

# Incremental Learning of Robot Dynamics using Random Features

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**Abstract**—Analytical models for robot dynamics often perform suboptimally in practice, due to various non-linearities and the difficulty of accurately estimating the dynamic parameters. Machine learning techniques are less sensitive to these problems and therefore an interesting alternative for modeling robot dynamics. We propose a learning method that combines a least squares algorithm with a non-linear feature mapping and an efficient update rule. Using data from five different robots, we show that the method can accurately model manipulator dynamics, either when trained in batch or incrementally. Furthermore, the update time and memory usage of the method are bounded, therefore allowing use in real-time control loops.

## I. INTRODUCTION

Dynamic modeling plays a significant role in various robotic applications, including computed torque control, zero-gravity control, and contact detection. Commonly, the (inverse) dynamics are modeled analytically using

$$\tau = D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q),$$

where  $D$ ,  $C$  and  $g$  are the inertial, Coriolis, and gravity terms, respectively [1]. Despite being analytically correct, this formulation has limited applicability on real-life robots, due to the difficulty of accurately determining the various kinematic and dynamic parameters. More importantly, the model is idealized and does not capture non-linearities due to friction, elasticity, flexibility, and vibrations.

Machine learning forms a viable alternative for modeling robot dynamics. In recent years, multiple studies have shown that learning techniques often outperform analytical models of the rigid body dynamics [2], [3]. Typically, these studies employ a batch learning setting, in which the model is trained offline prior to deployment on the robot. Batch learning methods assume i.i.d. sampling of the training samples, which conflicts with the nature of the dynamics problem. This is because samples are obtained sequentially from the robot at high sampling frequencies, causing consecutive samples to be dependent. A large body of training samples is therefore required to ensure that the probabilistic distributions of both training and subsequent test data coincide sufficiently. Furthermore, batch methods are static after the initial training phase and may perform suboptimally in case

of concept drift (e.g., sensor drift, mechanical wear, or temperature fluctuations).

Incremental learning methods, on the other hand, adapt their model continuously during online operation, therefore having a clear advantage over batch methods for sequential learning. Locally Weighted Projection Regression (LWPR) has been used extensively in robotics for incremental learning [4]. Unfortunately, LWPR usually does not perform as well as state of the art batch methods in terms of prediction accuracy. Moreover, its large number of hyperparameters are notoriously hard to tune, making it difficult to obtain optimal performance. Nguyen-Tuong *et al.* propose Local Gaussian Processes (LGP) to alleviate these problems by combining the local learning approach of LWPR with Gaussian Process Regression (GPR) [5], [6]. Though being efficient in a general sense, neither of these two incremental methods gives strict, theoretical guarantees on timing requirements, rendering them unsuitable for (hard) real-time control loops.

In this paper, we propose an incremental algorithm based on the well-known Ridge Regression (RR) algorithm [7], extended with a finite-dimensional kernel approximation to allow use on non-linear problems. The particular advantages over existing methods are (1) a constant time and space complexity for both predictions and model updates, (2) a tunable parameter that trades computational requirements for prediction accuracy, and (3) ease of implementation and use. We verify experimentally that the generalization performance of the method is at least comparable to existing methods, and that timing requirements are significantly lower for large scale problems.

The Ridge Regression method is explained in Section II, including the feature mapping for non-linearity and an incremental update routine for use in sequential learning. In Section III, we describe the setup used for our experiments, after which the corresponding results are covered in Section IV. Finally, we conclude the paper in Section V.

## II. INCREMENTAL RIDGE REGRESSION WITH RANDOM FEATURES

Real-time modeling of robotic dynamics requires a learning method that is non-linear and capable of performing fast predictions. Additionally, we restrict ourselves to methods that can learn incrementally during online operation of the robot. Our proposed algorithm satisfies these requirements by combining traditional RR with two additional extensions. First, a random feature mapping is used to approximate shift-invariant kernels in a finite number of dimensions. As a result, we obtain the non-linearity of kernels, while avoiding linear dependence on the number of training samples.

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Inspired by earlier work in signal processing, the second extension is an efficient update rule that allows the model to be computed incrementally.

### A. Ridge Regression

Let us define a set of  $m$  samples  $\mathcal{S} = \{\mathbf{x}_i, y_i\}_{i=1}^m$ , where  $\mathbf{x}_i \in \mathcal{X} \subseteq \mathbb{R}^n$  is the  $i$ -th  $n$ -dimensional input vector and  $y_i \in \mathcal{Y} \subseteq \mathbb{R}$  the associated output label. We consider an algorithm that constructs a linear hyperplane

$$f(\mathbf{x}) = \mathbf{w}^T \mathbf{x} , \quad (1)$$

where  $\mathbf{w}$  is the weight vector. In linear Ridge Regression [7] (RR, also known as regularized least squares), weight vector  $\mathbf{w}$  is chosen such that both the model complexity and the errors are minimized, i.e.,

$$\min_{\mathbf{w}} J(\mathbf{w}, \lambda) = \frac{\lambda}{2} \|\mathbf{w}\|^2 + \frac{1}{2} \|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2 , \quad (2)$$

where  $\lambda > 0$  is a regularization parameter that balances the tradeoff between minimizing both terms. Additionally,  $\mathbf{X}$  is an  $m \times n$  matrix of input samples defined as  $[\mathbf{x}_1, \dots, \mathbf{x}_m]^T$  and analogously  $\mathbf{y} = [y_1, \dots, y_m]^T$ . Setting the gradient of (2) to zero, we find that the optimal  $\mathbf{w}$  is given by

$$\mathbf{w} = (\lambda \mathbf{I} + \mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} . \quad (3)$$

Computing the optimal solution thus requires solving a system of  $n$  linear equations, and the time and space complexity of linear RR are  $\mathcal{O}(n^3 + mn^2)$  and  $\mathcal{O}(n^2 + mn)$ , respectively. Sample predictions are only in  $\mathcal{O}(n)$  and thus independent of the number of training samples.

### B. Kernel Ridge Regression

Practical use of RLS is limited due to its linearity. This limitation is circumvented in Kernel RR (KRR) by applying the kernel-trick to map samples implicitly into a (typically non-linear) feature space [8], [9]. A consequence of the kernel machinery, however, is that the prediction function is written as a kernel expansion

$$f(\mathbf{x}) = \sum_{i=1}^m c_i k(\mathbf{x}, \mathbf{x}_i) , \quad (4)$$

where  $k(\cdot, \cdot)$  is an admissible kernel function [10]. Defining kernel matrix  $\mathbf{K} = [k(\mathbf{x}_i, \mathbf{x}_j)]_{i,j=1}^m$ , we can compute the optimal  $m$ -dimensional vector of coefficients  $\mathbf{c}$  as

$$\mathbf{c} = (\mathbf{K} + \lambda \mathbf{I})^{-1} \mathbf{y} .$$

Contrary to linear RR, training KRR equates to solving a system of  $m$  linear equations, yielding an overall time complexity of  $\mathcal{O}(m^3 + nm^2)$ . Due to the kernel expansion in (4), the prediction function of KRLS has become linearly dependent on the number of training samples. This scaling behavior is computationally prohibitive when learning from a large number of samples.

### C. Approximating the Kernel with Random Features

The dependence on the number of training samples can be avoided by limiting the growth of the kernel expansion. Possible ways of achieving this include discarding old samples once a given budget has been reached [11], projecting linear dependent samples on the current support set [12], or dividing the global model into multiple local models [6]. Another approach is to approximate the kernel using a finite number of basis functions. Bochner's theorem states that a continuous shift-invariant kernel  $k(\mathbf{x} - \mathbf{y})$  on  $\mathbb{R}^d$  is positive definite if and only if  $k$  is the Fourier transform of a non-negative measure  $\mu(\boldsymbol{\omega})$  [13]. Rahimi and Recht use this theorem to find a random feature that gives an unbiased estimate of a shift-invariant kernel, that is

$$k(\mathbf{x}, \mathbf{y}) = \int_{\mathbb{R}^d} e^{-i\boldsymbol{\omega}^T(\mathbf{x}-\mathbf{y})} d\mu(\boldsymbol{\omega}) = \mathbb{E}[z_{\boldsymbol{\omega}}(\mathbf{x})z_{\boldsymbol{\omega}}(\mathbf{y})] , \quad (5)$$

where  $z_{\boldsymbol{\omega}}(\mathbf{x}) = \sqrt{2} \cos(\boldsymbol{\omega}^T \mathbf{x} + b)$ <sup>1</sup>,  $\boldsymbol{\omega}$  is drawn from  $\mu$ , and  $b$  is drawn from a uniform distribution from 0 to  $2\pi$  [14]. In the remainder of this text, we will focus on the Radial Basis Function (RBF) kernel

$$k(\mathbf{x}, \mathbf{y}) = e^{-\gamma \|\mathbf{x} - \mathbf{y}\|^2} \quad \text{for } \gamma > 0 ,$$

for which the corresponding measure  $\mu$  is a normal distribution with covariance  $2\gamma \mathbf{I}$ .

The variance of the approximation can be lowered arbitrarily using a (normalized) concatenation of  $D$  random features  $z_{\boldsymbol{\omega}}$ , where each feature draws an individual  $\boldsymbol{\omega}$  and  $b$ . In order to non-linearize RR, it suffices to replace each input vector  $\mathbf{x}$  with its random projection  $\mathbf{z} = \frac{1}{\sqrt{D}} [z_1(\mathbf{x}), \dots, z_D(\mathbf{x})]^T$  in (1) and (3). Rahimi and Recht show that RR with random features (RFRR) converges to KRR as  $\mathcal{O}\left(\frac{1}{\sqrt{D}}\right)$  [15]. Increasing  $D$  will therefore give a more accurate approximation, at the cost of computational efficiency.

### D. Solving Ridge Regression Incrementally

Besides applications in the fields of machine learning, optimization, and statistics, the RR algorithm has also been used as an adaptive filter in the field of signal processing. This particular use has led to the development of a wide array of efficient update rules, collectively known as Recursive Least Squares methods<sup>2</sup>. Commonly, updating a solution with a new sample only requires  $\mathcal{O}(n^2)$ , as compared to  $\mathcal{O}(n^3)$  needed to completely recompute the solution. The effect on space complexity is even more profound, as previous samples are not required to be stored in memory.

The easiest method to incrementally update an RR solution is to perform rank-1 updates on the inverse of the covariance matrix (cf. (3)) using the Sherman–Morrison formula. However, this update formula is known to be sensitive to rounding errors [16]. A numerically much more stable alternative is to update the Cholesky factor  $\mathbf{R}$  of the covariance matrix,

<sup>1</sup>There are other equivalent projections that satisfy (5).

<sup>2</sup>The terminology *recursive* is common-place in the field of signal processing. In this paper, however, we will consistently use *incremental*.

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**Algorithm 1** Incremental RLS with RBF Random Features

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**Require:**  $\lambda > 0, \gamma > 0, n > 0, D > 0$ 

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1:  $\mathbf{R} \leftarrow \sqrt{\lambda} \mathbf{I}_{D \times D}$ 
2:  $\mathbf{w} \leftarrow \mathbf{0}_{D \times 1}$ 
3:  $\boldsymbol{\beta} \leftarrow \mathbf{0}_{D \times 1}$ 
4:  $\boldsymbol{\Omega} \sim \mathcal{N}(0, 2\gamma)_{D \times n}$ 
5:  $\mathbf{b} \sim \mathcal{U}(0, 2\pi)_{D \times 1}$ 
6: for all  $(\mathbf{x}, y)$  do
7:    $\mathbf{z} \leftarrow \sqrt{\frac{2}{D}} \cos(\boldsymbol{\Omega} \mathbf{x} + \mathbf{b})$  // random feature mapping
8:    $\hat{y} \leftarrow \mathbf{w}^T \mathbf{z}$  // predict label
9:    $\boldsymbol{\beta} \leftarrow \boldsymbol{\beta} + \mathbf{z} y$ 
10:   $\mathbf{R} \leftarrow \text{CHOLESKYUPDATE}(\mathbf{R}, \mathbf{z})$ 
11:   $\mathbf{w} \leftarrow \text{CHOLESKYSOLVE}(\mathbf{R}, \boldsymbol{\beta})$  // update weights
12: yield  $\hat{y}$ 
13: end for
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such that  $\mathbf{R}^T \mathbf{R} = (\lambda \mathbf{I} + \mathbf{X}^T \mathbf{X})$ . This update strategy is known as the QR algorithm in the field of adaptive filtering [17]. Rank-1 updates of the Cholesky factor can be computed efficiently using a sequence of Givens rotations [16], while the weight vector  $\mathbf{w}$  can be obtained using back substitution. We want to emphasize that the solution obtained using incremental updates is identical to the batch solution when trained on the same samples.

Combining the incremental update with RFRR results in a non-linear, incremental learning method with bounded time and space complexity. An interesting perspective is to consider it an alternative to Kernel Recursive Least Squares [18], the main difference being that the latter algorithm projects linear dependent samples in order to limit the growth of the kernel expansion. In contrast to incremental RFRR, the computational cost of this linear dependence approximation, although bounded, cannot be known a priori. Algorithm 1 summarizes our proposed algorithm for an approximated RBF kernel, although other shift invariant kernels can be used by sampling from the appropriate probability distribution in line 4. A fundamental observation is that the  $\mathcal{O}(D^2)$  time and space complexity of incremental RFRR is tight and constant with respect to the number of training samples, allowing us to give strict real-time guarantees on computation time and memory storage.

*Remark 1:* We can observe in (2) that the level of regularization decreases with an increasing number of training samples. Regularization is primarily needed in the initial stage of the algorithm, when  $\mathbf{X}$  is not full column-rank. Subsequent rank-1 updates increase the smallest singular value of  $\mathbf{R}$  [16], therefore reducing the need for regularization in later stages.

### III. EXPERIMENTAL SETUP

Several experiments have been conducted to evaluate both batch and incremental variants of RFRR for modeling inverse dynamics of a range of distinct robot manipulators. Prior to presenting the results, we describe the datasets used for the experiments, as well as preprocessing and hyperparameter optimization procedures.

TABLE I  
DATASETS USED FOR THE EXPERIMENTS.

	#joints	output	#train	#test
Simulated Sarcos	7	$\tau \times 7$	14904	5520
Sarcos	7	$\tau \times 7$	13922	5569
Barrett	7	$\tau \times 7$	13572	5000
James	4	$[F, \tau]_{x,y,z}$	15000	195977
iCub	4	$[F, \tau]_{x,y,z}$	15000	72850

#### A. Datasets

A total of five datasets was used in our study, divided in two groups based on manipulator configuration (Table I). The first group consists of robot arms with 7 degrees of freedom, with a torque sensor located in each individual joint. The datasets for this category are taken from a Simulated Sarcos arm, a real Sarcos arm, and a Barrett WAM, and have been used previously in the context of learning dynamics [3], [6]. The manipulators were programmed to perform a periodic motion, during which samples were collected at a rate of approximately 500Hz. Training and test sets were obtained by subsampling the resulting dataset at regular intervals.

The second category contains robot arms of 4 degrees of freedom<sup>3</sup> that have a single force/torque sensor mounted in the upper arm, just below the shoulder joints. These datasets were obtained from the upper torso humanoid James [19] and the full body iCub humanoid [20]. Fumagalli *et al.* used the James dataset previously for a comparison between learning methods and an analytical model of the rigid body dynamics [2]. For both datasets<sup>4</sup>, the end effector was moved to random Cartesian coordinates in the robot's workspace at constant intervals, while samples were collected at a frequency of roughly 50Hz. In order to investigate predictive behavior over a large number of test samples, we restrict the training set to the first 15000 samples.

#### B. Preprocessing and Hyperparameter Selection

Data preprocessing and hyperparameter selection often has a profound effect on the performance of learning methods. For the dynamics datasets, it is crucial to properly scale the input features when using an isotropic RBF kernel, as to avoid large acceleration features to dominate lower-valued position features. In our setup, all input features are therefore divided by a characteristic length scale  $\ell_i > 0$  for  $1 \leq i \leq n$ . Optimizing these length scales using grid search is computationally infeasible due to its exponential scaling behavior. Instead, we exploit the similarity between GPR and KRR, and therefore indirectly also RFRR. It is well-known that the predictive mean of GPR is identical to the KRR formulation [5]. In GPR, hyperparameters can be optimized by maximizing the marginal likelihood. We use this method to optimize the hyperparameters of GPR trained on a random subset of 2000 training samples using an anisotropic RBF kernel. The formulation of this kernel in the context of GPR

<sup>3</sup>We limit ourselves to the shoulder and elbow joints; the actual robot arms have more than 4 degrees of freedom.

<sup>4</sup>These datasets can be obtained by contacting the authors.

is

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma_f^2 e^{(-\frac{1}{2}(\mathbf{x}_i - \mathbf{x}_j)^\top \mathbf{M}(\mathbf{x}_i - \mathbf{x}_j))} + \delta_{ij} \sigma_n^2,$$

where  $\delta_{ij}$  is the Kronecker delta,  $\sigma_n^2$  and  $\sigma_f^2$  respectively the noise and signal variance, and  $\mathbf{M}$  an  $n \times n$  scaling matrix with  $[\ell_1^{-2}, \dots, \ell_n^{-2}]$  on its diagonal. Furthermore, the locally optimal hyperparameters can be converted to KRR and RFRR with isotropic RBF kernel by scaling each  $i$ -th input feature by  $\ell_i^{-1}$  for  $1 \leq i \leq n$ , and subsequently setting  $\lambda = \sigma_n^2 / \sigma_f^2$  and  $\gamma = \frac{1}{2}$ .

The hyperparameters have to be optimized for each output separately. Preliminary experiments, however, showed that all outputs share similar dependencies on hyperparameters; configurations that perform well for one output are also likely to perform well on the other outputs. The computational cost of our method can therefore be reduced significantly by sharing a hyperparameter configuration for all outputs. In this case, we compute a single kernel or covariance matrix, which is subsequently used to solve multiple weight vectors concurrently (i.e., one for each output) at negligible additional cost. For RFRR, the gain in computation time could be used to improve prediction accuracy by increasing the number of random features. From the optimal hyperparameter configurations for each output, we select the configuration with the lowest mean error as measured using cross validation on the remaining training samples. Identical configurations were used both for KRR and RFRR.

#### IV. RESULTS

The experimental results are divided in two sets. In the first set of experiments, RFRR is evaluated terms of prediction errors and scalability in a batch setting. Results on these datasets will give insight in how well our method performs with respect to other state of the art learning methods. The second set of experiments aims to verify the benefits of incremental learning over batch learning on two large scale datasets. For these experiments, we compare batch KRR with incremental RFRR.

##### A. Batch Learning

The Simulated Sarcos, Sarcos, and Barrett datasets have been used previously for learning experiments (e.g., [3], [6]), and therefore form an excellent benchmark setting for RFRR. In particular, we focus on the results for LWPR, GPR, and LGP<sup>5</sup> as presented by Nguyen-Tuong *et al.* [6]. Identical experiments were performed with batch RFRR using 500, 1000, and 2000 random features (i.e., parameter  $D$ ). Additionally, we included KRR as a reference method, since it is an (approximate) upper bound on the performance of RFRR. A minor deviation from the experimental setup, as described in Section III-B, is that for RFRR<sup>500</sup> on the Barrett dataset we report the results using optimal hyperparameter configurations for each distinct output. With this limited number of random features, results using a shared hyperparameter configuration for all outputs were unsatisfactory.

<sup>5</sup>Published results for other methods (e.g.,  $\nu$ -SVR) are comparable to the considered methods and have been left out for conciseness.

The results in Fig. 1 show that, contrary to our expectations, KRR often outperforms GPR by a significant margin, even though both methods have identical formulations for the predictive mean and KRR was optimized using GPR. These deviations indicate that different hyperparameter configurations were used in both experiments. This is a common problem with GPR in comparative studies: the marginal likelihood is non-convex and its optimization often results in a local optimum that depends on the initial configuration [5]. Hence, we have to be cautious when interpreting the comparative results on these datasets with respect to generalization performance. The comparison between KRR and RFRR, trained using identical hyperparameters, remains valid and gives an indication of the approximation quality of RFRR. As expected, the performance of RFRR steadily improves as the number of random features increases. Furthermore, RFRR<sup>1000</sup> is often sufficient to obtain satisfactory predictions on all datasets. RFRR<sup>500</sup>, on the other hand, performs poorly on the Barrett dataset, despite using distinct hyperparameter configurations for each degree of freedom. In this case, RFRR<sup>1000</sup> with a shared hyperparameter configuration is more accurate and requires overall less time for prediction.

Fig. 2 shows the prediction times for all 7 outputs. Note that the timing results for GPR, LWPR, and LGP were measured on different hardware, and consequently may vary up to a constant factor when compared to our timing results. Nevertheless, Fig. 2 shows that the standard kernel methods (i.e., GPR and KRR) scale linearly with the number of training samples. Both LWPR and LGP scale sublinearly, though still outperformed by RFRR due to its constant time complexity. Moreover, the prediction times of RFRR are in the worst case only 200 $\mu$ s (i.e.,  $D = 2000$ ), which is several times faster than all competing methods. When combined with the prediction results from Fig. 1, we can confirm that parameter  $D$  effectively trades computation for prediction accuracy. This property is particularly useful for real-time control, as it allows to maximize accuracy given strict timing constraints. Although typically less relevant in a batch setting, the training time of RFRR on the Simulated Sarcos dataset ranges from approximately 1.6s to 12s for  $D = 500$  and  $D = 2000$ , respectively. This compares favorably to 60s needed to train KRR using identical hardware.

*Remark 2:* The low prediction errors of KRR in general are due to training and test sets being subsampled interleavingly from the same motion, and they are therefore strongly correlated. Combined with dense sampling, it is inevitable that there is a similar training sample for each test sample. To investigate this issue further, we performed additional experiments using Kernel Nearest Neighbor (1-KNN) [21]. This method simply returns as predicted output the label associated with the most similar training sample and therefore lacks generalization ability. Consequently, 1-KNN is critically dependent on congruence between training and test distributions, dense sampling, and noise-free outputs. Also 1-KNN, when used with the same kernel as KRR, outperforms previously published results in nearly all cases. This confirms that the similarity between training and test

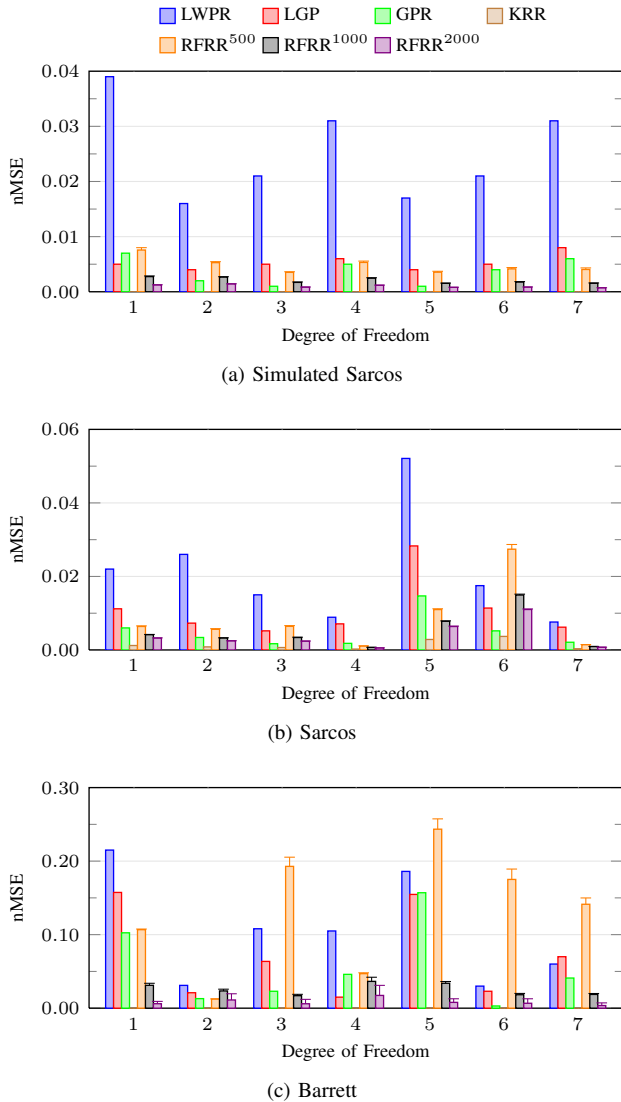


Fig. 1. Prediction error per degree of freedom for the (a) Simulated Sarcos, (b) Sarcos, and (c) Barrett datasets. The results for LWPR, GPR, and LGP are taken from Nguyen-Tuong *et al.* [6]. The mean error over 25 runs is reported for RFRR with  $D \in \{500, 1000, 2000\}$ , whereas error bars mark a distance of one standard deviation. Note that in some cases the prediction errors for KRR are very close to zero and therefore barely noticeable.

samples is very high, given correct scaling of the input features. Small prediction errors therefore do not necessarily indicate good generalization performance.

### B. Incremental Learning

A primary advantage of incremental learning over the more common batch approach is the ability to adapt continuously to changing conditions. This is particularly important if the i.i.d. sampling assumption is violated, which is often the case for real-life (robotic) problems. We therefore compare incremental learning with the more common batch learning paradigm on the James and iCub datasets. For the batch case, we trained KRR with RBF kernel on the initial 15000 training samples. This subdivision is representative for realistic use of batch methods, in which all training samples are necessarily collected *prior* to testing. Random or

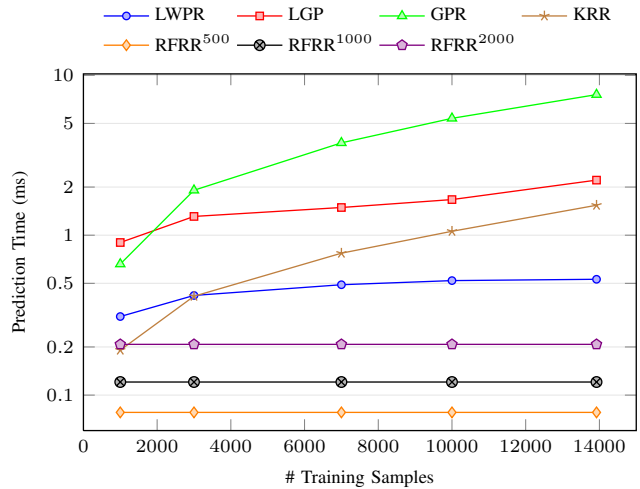


Fig. 2. Prediction time for all seven output labels with respect to the number of training samples. The results for LWPR, GPR, and LGP are taken from Nguyen-Tuong *et al.* [6] and could vary by a constant factor as compared to KRR and RFRR. Note the log scale on the  $y$ -axis.

interleaved subsampling conflicts with this principle of batch learning and is therefore of limited interest. Moreover, this amount of training samples is not overly restrictive, as it is roughly equal to the previous three datasets. We compare this batch method with incremental RFRR as detailed in Algorithm 1, which uses the training samples exclusively for hyperparameter optimization (cf. Section III-B). During testing, analogous to real-life employment of the learning method, the incremental method thus starts without any learned knowledge.

Fig. 3 shows how the average nMSE develops as test samples are predicted in sequential order using both methods. RFRR requires between 5000 and 10000 samples to achieve performance comparable to KRR. The performance of KRR, on the other hand, decreases over time. In particular on the iCub dataset it suffers a number of large errors, causing the average nMSE to show sudden jumps. This is a direct consequence of the unavoidable fact that train and test samples are not guaranteed to be drawn from the same distribution. Incremental RFRR, on the other hand, is largely unaffected by these changes and demonstrates stable predictive performance. This is not surprising, as (1) it is able to adapt to changing conditions, and (2) it eventually has trained on significantly more samples than KRR. Furthermore, Fig. 3 shows that 200 random features are sufficient to achieve satisfactory performance on either dataset. In this case, model updates of RFRR require only 400 $\mu$ s, as compared to 2ms and 7ms when using 500 or 1000 random features, respectively. These timing figures make incremental RFRR suitable for high frequency control loops.

## V. CONCLUSIONS

Machine learning is an interesting alternative to modeling robot dynamics analytically, as it avoids the need to estimate kinematic and dynamic parameters and can handle non-linearities inherent to real robotic systems. In this paper,

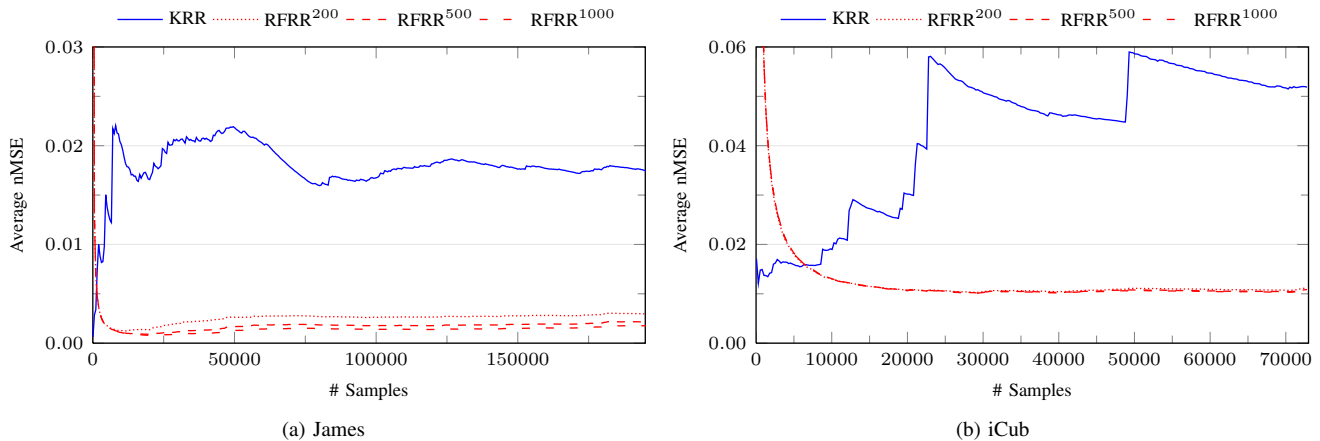


Fig. 3. Average prediction error with respect to the number of test samples of KRR and incremental RFRR with  $D \in \{200, 500, 1000\}$  on the (a) James and (b) iCub datasets. The error is measured as the nMSE averaged over the force and torque output components. The standard deviation over 25 runs of RFRR is negligible in all cases, for clarity we report only the mean without error bars.

we propose the RFRR algorithm to learn dynamics models incrementally. An experimental comparison with state of the art batch and incremental methods shows that RFRR is highly competitive in terms of prediction accuracy and superior in terms of computational requirements. A primary advantage of our method compared to other incremental methods, such as LWPR or LGP, is the constant update complexity with respect to the number of samples. Furthermore, the tradeoff between accuracy and computational cost can be regulated using a single parameter. These properties make RFRR particularly suited for use in control loops with strict timing requirements.

In addition, we demonstrate the advantage of incremental learning over traditional batch learning for modeling robot dynamics. In this problem, consecutive samples are strongly dependent and there may additionally be concept drift. This lack of i.i.d. sampling is particularly problematic for batch learners, as past training samples may not be entirely representative for future test data. Incremental methods, on the other hand, alleviate this problem by learning from all available samples. The resulting training data (i.e., the past test data) is therefore more likely to be representative for future samples. Experiments on two large scale datasets show that prediction errors of incremental RFRR are indeed significantly lower as compared to batch KRR.

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