Basic Engineering

Electrical Units

F Hamer, A Khvedelidze & M Lavelle

The aim of this package is to provide a short self assessment programme for students who want to understand some of the units used in electricity.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Introduction

In the package on **Units** some Système International (SI) units such as the metre, kilogram and second were introduced. *Derived units* such as newtons (for forces) and joules (for energy) which may be expressed in terms of the basic units were also discussed.

Example 1 Newton's equation expresses force $F = ma$ in terms of mass (m) and acceleration (a). So the units of force are

$$\begin{aligned} \text{units of force} &= \text{units of mass} \times \text{units of acceleration} \\ &= \mathrm{kg\,m\,s^{-2}}\,. \end{aligned}$$

For clarity "3 $\mathrm{kg\,m\,s^{-2}}$" of force is called 3 newtons of force.

Exercise 1. (Click on the **green** letters for solutions.)

(a) Gravitational potential energy, E_p is given in terms of mass, m, acceleration due to gravity g and height h by $E_p = mgh$. Express, joules, the units of energy, in terms of basic units.

(b) Thus show that $1\,\mathrm{N} = 1\,\mathrm{J\,m^{-1}}$ ("a newton is a joule per metre").

2. Force between Charges (Electric Field)

Electricity is based on the existence of charges and their interactions. The SI **unit of charge** is the **coulomb** (C).

$$1\,\text{coulomb} = \text{charge of } 6.2 \times 10^{18} \text{ electrons}$$

Quiz What is the charge of a single electron (in Coulombs)?

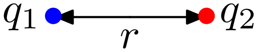
(a) 1 C (b) 1.6×10^{-19} C (c) 1.6×10^{-18} C (d) 6.2×10^{-18} C

N.B. the symbol C for the unit of charge should *not* be confused with the use of C for the physical quantity called capacitance. The **symbols for quantities and their units are normally different**. Here are some examples.

Quantity	charge, Q	current, I	capacitance, C	resistance, R
Unit	coulomb, C	ampere, A	farad, F	ohm, Ω

All of the quantities above, and more, will be discussed in this package.

By convention electrons have negative charge (and protons or atomic nuclei have positive charge). If two static charges are separated by a distance r



the size of the **force** between them is experimentally described by

$$F = k\frac{q_1 q_2}{r^2}$$

where k is a constant. This is an *inverse square law* (the gravitational force between two masses also follows an inverse square law).

- Charges of the same sign repel each other (e.g., two electrons)
- Opposite charges attract (e.g., electron and atomic nucleus).

EXERCISE 2.

(a) Rewrite the above equation for the force between static charges in the form $k = \ldots$.

(b) So find the units of k in terms of newtons, metres and coulombs.

In the **SI system of units**, we represent the constant k using the Greek symbol ϵ as follows

$$F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2}$$

where ϵ is called the **permittivity** of the region between them.

Example 2 The permittivity, ϵ, is given from rewriting the above equation by

$$\epsilon = \frac{q_1 q_2}{4\pi F r^2}\,,$$

so the **units of permittivity** are (4π is a number and has no units)

$$\text{units of permittivity } = \frac{\mathrm{C}^2}{\mathrm{N\,m}^2} = \mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}\,.$$

This is complicated and does not bring out any physical meaning. It is usual to express permittivity in $\mathrm{F\,m}^{-1}$ ("farads per metre"), where the farad, symbol F, is the (derived) unit of capacitance (see Section 4).

The **permittivity of free space** is written as ϵ_0. Permittivities in materials are often written as $\epsilon = \epsilon_r \epsilon_0$, where ϵ_r is the **relative permittivity**, which is a dimensionless ratio, i.e., a number which is independent of the units chosen.

Quiz The permittivity of free space is $8.9 \times 10^{-12}\,\mathrm{F\,m^{-1}}$. If ϵ_r in glass is 8, what is the permittivity ϵ of glass (in $\mathrm{F\,m^{-1}}$)?

(a) 7×10^{-11} (b) 9×10^{-11} (c) 9×10^{12} (d) 8×10^{13}

Quiz The permittivity of water is $7 \times 10^{-10}\,\mathrm{F\,m^{-1}}$. What is ϵ_r in water (to one significant figure)?

(a) 8 (b) 70 (c) 0.01 (d) 80

A charge q will experience a force from other charges attracting (or repelling it). The **electric field**, E, is the **force per unit charge**

$$E = \frac{F}{q}\,.$$

The units of E are thus $\mathrm{N\,C^{-1}}$ ("newtons per coulomb"). Charges in large electric fields experience stronger forces.

Exercise 3. (Click on the **green** letters for solutions.)

(a) If a charge of $3\,\mathrm{C}$ experiences a force of $6\,\mathrm{N}$ in an electric field, what is the size of the electric field E?

(b) The typical electric field produced by rubbing a plastic comb is of the order of $1,000\,\mathrm{N\,C^{-1}}$. Calculate the force (in newtons) on an electron (charge $1.6 \times 10^{-19}\,\mathrm{C}$) in such a field.

(c) The mass of an electron is $9 \times 10^{-31}\,\mathrm{kg}$. Calculate the force due to gravity on the electron. (Use $F = mg$ where $g = 10\,\mathrm{ms^{-2}}$ is the acceleration due to gravity.)

3. Electric Potential and Capacitance

When an object is lifted up/dropped on Earth (moved through a gravitational field) it gains/loses potential energy. Similarly a charge moved through an electric field changes its electric potential energy. The **electric potential** is defined as *electric potential energy per unit of charge* (unit **volts**, symbol V).

This means that if a charge Q moves through a potential V it gains or loses electrical potential energy, $E_p = QV$.

Quiz From this definition, express volts in terms of joules and coulombs.

(a) $1\mathrm{V} = 1\mathrm{J\,C}$ (b) $1\mathrm{V} = 1\mathrm{J}^2\,\mathrm{C}^2$ (c) $1\mathrm{V} = 1\mathrm{J\,C}^{-1}$ (d) $1\mathrm{V} = 1\mathrm{J}^{-1}\,\mathrm{C}$

Quiz The **electron volt**, eV, is a widely used unit of energy in modern physics. It is the energy change when an electron (charge 1.6×10^{-19} C) moves through a potential difference of one volt. What is one electron volt in joules?

(a) 6.2×10^{18} J (b) 1.6×10^{-19} J (c) 10^6 J (d) 9.1×10^{-31} J

In the Introduction we saw that $1\,\mathrm{N} = 1\mathrm{J}\,\mathrm{m}^{-1}$ ("a newton is a joule per metre"). The units of electric field E, which from the definition are $\mathrm{N}\,\mathrm{C}^{-1}$, can thus be written as:

$$\text{units of } E = \mathrm{N}\,\mathrm{C}^{-1} = \mathrm{J}\,\mathrm{m}^{-1}\,\mathrm{C}^{-1} = \mathrm{V}\,\mathrm{m}^{-1}\,,$$

where we used that $1\mathrm{V} = 1\mathrm{J}\,\mathrm{C}^{-1}$.

This result shows that the **electric field is a potential gradient** (if the potential changes rapidly over a short distance then E is large).

Quiz If the electric potential changes at a constant rate by 12 joules per coulomb over a distance of 2 metres what is the electric field strength in this region?

(a) $14\,\mathrm{V}\,\mathrm{m}^{-1}$ (b) $10\,\mathrm{V}\,\mathrm{m}^{-1}$ (c) $24\,\mathrm{V}\,\mathrm{m}^{-1}$ (d) $6\,\mathrm{V}\,\mathrm{m}^{-1}$

Quiz Using the relation between joules and newtons, select the equivalent expression for a volt.

(a) $\mathrm{N}\,\mathrm{m}^{-1}\,\mathrm{C}$ (b) $\mathrm{N}\,\mathrm{m}^{-1}\,\mathrm{C}^{-1}$ (c) $\mathrm{N}\,\mathrm{m}\,\mathrm{C}$ (d) $\mathrm{N}\,\mathrm{m}\,\mathrm{C}^{-1}$

A **capacitor** stores electric charge. This means it is a store of electric potential energy. Capacitance, variable symbol C, is measured in farads (symbol F).

A capacitor of one farad stores one coulomb of charge when one volt is applied. The general relation is charge = capacitance × potential:

$$Q = CV$$

Rewriting this as $C = Q/V$, we see that

$$\text{units of capacitance} = \frac{coulombs}{volts} = \mathrm{C\,V^{-1}}$$

Thus a farad is a coulomb per volt.

Exercise 4. (Click on the **green** letters for solutions.)

(a) If a charge $2\,\mathrm{C}$ is produced by applying a voltage $3\,\mathrm{V}$, what is the capacitance?

(b) Show that, as stated on page 6, $\mathrm{C^2\,N^{-1}\,m^{-2} = F\,m^{-1}}$.

4. Current and Resistance

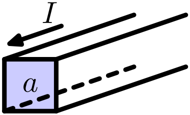
When charges move past a point, we say that a **current**, I, flows.

> One ampere (unit A) of current corresponds to one coulomb of charge flowing past a point per second.

Quiz Select the correct expression for an ampere (amp) below.

(a) $\mathrm{C\,s}$ (b) $\mathrm{C\,s^{-2}}$ (c) $\mathrm{C\,s^{-1}}$ (d) $\mathrm{s\,C^{-1}}$

Example 3 If one ampere flows through the copper wire in the picture



then a coulomb of charge, 6.2×10^{18} electrons, flows through the area a every second.

For historical reasons, the flow of current is conventionally taken in the opposite direction to that which electrons are really moving in.

Current depends upon the voltage applied and upon the material (copper is a better conductor than paper) but it also depends upon the shape of the material (there is a greater current if the area a is larger).

To remove this dependence upon the area a, we define the **current density** J by

$$J = \frac{I}{a}\,, \qquad \text{units } \mathrm{Am^{-2}} \text{ read as “amps per metre squared”}\,.$$

Quiz Use the expression above to select an alternative expression for the unit of current density.

(a) $\mathrm{C\,s^{-1}\,m^{-2}}$ (b) $\mathrm{C\,s\,m^{2}}$ (c) $\mathrm{C\,s^{-1}\,m^{2}}$ (d) $\mathrm{C\,s\,m^{-2}}$

The current density J is related to the electric field strength in the conductor by

$$J = \sigma E$$

where σ is called the **conductivity** of the wire.

Conductivity is a property of the material. From the last equation if σ is large then the current density in a material will be higher. The (derived) unit of σ is $\mathrm{Sm^{-1}}$ (read as "siemens per metre"). The conductivity of copper is $5.8 \times 10^{7}\,\mathrm{Sm^{-1}}$

Quiz The units of conductivity can also be expressed in terms of other units from rewriting its definition as

$$\sigma = \frac{J}{E}$$

Select the correct result from the possibilities below.

(a) $\mathrm{A\,V^{-1}\,m^{-1}}$ (b) $\mathrm{A\,V\,m^{-1}}$ (c) $\mathrm{A\,V^{-1}m^{-3}}$ (d) $\mathrm{A\,V\,m^{-3}}$

Example 4 Ohm's Law In a resistor with a constant electric field E and length ℓ (metres) the potential difference is $V = E\ell$ (volts).

Similarly the current in the conductor is $I = Ja$ (amps). Hence from Ohm's law $V = IR$ we have that resistance R is given by

$$R = \frac{V}{I} \quad \Rightarrow \quad R = \frac{E\ell}{Ja} = \frac{\ell}{\sigma a}\,.$$

To understand this equation note that:

- Doubling the length ℓ doubles the resistance.
- Increasing the cross-sectional area a decreases the resistance.
- For two identically shaped resistors, the one made from material with a higher conductivity σ will have the lower resistance.

Quiz The units of resistance R are **ohms** (symbol Ω). From the above equation select the correct expression for an ohm.

(a) $\mathrm{S\,m}$ (b) $\mathrm{m\,S^{-1}}$ (c) $\mathrm{S^{-1}}$ (d) $\mathrm{S^{-1}\,m^{-3}}$

Exercise 5. (Click on the **green** letters for solutions.)

(a) From Ohm's law, express the unit of voltage, the volt in terms of amperes and ohms.

(b) By rewriting Ohm's law, express the unit of resistance, the ohm in terms of volts and amperes.

(c) From

$$R = \frac{\ell}{\sigma a}$$

and the conductivity of copper stated above, calculate the resistance of a copper wire of length $10\,\text{m}$ and radius $2\,\text{mm}$.

5. Final Quiz

Begin Quiz

1. Coulomb volts, $\mathrm{C\,V}$, correspond to which physical quantity?
(a) Capacitance (b) Resistance (c) Current (d) Energy .

2. Express the siemens, S, in terms of ohms, Ω?
(a) $\mathrm{S} = \Omega\,\mathrm{m}^{-2}$ (b) $\mathrm{S} = \mathrm{A}\,\Omega$ (c) $\mathrm{S} = \mathrm{V}\,\Omega^{-1}$ (d) $\mathrm{S} = \Omega^{-1}$

3. Using Ohm's law, select the equivalent expression for the unit of current, the ampere.
(a) $\mathrm{V}\,\Omega$ (b) $\mathrm{V}^{-1}\,\Omega^{-1}$ (c) $\mathrm{V}\,\Omega^{-1}$ (d) $\Omega\,\mathrm{V}^{-1}$

4. Select the electric field value which produces a force on an electron similar to its weight on Earth.
(a) $10^{11}\,\mathrm{N\,C^{-1}}$ (b) $10^{-13}\,\mathrm{N\,C^{-1}}$ (c) $10^{48}\,\mathrm{N\,C^{-1}}$ (d) $10^{-10}\,\mathrm{N\,C^{-1}}$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a) The **SI unit of energy**, the joule, may be **written in terms of basic units** from the equation $E_p = mgh$, where m is the mass, g is the acceleration due to gravity and h is a height. This gives

$$\text{the joule, } \mathrm{J} = \mathrm{kg} \times \mathrm{m\,s^{-2}} \times \mathrm{m}\,,$$

which simplifies to

$$\mathrm{J} = \mathrm{kg\,m^2\,s^{-2}}\,.$$

Click on the **green** square to return

□

Exercise 1(b) To show that a newton is a joule per metre, $1\,\mathrm{N} = 1\,\mathrm{J\,m^{-1}}$, recall from Example 1 that a newton is

$$1\,\mathrm{N} = 1\,\mathrm{kg\,m\,s^{-2}}\,,$$

and that a joule is

$$1\,\mathrm{J} = 1\,\mathrm{kg\,m^2\,s^{-2}}\,.$$

Hence

$$\frac{\mathrm{J}}{\mathrm{m}} = \frac{\mathrm{kg\,m^2\,s^{-2}}}{\mathrm{m}} = \mathrm{kg\,m\,s^{-2}}\,.$$

which is indeed a newton.

This means that if an object is accelerated by a force of one newton over a distance of one metre, it will acquire one joule of energy.

Click on the **green** square to return

□

Exercise 2(a) To find an equation for the constant k, multiply both sides of

$$F = k\frac{q_1 q_2}{r^2}$$

by r^2. This gives

$$Fr^2 = kq_1 q_2$$

and then divide both sides by $q_1 q_2$ to get

$$\frac{Fr^2}{q_1 q_2} = k\,.$$

Click on the **green** square to return

□

Exercise 2(b) To find the units of k, use the equation derived above

$$k = \frac{Fr^2}{q_1 q_2}\,.$$

Since F is a force (unit newtons), r is a distance (unit metres), and q_1 & q_2 are both charges (units coulombs), this means

$$\text{units of } k = \frac{\mathrm{N\,m^2}}{\mathrm{C^2}} = \mathrm{N\,m^2\,C^{-2}}\,.$$

This is read as “newtons, metres squared per square coulomb”.

The constant k ***measures the strength of the force between charges***. The larger the value of k, the larger the force between two charges at a fixed separation would be.

Click on the **green** square to return □

Exercise 3(a) If 3 C of charge experiences a force of 6 N from an electric field, the **value of the electric field** must be

$$E = \frac{F}{Q} = \frac{6}{3} = 2\,\mathrm{N\,C^{-1}}\,,$$

which is read as "two newtons per coulomb".

Click on the **green** square to return

□

Exercise 3(b) If an electron is in an electric field of $1,000\,\mathrm{N\,C^{-1}} = 10^3\,\mathrm{N\,C^{-1}}$, then since the charge on the electron is $1.6 \times 10^{-19}\,\mathrm{C}$, the electric force on it can be found by rearranging $E = F/Q$ to read

$$\begin{aligned} F &= EQ \\ &= 10^3\,\mathrm{N\,C^{-1}} \times 1.6 \times 10^{-19}\,\mathrm{C} \\ &= 1.6 \times 10^{-19+3}\,\mathrm{N}\,, \\ &= 1.6 \times 10^{-16}\,\mathrm{N}\,. \end{aligned}$$

This force acts on each individual electron in such an electric field.

Click on the **green** square to return

□

Exercise 3(c) If the mass of an electron is $9 \times 10^{-31}\,\text{kg}$, then the force on it due to gravity (i.e., its weight) on Earth where the acceleration due to gravity is $g = 10\,\text{ms}^{-2}$ is given by

$$\begin{aligned} F &= mg \\ &= 9 \times 10^{-31}\,\text{kg} \times 10\,\text{ms}^{-2}\,, \\ &= 9 \times 10^{-30}\,\text{kg}\,\text{ms}^{-2}\,, \\ &= 9 \times 10^{-30}\,\text{N} \end{aligned}$$

since ***one newton is one kilogram metre per second squared.***

It is clear that the force due to gravity (the weight of the electron) is much smaller than the electric force of part **3(b)**. This indicates why it is possible to lift small pieces of paper by rubbing a comb (although the weight of the paper is much more than just the weight of the electrons).

Click on the **green** square to return

□

Exercise 4(a) If a charge of $2\,\mathrm{C}$ (two coulombs) is produced on a capacitor by applying a voltage of $3\,\mathrm{V}$, the capacitance C can be found as follows. From

$$Q = CV$$

we have

$$C = \frac{Q}{V}$$

so substituting the data

$$C = \frac{2\,\mathrm{C}}{3\,\mathrm{V}} = 0.7\,\mathrm{F}\,,$$

to one significant figure. In the final step the definition of the farad, $\mathrm{F} = \mathrm{C\,V^{-1}}$ was used. Note the distinction between C (the symbol for the physical quantity capacitance) and C (the symbol for the coulomb, the unit of charge).

Click on the **green** square to return

□

Exercise 4(b) To prove that

$$\mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2} = \mathrm{F}\,\mathrm{m}^{-1}$$

recall the definition of capacitance

$$C = \frac{Q}{V}$$

which means that the units are related by

$$\mathrm{F} = \frac{\mathrm{C}}{\mathrm{V}}\,.$$

However, it was shown in the second quiz on page 10 that

$$1\,\mathrm{V} = 1\,\mathrm{N}\,\mathrm{m}\,\mathrm{C}^{-1}$$

so that

$$\begin{aligned}\mathrm{F}\,\mathrm{m}^{-1} &= \frac{\mathrm{C}\,\mathrm{m}^{-1}}{\mathrm{N}\,\mathrm{m}\,\mathrm{C}^{-1}} \\ &= \mathrm{C}^{1-(-1)}\,\mathrm{m}^{-1-1}\mathrm{N}^{-1} = \mathrm{C}^2\,\mathrm{N}^{-1}\,\mathrm{m}^{-2}\end{aligned}$$

□

Click on the **green** square to return

Exercise 5(a) The unit of voltage, the volt, may be rewritten in terms of amperes and ohms by using Ohm's law, $V = IR$. Thus

$$\text{the units of voltage } = \text{amperes} \times \text{ohms} = \mathrm{A}\,\Omega\,.$$

We read that ***a volt is a "ampere ohm"***.

Thus a four ohm resistor with five amps flowing through it must have a potential difference of twenty volts across it.

Click on the **green** square to return

□

Exercise 5(b) The unit of resistance, the ohm, may be rewritten in terms of volts and amperes by using ohm's law, $V = IR$, as

$$R = \frac{V}{I}\,.$$

Thus

$$\text{the units of resistance } = \frac{\text{volts}}{\text{amperes}} = \frac{\text{V}}{\text{A}} = \text{V}\,\text{A}^{-1}\,.$$

We read that ***an ohm is a "volt per ampere"***.

Thus a three ohm resistor needs three volts applied to it to produce one amp of current.

Click on the **green** square to return

□

Exercise 5(c) To find the resistance, R, of copper wire of length $\ell = 10\,\mathrm{m}$ and radius $2\,\mathrm{mm}$ $(= 2 \times 10^{-3}\,\mathrm{m})$, use

$$R = \frac{\ell}{\sigma a}$$

and the value $\sigma = 5.8 \times 10^{7}\,\mathrm{S\,m^{-1}}$ for copper. The wire's cross-sectional area is $a = \pi r^2 = \pi \times (2 \times 10^{-3})^2\,\mathrm{m}^2 = 4\pi \times 10^{-6}\,\mathrm{m}^2$. Thus

$$\begin{aligned} R &= \frac{10\,\mathrm{m}}{5.8 \times 10^{7}\,\mathrm{S\,m^{-1}} \times 4\pi \times 10^{-6}\,\mathrm{m}^2} \\ &= \frac{10\,\mathrm{m}}{4\pi \times 5.8 \times 10^{7-6}\,\mathrm{S\,m^{-1+2}}} \\ &= \frac{10\,\mathrm{m}}{4\pi \times 5.8 \times 10\,\mathrm{S\,m}} = \frac{1}{4\pi \times 5.8\,\mathrm{S}} = 0.01\,\Omega\,, \end{aligned}$$

where in the last step we used that $\mathrm{S}^{-1} = \Omega$. The tiny result is because copper wire is a good conductor.

Click on the **green** square to return □

Solutions to Quizzes

Solution to Quiz: A coulomb is the charge of 6.2×10^{18} electrons.

$$1\text{C} = 6.2 \times 10^{18}\,\text{electron charges}$$

Therefore

$$\frac{1}{6.2 \times 10^{18}}\text{C} = 1\,\text{electron charge}\,.$$

So the charge of a single electron is

$$\frac{1}{6.2 \times 10^{18}}\text{C} = 1.6 \times 10^{-19}\,\text{C}\,.$$

To check this, note that the charge of 6.2×10^{18} electrons is indeed

$$6.2 \times 10^{18} \times 1.6 \times 10^{-19}\,\text{C} = 1\,\text{C}\,.$$

End Quiz

Solution to Quiz: To find the permittivity in glass, ϵ, given that the relative permittivity is $\epsilon_r = 8$ use

$$\epsilon = \epsilon_r \epsilon_0 \, ,$$

where $\epsilon_0 = 8.9 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ is the permittivity of free space. Thus

$$\epsilon = 8 \times 8.9 \times 10^{-12}\,\mathrm{F\,m^{-1}} = 7 \times 10^{-11}\,\mathrm{F\,m^{-1}} \, ,$$

to one significant figure. End Quiz

Solution to Quiz: To find the relative permittivity of water, ϵ_r, given that its permittivity is $\epsilon = 7 \times 10^{-10}\,\mathrm{F\,m^{-1}}$ rearrange $\epsilon = \epsilon_r \epsilon_0$ to read

$$\epsilon_r = \frac{\epsilon}{\epsilon_0}$$

where $\epsilon_0 = 8.9 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ is the permittivity of free space. Thus

$$\epsilon_r = \frac{7 \times 10^{-10}\,\mathrm{F\,m^{-1}}}{8.9 \times 10^{-12}\,\mathrm{F\,m^{-1}}}\,,$$

which shows that ϵ_r is a dimensionless ratio (a number) and does not have any units. Its numerical value is

$$\begin{aligned} \epsilon_r &= \frac{7 \times 10^{-10}}{8.9 \times 10^{-12}} \\ &= \frac{7 \times 10^{-10-(-12)}}{8.9} = \frac{7 \times 10^{2}}{8.9} \\ &= 80\,, \end{aligned}$$

to one significant figure.

End Quiz

Solution to Quiz: To express **volts in terms of joules and coulombs**, use

$$E_p = QV\,,$$

which implies that

$$V = \frac{E_p}{Q}\,.$$

Thus the units of volts are:

$$\text{units of volts, } \mathrm{V} = \frac{\text{joules}}{\text{coulombs}} = \frac{\mathrm{J}}{\mathrm{C}}\,.$$

In other words $\mathrm{V} = \mathrm{J\,C^{-1}}$ ("a volt is a joule per coulomb").

Thus if a coulomb of charge passes through a potential difference of one volt, it will gain or lose one joule of electrical potential energy.

End Quiz

Solution to Quiz: The charge of an electron is $1.6 \times 10^{-19}\,\mathrm{C}$. Thus an electron volt (eV), the change in energy of an electron moving through a one volt potential difference is

$$\begin{aligned} E_p &= QV \\ &= 1.6 \times 10^{-19}\,\mathrm{C} \times 1\,\mathrm{V} \\ &= 1.6 \times 10^{-19}\,\mathrm{J}\,. \end{aligned}$$

In the last step the relation between the units $\mathrm{V} = \mathrm{J\,C^{-1}}$ was used.

End Quiz

Solution to Quiz: The electric field, E, is a potential gradient

$$E = \frac{\text{change in } V}{\text{distance}} \, .$$

Thus if the change in V is $12\,\text{V}$ over a distance of $2\,\text{m}$

$$E = \frac{12\,\text{V}}{2\,\text{m}} = 6\,\text{V}\,\text{m}^{-1} \, .$$

End Quiz

Solution to Quiz: To find an equivalent expression for a volt, we have:

$$\mathrm{N} = \mathrm{J\,m^{-1}}$$

and

$$\mathrm{V} = \mathrm{J\,C^{-1}}\,.$$

From the first equation

$$\mathrm{J} = \mathrm{N\,m}$$

which may be inserted into the second equation to give

$$\mathrm{V} = \mathrm{N\,m\,C^{-1}}\,.$$

End Quiz

Solution to Quiz: An ampere is related to a coulomb by

$$\text{one amp} = \text{one coulomb per second}$$

which means

$$1\,\mathrm{A} = 1\,\mathrm{C\,s^{-1}}\,.$$

End Quiz

Solution to Quiz: To find an expression for the units of current density, J, use the definition

$$J = \frac{I}{a}\,.$$

Since the unit of current, the ampere, can be written as $\mathrm{C\,s^{-1}}$ and the units of area are $\mathrm{m^2}$, the units of J are

$$\frac{\mathrm{C\,s^{-1}}}{\mathrm{m^2}} = \mathrm{C\,s^{-1}\,m^{-2}}\,.$$

This is read as "coulombs per second per square metre".

End Quiz

Solution to Quiz: To find an alternative expression for the units of conductivity, σ, recall its definition

$$\sigma = \frac{J}{E}\,.$$

The units of current density, J, are $\mathrm{A\,m^{-2}}$ and the units of electric field, E, may be written as either $\mathrm{N\,C^{-1}}$ or, equivalently, $\mathrm{V\,m^{-1}}$.

From the possible answers given, we see we have to use the expression $\mathrm{V\,m^{-1}}$. Thus

$$\begin{aligned}\text{units of } \sigma &= \frac{\mathrm{A\,m^{-2}}}{\mathrm{V\,m^{-1}}}\\ &= \mathrm{A\,V^{-1}\,m^{-2-(-1)}}\\ &= \mathrm{A\,V^{-1}\,m^{-1}}\,.\end{aligned}$$

End Quiz

Solution to Quiz: The **units ohms and siemens are related as follows**. Resistance is related to conductivity by

$$R = \frac{\ell}{\sigma a}\,,$$

and ℓ is a length (unit *metres*), a is an area (unit *square metres*) and the units of conductivity, σ are siemens per metre ($\mathrm{S\,m^{-1}}$). Thus

$$\text{the units of resistance } R = \frac{\mathrm{m}}{\mathrm{S\,m^{-1}} \times \mathrm{m^2}} = \frac{\mathrm{m}}{\mathrm{S\,m}} = \mathrm{S}^{-1}\,.$$

Thus **an ohm is an inverse siemen**.

End Quiz