are flexibly integrated

Emergence of functional hierarchy

into novel sequences. In experiments, the proposed network model, coordinating the physical body of a humanoid robot through high-dimensional sensori-motor control, also successfully situated itself within a noisy environment. The idea proposed here of a functional hierarchy which self-organizes through multiple timescales in neural activity, in addition to contributing to further neurophysiological investigations of the motor control system, could also contribute to the study of various regions of the brain where similar mechanisms may be observed. In addition the extension of the current model to bind behavior and linguistic modalities are also introduced.

Introduction

Functional hierarchy, defined broadly as the principle that complex entities may be segmented into simpler elements and that simple elements may be integrated into a complex entity, is a ubiquitous feature of information processing in biological neural systems [1-4]. The motor control system governing skilled behavior is a representative example of a system with functional hierarchy. Humans acquire a number of skilled behaviors through the experience of repeatedly carrying out the same movements. Certain components of such movements, through repetitive experiences, are segmented into reusable elements referred to as "primitives". In adapting to various situations, series of motor primitives are in turn also integrated into complex sequential behavior [5, 6].

The action of drinking a cup of coffee, for example, may be broken down into a combination of motor primitives such as the motion of reaching for a cup on the table, and the motion of grasping the cup and bringing it to one's mouth. Ideally, these motor primitives should be represented in generalized manner, in the sense that the representation should be adaptive for differences of locations and shapes of the cup. Primitives must also be flexible with respect to changes in the sequence of actions; for example, after grasping a cup, one sometimes brings the cup to one's mouth to drink, but one also sometimes takes the cup off the table to wash up. It is this adaptability (intra-primitive level) and flexibility (inter-primitive level) of primitives that allow humans to generate countless patterns of complex behavior.

The observations of human reaching motions [7][8] and animal muscle movements [9][10] strongly suggest that the complex movements of animals are made up of flexible combinations of reusable movement elements, i.e. motor primitives. What is not yet clear, however, is what underlying neural mechanisms govern the segmentation of continuous sensori-motor flows into primitives, and how series of primitives are combined into complex sequential behavior.

To address this issue, we propose a neural network model for describing the neural mechanisms of segmentation and integration in complex sensori-motor flows. In experiments, the proposed network model was tested through the interaction of a humanoid robot with a noisy environment, the robot requiring high-dimensional complex sensori-motor control. The robotics experiment is important when one considers the idea of the embodied mind by Varela [11], who explained that cognitive functions of neural systems emerge not only in the brain, but also in dynamic interactions between the physical body and the environment.

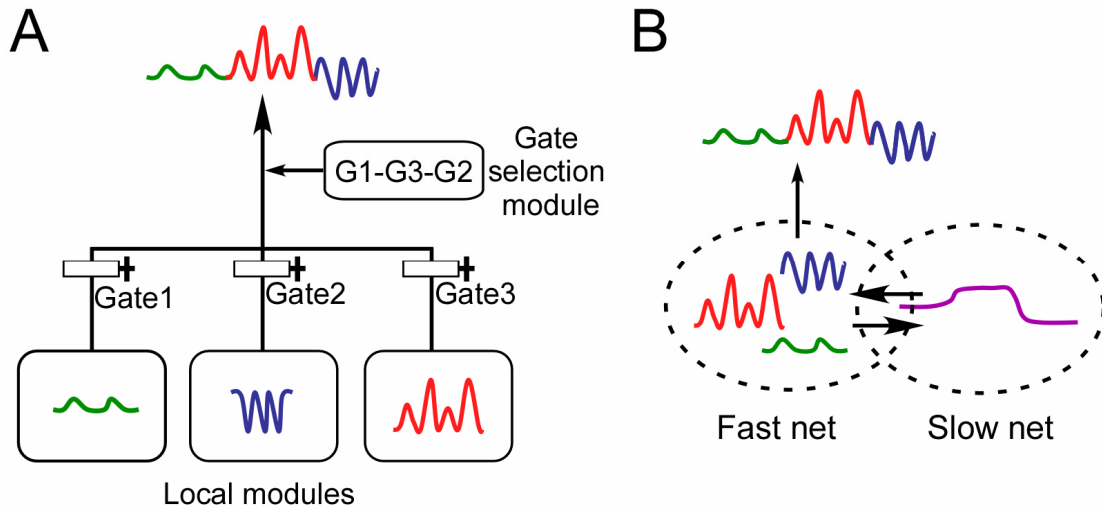


Figure1. Schematic drawings of (A) Local representation model and (B) Multiple timescale model.

Previous studies exist on the computational modeling of functional hierarchy in sequences of motor primitives [12, 13]. In these studies, functional hierarchy is realized through the use of explicit hierarchical structure, with local modules representing motor primitives in the lower level, and a higher module representing the order of motor primitives switched via additional mechanisms such as gate-selection (Figure 1A). We refer to this type of model as the "local representation" model.

There are a number of possible advantages of the local representation. First, the learning of one module would seem not to affect other modules. Second, based on this independence in the learning process, it would seem that increasing the number of local modules would lead to an increase in the number of acquirable primitives.

An earlier study using multiple complex sensori-motor sequences, however, demonstrated that difficult problems arise in the local representation model as a result of its local nature [14]. Similarities in learned sensori-motor sequences create competition in the learning process between corresponding modules. Generalization requires similar patterns to be represented in the same module as the same primitive, even if there are subtle differences between such patterns. On the other hand, for the purposes of achieving "crisp" segmentation of sensory-motor flow, different patterns must be represented as separate primitives in distinct modules. This conflict between generalization and segmentation poses serious problems for the treatment of complex sensori-motor sequences. Due to the difficulty of this problem, it is not possible to increase the number of acquirable primitives simply by increasing the number of local modules [14]. In addition, due to the explicit hierarchical structure of the local

representation, learning of the lower module (primitives) and learning of the higher module (sequences of primitives) have to be explicitly separated through subgoals arbitrarily set by the experimenter [12, 15].

In order to overcome difficulties associated with the local representation model, we introduce in the current study a different type of functional hierarchy representation. The representation we use neither makes use of separate local modules to represent primitives, nor introduces explicit hierarchical structure to manipulate these primitives. Instead of setting up an explicit hierarchy, we attempt to realize the self-organization of a functional hierarchy by means of neural activity with multiple timescales. This functional hierarchy is made possible through the use of two distinct types of neurons, each with different time properties. The first type of neuron is the "fast" unit, whose activity changes quickly over the short term. The second type of neuron is the "slow" unit, whose activity changes over the long term (Figure 1B).

The use of timescale variation in representing different levels of function has been attempted in earlier computational studies. In a study of reinforcement learning among agents solving a maze game, for example, Sutton's group showed that learning speed can be increased by using not only short-term rewards, but also long-term rewards [16]. In a study of an evolutionary neural network model using a mobile robot, Nolfi [17] showed that a model with differing timescales is superior to the normal model in cases in which the robot is required to achieve two different tasks: collision avoidance, which requires short-term sensori-motor control, and self-localization, which requires long-term sensory integration. Although advantages in learning were shown, these studies did not demonstrate how continuous sensori-motor experiences are segmented into primitives, and how series of primitives are integrated into complex behaviors.

The current study focuses not only on demonstrating advantages in learning, but also on showing that hierarchical differentiation of functions can be self-organized through multiple timescales in neural activities. In the proposed model, through repetitive execution of skilled behavioral tasks, continuous sensori-motor flows are segmented into reusable motor primitives, and segmented primitives are integrated into new behavior sequences. The model does this without setting up an explicit sub-goal or functions such as gate-selection for manipulating primitives in the lower module.

The main component of the current model is a continuous time recurrent neural network (RNN). Thanks to its capacity to preserve the internal state, which enables it to reproduce complex dynamics, the RNN is often used for modeling temporal sequence learning [18-20]. The continuous time RNN (CTRNN) is a type of RNN which

implements a feature of biological neurons, namely that the activities of neurons are determined not only by current synaptic inputs but also by the past history of neural states. Due to this characteristic, according to which activation changes continuously, the CTRNN is superior in describing continuous sensori-motor sequences [21, 22].

It is not our intention in the current study to map directly between model components and actual brain structures. However, at the macro-level, possible correspondence between the actual brain and the proposed computational model, and possible contributions for the future research in the field of neuroscience, will be discussed.

Task design

A small humanoid robot was used in the role of a physical body interacting with actual environment. A workbench was set up in front of the robot, and a cubic object (approximately 9x9x9 cm) placed on the workbench served as the goal object. The task for the robot was to autonomously generate five different types of behavior (referred to as the "basic" behavior patterns): (1) move the object up and down three times, (2) move the object left and right three times, (3) move the object backward and forward three times, (4) touch the object with one hand and (5) clap hands three times. For each behavior, the robot's task began from the same home position and ended with the home position (Figure 2A).

As shown in Figure 2A, task trajectories followed tree-like branching structures, although there was no explicit trigger for branching. From the home position, trajectories branched three ways, each corresponding to different actions: reaching for the object, touching with a single hand, and clapping. After reaching for the object, trajectories again branched three ways for different possible actions: moving the object up and down, moving it left and right, and moving it backward and forward. Even with repetitive movement such as moving the object up and down, there was potential branching in the possibility of either repeating the up-down movement one more time, or going back to the home position. This structure of task sequences exhibited a form of complexity in which sensori-motor state changed rapidly over the short term, while task sequences followed a tree-like branching structure over the long term.

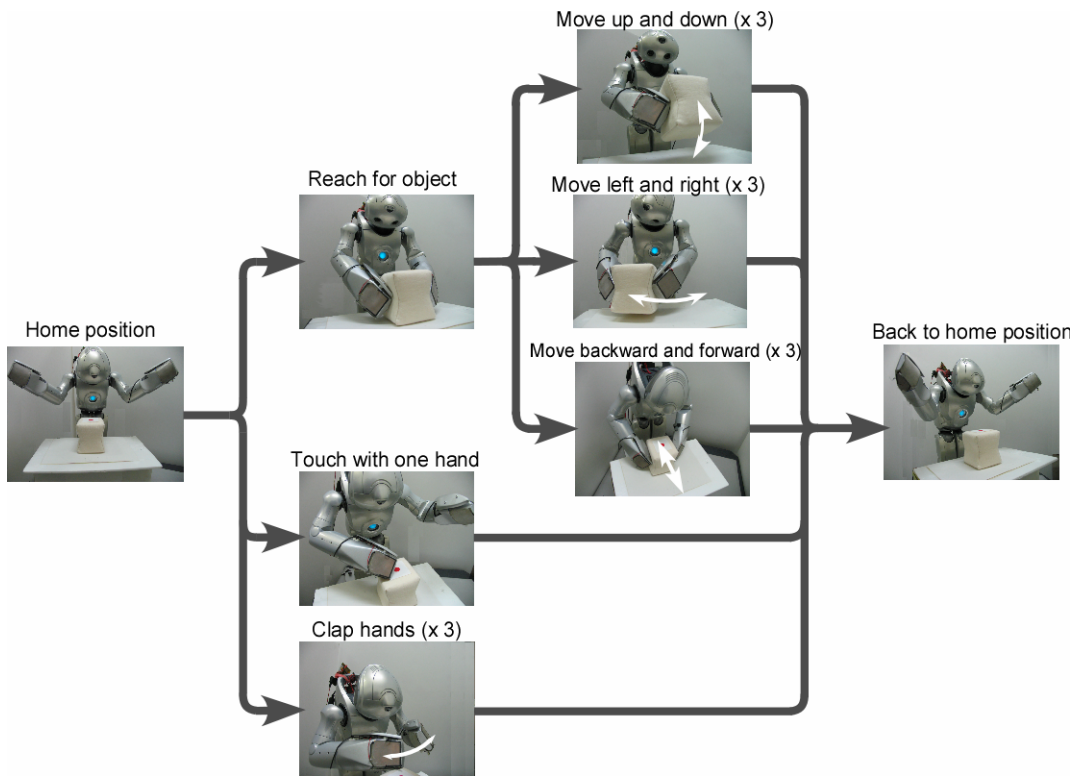


Fig2. Task design

A humanoid robot was fixed to a stand. In front of the robot, a workbench was set up, and a cubic object (approximately 9x9x9 cm) was placed on the workbench to serve as the goal object. The task for the robot was to autonomously generate five different types of behavior: (1) move the object up and down three times, (2) move the object left and right three times, (3) move the object backward and forward three times, (4) touch the object with one hand, and (5) clap hands three times. For each behavior, the robot began from the home position and ended at the same home position.

System overview

Inputs to the system were the proprioception \hat{m}_t (8 dimensional vector representing the angles of arm joints) and the vision sense \hat{s}_t (2 dimensional vector representing object position) (Figure 3). Based on the current \hat{m}_t and \hat{s}_t , the system generated predictions of proprioception m_{t+1} and the vision sense s_{t+1} for the next time step. This prediction of the proprioception m_{t+1} was sent to the robot in the form of target joint angles, which acted as motor commands for the robot in generating movements and interacting with the physical environment. This inverse computation, in which necessary motor torques were computed from the desired state, is called the inverse model. The inverse model is thought to exist in the cerebellum or motor cortex of real brains [23]. The inverse

computation process was preprogrammed in the current system within the robot control system. Changes in the environment, including changes in object position and changes in the actual position of joints, were sent back to the system as sensory feedback.

The main component of the system modeled by the CTRNN received two different modality inputs, proprioceptive somato-sensory input and vision input. These different modality sensations came together in the CTRNN to generate predictions of the future state. These predictions were made possible by the capacity of the CTRNN to preserve the internal state, which enables it to reproduce complex dynamics. It is thought that this type of prediction mechanism is realized in animal and human brains through a simple forward model [24, 25].

In the CTRNN, proprioception and vision inputs were sparsely encoded with the preserving topology of the input space (see Method for details). This topology preserving sparse encoding of sensori-motor trajectories, which resembles information processing in the primary sensory cortices such as VI and SI [26, 27], reduced overlap between sensori-motor sequences and improved the learning capacity of the CTRNN.

A conventional firing rate model, in which each unit's activity represents the average firing rate over a group of neurons, was used to model neurons in the CTRNN. In addition, every unit's membrane potential was assumed to be influenced not only by current synaptic inputs, but also by their previous state. This characteristic is described by the following differential equation, which uses a parameter τ referred to as the time constant:

$$\tau_i \dot{u}_{i,t} = -u_{i,t} + \sum_j w_{ij} x_{j,t} \quad (1)$$

where $u_{i,t}$ is the membrane potential, $x_{i,t}$ is the neural state of the i th unit, and w_{ij} is synaptic weight from the j th unit to the i th unit. The second term of equation (1) corresponds to synaptic inputs to the i th unit. The time constant τ is defined as the decay rate of the unit's membrane potential, analogous to the leak current of membrane potential in real neurons. One might consider this decay rate to correspond to an integrating time window of the neuron, in the sense that the decay rate indicates the degree to which the earlier history of synaptic inputs affects the current state. When the τ value of a unit is large, the activation of the unit changes slowly, because the internal state potential is strongly affected by the history of the unit's potential. On the other hand, when the τ value of a unit is small, the effect of the history of the unit's potential is also small, and thus it is possible for activation of the unit to change quickly.

The network that was used in the current model consisted of input-output and non-input-output units, the latter referred to as context units. Context units were

divided into two groups based on the value of time constant τ . The first group consisted of fast context units with small time constant ($\tau=5$) whose activity changed quickly, whereas the second group consisted of slow context unit with a large time constant ($\tau=70$) whose activity, in contrast, changed much more slowly.

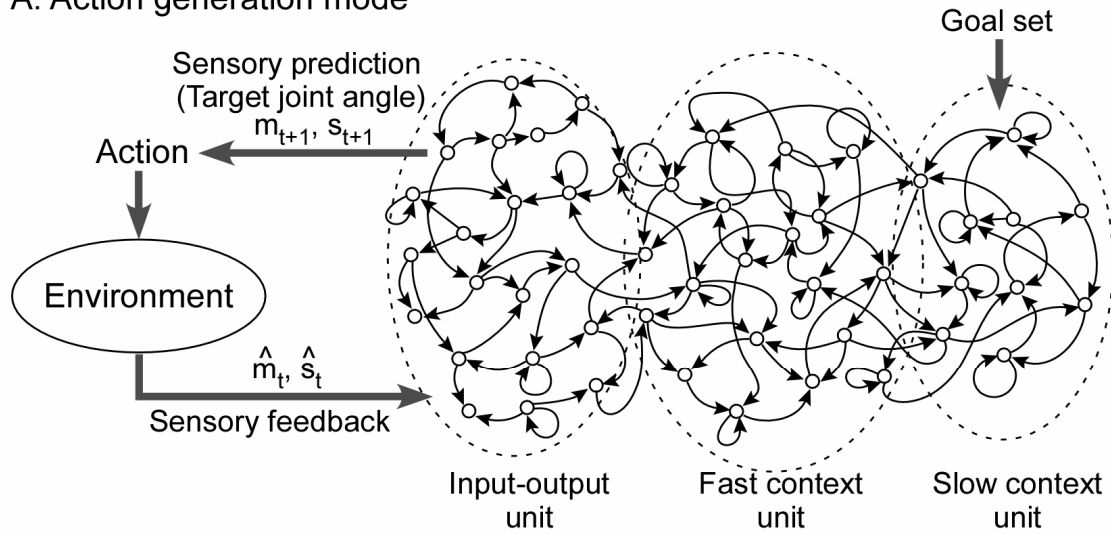
Among the input-output units, units corresponding to proprioception and units corresponding to vision were not connected to each other. This topology was modeled on the actual brain, in which primary somato-sensory inputs and primary visual inputs are not directly connected. In addition, inputs were also not directly connected to slow context units, a restriction employed in order to reduce computation time required for real-time control of the robot.

Training

In order to obtain a teaching signal, the experimenter guided both hands of the robot along the trajectory of the goal action. As the robot hands were guided along the trajectory, encoder values of each joint were recorded, and recorded sensori-motor trajectories were used as teaching sequences. For each behavior task other than the clapping of hands, the object was located in five different positions (position 1 to position 5 in Figure 2B). Since the action of clapping hands was independent of object location, the object was always located at the center of the workbench for this task (position 3).

The objective of learning was to find optimal values of connective weights minimizing the error between teaching sequences and model outputs. At the beginning of training, synaptic weights of the network were set randomly, resulting in the network generating random sequences. Synaptic weights were modified based on the error between

A. Action generation mode



B. Training mode

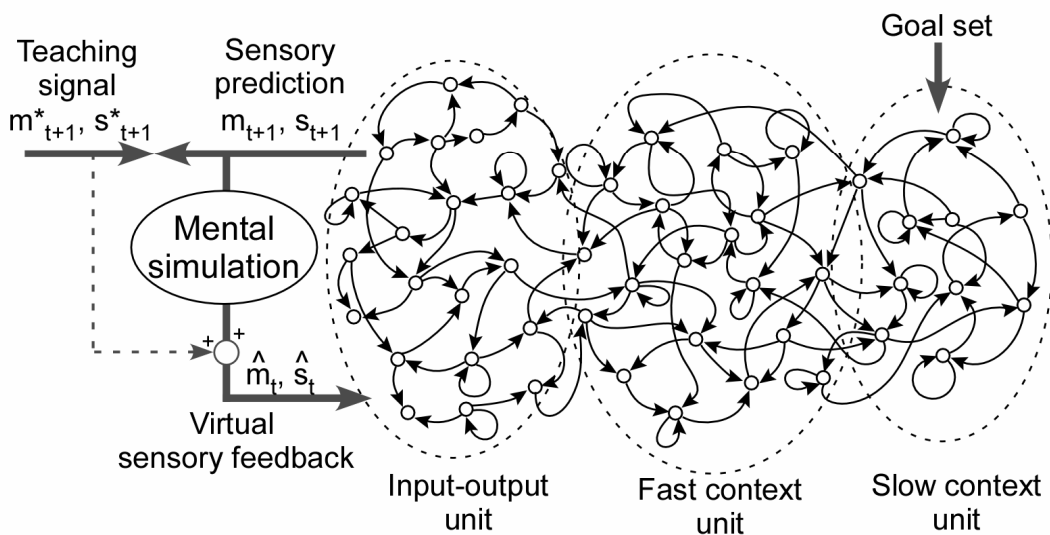


Fig3. System overview

(A) Action generation mode and (B) training mode.

teaching signals and generated sequences. After many repetitions of this process, the error between teaching sequences and model outputs eventually reached a minimum level.

This training process was conducted in an off-line manner, in the sense that the prediction of the sensory-motor trajectories were generated by means of so-called closed-loop operations (Figure 3B) in which the current prediction of the proprioception and vision state are used as input for the next time step.

Nishimoto [28] demonstrated that CTRNN can learn to generate multiple sequences

starting from different initial states through an association between initial states and corresponding sequences. Utilizing this characteristic of initial sensitivity, the CTRNN was trained to generate multiple behavior sequences through the selection of corresponding initial states, defined by the experimenter.

In the proposed model, a network was trained by means of supervised learning using teaching sequences obtained through tutoring by the experimenter. The conventional back-propagation through time (BPTT) algorithm was used for learning of the model network [29]. In the current study, the BPTT was used not for mimicking the learning process of biological neural systems, but rather as a general learning rule. Results obtained reflect characteristic features of the proposed network architecture, and not of the learning algorithm. Similar results could be obtained using a different learning algorithm, such as for example the biologically plausible algorithm proposed by Seung's group [30, 31], a kind of reinforcement learning.

Action generation in physical environment and mental simulation

Through the training process, the network learned to predict sensory feedback for the next time step. This prediction of sensory feedback was treated as a target joint angle, and was sent to the robot. Following this target joint angle, the robot was in turn able to generate learned movements even in a noisy physical environment.

Moreover, by using the prediction of sensory feedback as input to the next time step (closed loop generation), the network was able to autonomously generate sensori-motor trajectories without producing actual movements. This process of closed loop generation was treated as corresponding to the mental simulation of actions [24, 32].

Performance of robot

Five learning trials were conducted with different initial values for synaptic weights. The BPTT was conducted over 5000 iterations, with optimal performance weights taken as the set of weight values for which error was minimized. Model networks with these optimal weights were tested through the interaction of the robot with a noisy physical environment. Learning error and performance of the robot, for all types of behavior and for all different object positions, is summarized in the Table 1. Interacting with the physical environment, the robot was able to nearly perfectly reproduce learned behavior, and also successfully adapted to differences in the location of objects. Success or failure was judged according to criteria described later in this paper (see Method for details).

Figure 4 and Figure 5 illustrate examples of sensori-motor sequences, as well as

Emergence of functional hierarchy

examples of teaching signals and trained model network interacting with a physical environment through the body of the robot. Figure 4 also includes examples sequences generated by mental simulation. Both in mental simulation and in the context of the robot interacting with a physical environment, the trained network reproduced target behavior sequence successfully.

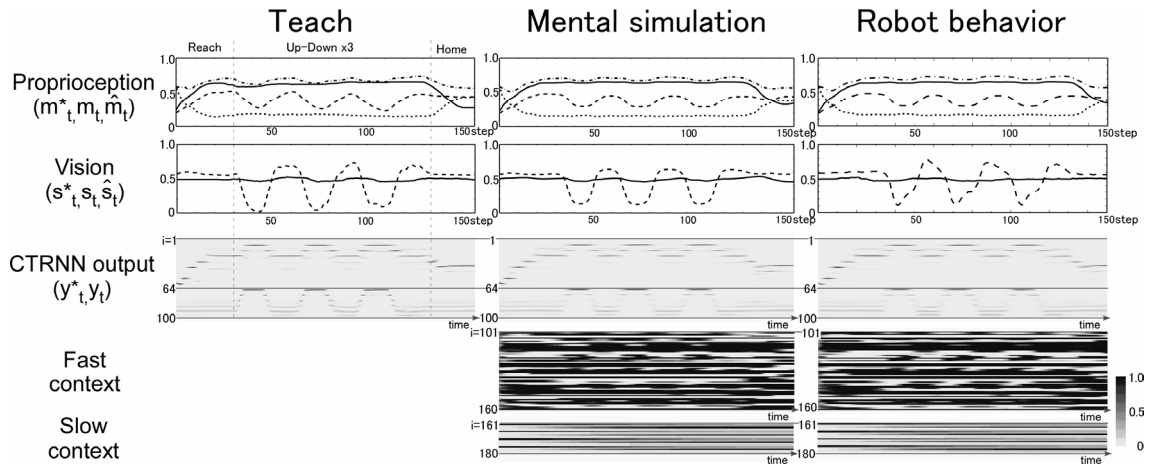


Figure 4. Example of behavior sequence for up-down behavior

Proprioception (first row), vision (second row), sparsely encoded RNN activation (third row), fast and slow context activation (forth and fifth row) of teaching signal (left column), mental simulation of trained network (center column) and actual sensory feedback in physical environment (right column) during up-down behavior at position 3 are shown.

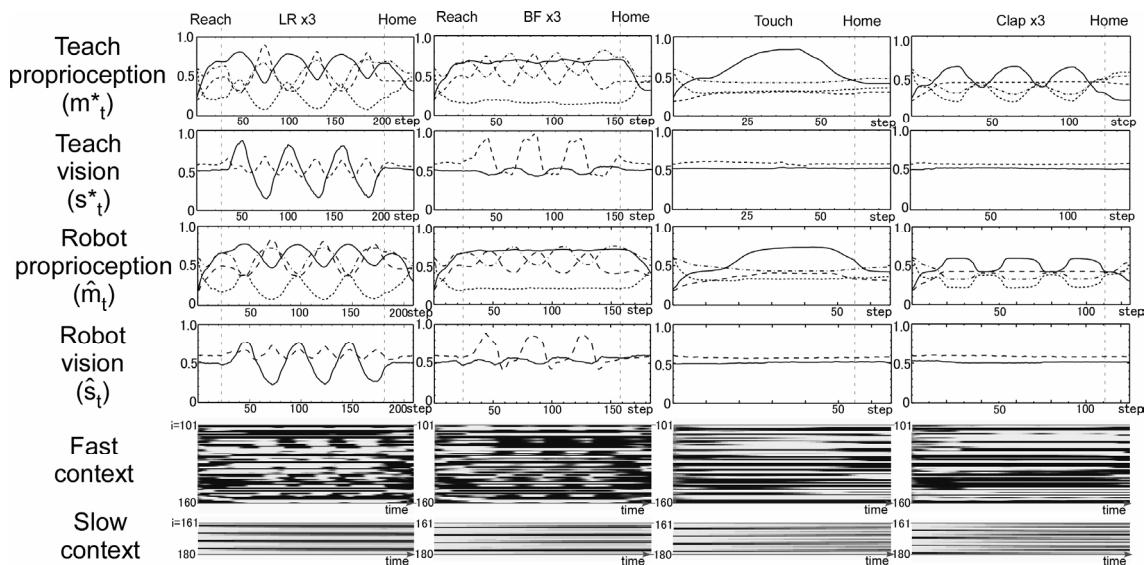


Figure 5. Example of behavior sequences for other basic behavior

Emergence of functional hierarchy

Proprioception, vision, fast and slow context activation of teaching signal and actual values in physical environment during left-right (LR: first column), backward-forward (BF: second column) touch with single hand (touch: third column) and clapping hands (clap: forth column) behavior at position 3 are shown.

Representations in the fast and slow context unit

When the robot generated repetitive movements such as moving the object up and down three times, repetitions of similar patterns were observed in activities of the fast context units. The slow context units, in contrast, changed gradually, and no such repetitive patterns were observed (Figure 4, 5). Changes in the value of slow context units seemed to drive switching between movements, for example between repetitive movements and the action of going back to the home position. These patterns in the activation of context units suggest that the fast context units encoded reusable movement segments ("primitives"), whereas the slow context units encoded the switching between these primitives.

In order to confirm this hypothesis, internal network representations for each pattern of behavior were investigated by analyzing the activation of context units for different behavior and for different positions. For every behavior at every position,

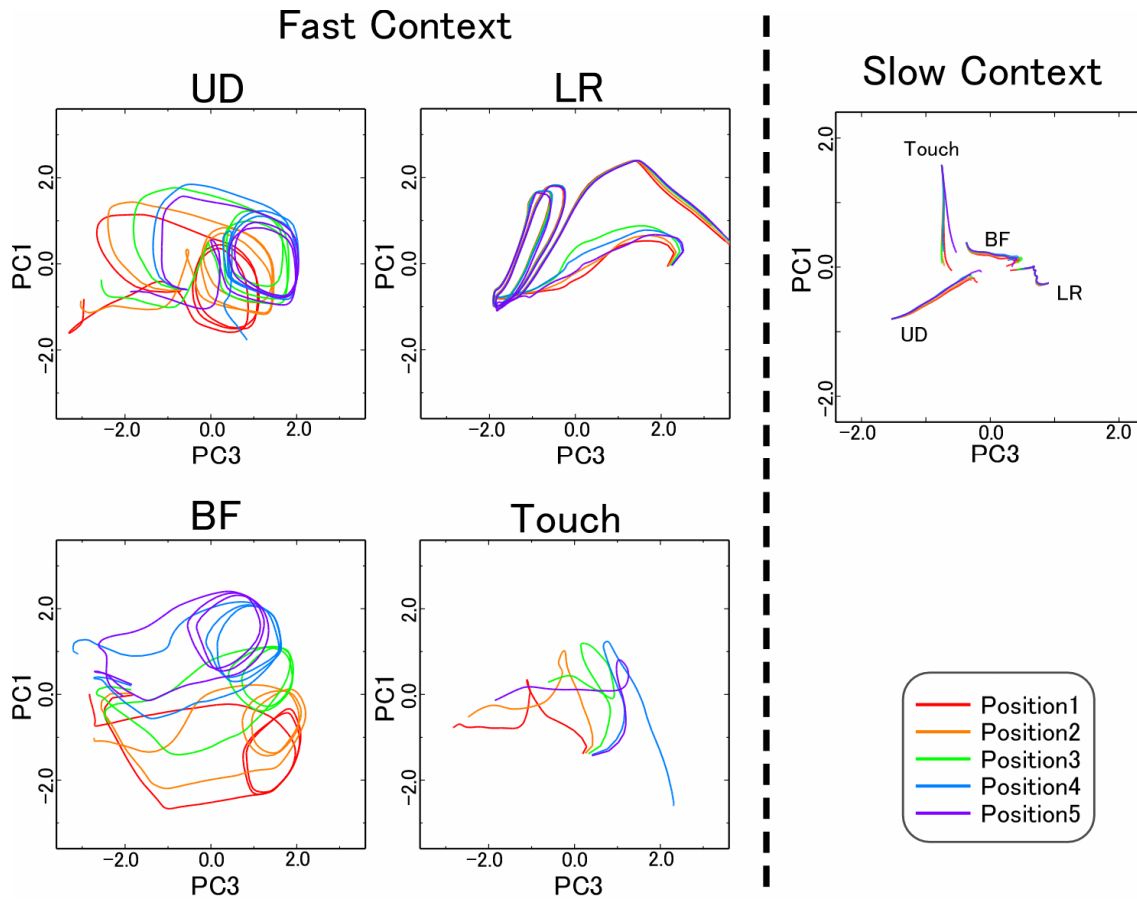


Figure 6. Position-dependent changes in context state space

Changes of context activation during each behavior at every position are shown in a 2 dimensional space based on the results of PCA analysis. The four graphs on the left side and single graph on the right side correspond to fast context activities and slow context activities, respectively. State changes of the fast context units for each behavior exhibit a particular structure which shifts with the object position. On the other hand, activity of the slow context units for a particular behavioral task exhibited very little location-dependent variation. UD: up-down, LR: left-right, BF: backward-forward and Touch: touch with single hand.

context unit activation values were recorded as sequences of sixty dimensional vectors (fast context) and twenty dimensional vectors (slow context). The dimensionality of these multidimensional data sets was reduced using principal component analysis (PCA).

In order to visualize changes of state in the network during execution of behavioral tasks, two principal components of context unit activation values were plotted in Figure 6 for every behavior and at every position. The clapping hand behavior was not plotted

as this behavior was independent of object position.

In the fast context units, state changes for each behavior exhibited a particular structure which shifted with the object position. This structure also exhibited cyclic patterns corresponding to repetitive movements such as up-and-down motion, left-and-right motion, and backward-and-forward motion. Activity of the slow context units, on the other hand, exhibited no periodic patterns corresponding to repetitive movements, and very little location-dependent variation was observed.

These observations suggest the existence of functional hierarchy in the model network. Specifically, reusable segmented movement patterns were represented in fast context units in a form that was generalized across object locations, whereas the slow context units appeared to be more abstract in nature, representing sequences of primitives in a way that was independent of the object location.

Future studies for binding linguistic modality

We are currently working on an extension of the proposed architecture for binding linguistic and behavior modalities. The extended architecture is shown in Figure 7. We consider a set of simple sentences such as “Put red on green”, “Push green to left”, “Put red in front of green”. Each sentence in the form of phono sequences without segmentations is input to the lower end of the linguistic module while the robot is in the preparatory period to move. The inputs of phono sequences initiate certain activations in the fast context units in the linguistic module which further propagate to the slow context units in the behavior module. After the preparatory period, the robot starts to behave as corresponding to the contents of the linguistic inputs. Our preliminary experiments have shown that different linguistic inputs can organize different activation patterns in the slow context units during the preparatory period if the error signal from the behavior module is back-propagated to the linguistic module. These different activations seem to play the equivalent role of setting the initial state in the previous behavior experiment shown in the current paper. We are currently comparing the generalization capability of the current model with Sugita and Tani’s model[33] utilizing the parametric biases for the binding

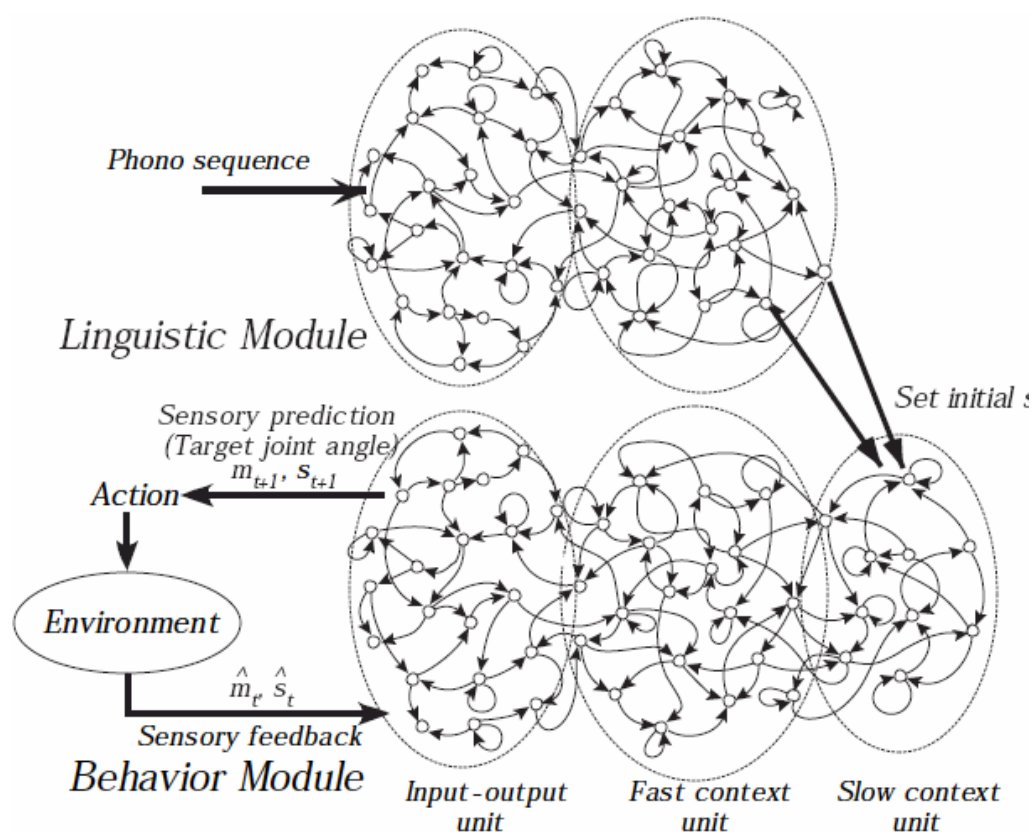


Figure 7. Binding of linguistic module and behavior module

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