



# Fractions

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic competence in the use of fractions.

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## 1. Fractions (Introduction)

The first thing to note is that all fractions can be represented in many different ways. Thus  $3/12$ ,  $4/16$ ,  $5/20$  represent the same fraction which, in its *lowest terms*, is  $1/4$ .

**Example 1** To show that each of the above fractions is equal to  $1/4$  we proceed as follows:

$$\frac{3}{12} = \frac{1 \times 3}{4 \times 3} = \frac{1 \times \cancel{3}}{4 \times \cancel{3}} = \frac{1}{4},$$

$$\frac{4}{16} = \frac{1 \times 4}{4 \times 4} = \frac{1 \times \cancel{4}}{4 \times \cancel{4}} = \frac{1}{4},$$

$$\frac{5}{20} = \frac{1 \times 5}{4 \times 5} = \frac{1 \times \cancel{5}}{4 \times \cancel{5}} = \frac{1}{4}.$$

The fractions in the left hand column each have a factor common to the **numerator** (top) and **denominator** (bottom) of that fraction, which is cancelled to give the fraction in its *lowest terms*.

In many problems it is necessary to carry out the reverse procedure, i.e. multiplying the **numerator** and **denominator** of a fraction by a common factor to obtain an *equivalent* fraction.

**Example 2** Arrange each of the following fractions in the order of size.

$$(a) \frac{3}{4}, \quad (b) \frac{2}{3}, \quad (c) \frac{5}{6}.$$

**Solution** To determine their relative order, each fraction must be written with the same **denominator**. The smallest such number is 12.

$$(a) \quad \frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12},$$

$$(b) \quad \frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12},$$

$$(c) \quad \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}.$$

The order of size, starting with the smallest, is  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{5}{6}$ .

**EXERCISE 1.** In each case below, arrange the fractions in increasing order of size. (Click on **green** letters for solution.)

(a)  $7/12$ ,  $1/2$ ,  $2/3$ .      (b)  $1/3$ ,  $5/8$ ,  $3/4$ .

(c)  $5/6$ ,  $8/9$ ,  $11/12$ .      (d)  $2/3$ ,  $3/5$ ,  $7/10$ .

Now try this short quiz. (Work out your solution and click on the **green** letter to see if you are correct.)

**Quiz** Of the following sets of fractions, exactly one is in ascending order of size. Which one?

(a)  $1/3$ ,  $5/8$ ,  $3/5$ .      (b)  $3/8$ ,  $2/3$ ,  $3/4$ .

(c)  $2/3$ ,  $3/4$ ,  $1/2$ .      (d)  $3/5$ ,  $5/7$ ,  $1/2$ .

## 2. Addition of Fractions

In this section we look at the addition (and subtraction) of fractions. If fractions are to be added then they must have the same **denominators**.

**Example 3** Write the following sums of fractions as single fractions.

$$(a) \quad \frac{3}{8} + \frac{2}{3} - \frac{3}{4}, \quad (b) \quad \frac{2}{3} + \frac{1}{2} - \frac{3}{4}.$$

### Solution

(a) Taking all the fractions with **denominator** 24,

$$\begin{aligned} \frac{3}{8} + \frac{2}{3} - \frac{3}{4} &= \frac{3 \times 3}{8 \times 3} + \frac{2 \times 8}{3 \times 8} - \frac{3 \times 6}{4 \times 6} \\ &= \frac{9}{24} + \frac{16}{24} - \frac{18}{24} \\ &= \frac{9 + 16 - 18}{24} = \frac{7}{24}. \end{aligned}$$

(b) This time, taking all the fractions with denominator 12,

$$\begin{aligned}\frac{2}{3} + \frac{1}{2} - \frac{3}{4} &= \frac{2 \times 4}{3 \times 4} + \frac{1 \times 6}{2 \times 6} - \frac{3 \times 3}{4 \times 3} \\ &= \frac{8}{12} + \frac{6}{12} - \frac{9}{12} \\ &= \frac{8 + 6 - 9}{12} = \frac{5}{12}.\end{aligned}$$

The exercise below is designed to give you some practice at addition and subtraction of fractions.

**EXERCISE 2.** Evaluate the following, putting the final answer in its lowest terms. (Click on green letters for solution.)

$$\begin{array}{lll} \text{(a)} & \frac{1}{3} + \frac{1}{8}, & \text{(b)} & \frac{5}{6} + \frac{2}{4}, & \text{(c)} & \frac{3}{4} + \frac{4}{5}, \\ \text{(d)} & \frac{4}{3} - \frac{1}{4}, & \text{(e)} & \frac{5}{6} - \frac{3}{2}, & \text{(f)} & \frac{5}{6} - \frac{3}{10}. \end{array}$$

To finish this section there follows two simple quizzes.

**Quiz** Which of the following fractions is the result of evaluating the sum

$$\frac{3}{4} - \frac{2}{3} + \frac{1}{6}?$$

- (a)  $1/4$ ,      (b)  $1/3$ ,      (c)  $1/5$ ,      (d)  $3/8$ .

**Quiz** From the fraction given below, choose the one which is mid-way between  $2/3$  and  $4/5$ .

- (a)  $3/4$ ,      (b)  $3/5$ ,      (c)  $10/15$       (d)  $11/15$ .

### 3. Multiplication and Division of Fractions

Multiplication of fractions is straightforward, as illustrated by the following.

**Example 4** Evaluate the following products.

$$(a) \frac{2}{5} \times \frac{3}{4}, \quad (b) \frac{3}{4} \times \frac{1}{3}.$$

#### Solution

It is simply a matter of multiplying the **numerators** and **denominators**.

$$(a) \frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{6}{20} = \frac{3}{10}, \quad (b) \frac{3}{4} \times \frac{1}{3} = \frac{3 \times 1}{4 \times 3} = \frac{1}{4}.$$

In each case the **common factors** have been cancelled to leave the fraction in its **lowest form**.

Division of fractions is straightforward once we note that **division** by a fraction of the form  $a/b$  is equivalent to **multiplication** by the fraction  $b/a$ . This can be seen in the following example.

**Example 5** Write the following as fractions in the usual way.

$$\text{(a)} \quad 1 \div \left(\frac{3}{4}\right), \quad \text{(b)} \quad \left(\frac{1}{8}\right) \div \left(\frac{3}{4}\right).$$

**Solution**

(a) First note that

$$\left(\frac{3}{4}\right) \times \left(\frac{4}{3}\right) = \frac{3 \times 4}{4 \times 3} = \frac{12}{12} = 1.$$

Divide both sides of this equation by  $(3/4)$ .

$$\begin{aligned} \left(\frac{3}{4}\right) \times \left(\frac{4}{3}\right) &= 1 \\ \therefore \left(\frac{4}{3}\right) &= 1 / \left(\frac{3}{4}\right) = 1 \div \left(\frac{3}{4}\right). \end{aligned}$$

(b) The previous result may be used.

$$\begin{aligned}\left(\frac{1}{8}\right) \div \left(\frac{3}{4}\right) &= \frac{\left(\frac{1}{8}\right)}{\left(\frac{3}{4}\right)} = \frac{\left(\frac{1}{8}\right) \times 1}{1 \times \left(\frac{3}{4}\right)} = \frac{\left(\frac{1}{8}\right)}{1} \times \frac{1}{\left(\frac{3}{4}\right)} \\ &= \left(\frac{1}{8}\right) \times \left(\frac{4}{3}\right) \text{ using (a)} \\ &= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}.\end{aligned}$$

where a **common factor** 4 has been cancelled.

**EXERCISE 3.** Evaluate each of the following in their lowest terms.

(a)  $\left(\frac{1}{3} + \frac{1}{8}\right) \times \frac{2}{11},$

(b)  $\left(\frac{5}{6} + \frac{1}{2}\right) \div \frac{1}{3},$

(c)  $\frac{\left(\frac{3}{4} + \frac{4}{5}\right)}{\left(\frac{1}{4} - \frac{1}{5}\right)},$

(d)  $\frac{\left(\frac{5}{6} - \frac{3}{2}\right)}{\left(\frac{4}{3} - \frac{1}{4}\right)}.$

## 4. Quiz on Fractions

**Begin Quiz** Evaluate each of the following and choose the answer from those given below.

1.  $\left(\frac{3}{4} - \frac{2}{3}\right) \div \left(\frac{3}{4}\right)$

(a)  $2/9$ , (b)  $1/6$ ,  $5/12$ , (c)  $5/12$ , (d)  $1/9$ .

2.  $\left(\frac{3}{5} + \frac{1}{3}\right) \times \left(\frac{5}{7}\right)$

(a)  $1/7$ , (b)  $2/3$ , (c)  $2/15$ , (d)  $4/21$ .

3.  $\left(\frac{3}{8} - \frac{1}{5}\right) \div \left(\frac{7}{10} - \frac{2}{3}\right)$

(a)  $4/15$ , (b)  $13/25$ , (c)  $13/20$ , (d)  $21/4$ .

**End Quiz**

## Solutions to Exercises

**Exercise 1(a)** The *least common denominator* of the fractions is 12, so

$$\frac{1}{2} = \frac{1 \times 6}{2 \times 6} = \frac{6}{12},$$

$$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}.$$

Since 7/12 already has 12 as denominator, the required order of the fractions is

$$\frac{1}{2}, \frac{7}{12}, \frac{2}{3}.$$

Click on green square to return



**Exercise 1(b)** In this case, the *least common denominator* of the fractions is 24.

$$\frac{1}{3} = \frac{1 \times 8}{3 \times 8} = \frac{8}{24},$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24},$$

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}.$$

The fractions are thus increasing in the order in which they appear.

Click on green square to return



**Exercise 1(c)** In this case the *least common denominator* of the fractions is 36.

$$\frac{5}{6} = \frac{5 \times 6}{6 \times 6} = \frac{30}{36},$$

$$\frac{8}{9} = \frac{8 \times 4}{9 \times 4} = \frac{32}{36},$$

$$\frac{11}{12} = \frac{11 \times 3}{12 \times 3} = \frac{33}{36},$$

The fractions are thus increasing in the order in which they appear.

Click on green square to return



**Exercise 1(d)** In this case the *least common denominator* of the fractions is 30.

$$\frac{2}{3} = \frac{2 \times 10}{3 \times 10} = \frac{20}{30},$$

$$\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30},$$

$$\frac{7}{10} = \frac{7 \times 3}{10 \times 3} = \frac{21}{30}.$$

The fractions, in increasing order of size, are  $3/5$ ,  $2/3$ ,  $7/10$ .

Click on green square to return



**Exercise 2(a)** The *lowest common denominator* is 24, so

$$\begin{aligned}\frac{1}{3} + \frac{1}{8} &= \frac{1 \times 8}{3 \times 8} + \frac{1 \times 3}{8 \times 3} \\ &= \frac{8}{24} + \frac{3}{24} \\ &= \frac{8+3}{24} = \frac{11}{24}.\end{aligned}$$

Click on green square to return



**Exercise 2(b)** Before proceeding, note that the **second** fraction is not in its lowest terms. Since  $2/4=(1 \times 2)/(2 \times 2)=1/2$ ,

$$\begin{aligned}\frac{5}{6} + \frac{2}{4} &= \frac{5}{6} + \frac{1}{2} \\ &= \frac{5}{6} + \frac{1 \times 3}{2 \times 3} \\ &= \frac{5}{6} + \frac{3}{6} \\ &= \frac{5+3}{6} = \frac{8}{6} \\ &= \frac{4 \times 2}{3 \times 2} = \frac{4}{3}.\end{aligned}$$

This fraction is called an *improper fraction* since the **numerator** is larger than the **denominator**. It is perfectly acceptable as a fraction but it may also be written as  $1\frac{1}{3}$ .

Click on green square to return



**Exercise 2(c)**

The *least common denominator* of the two fractions is 20 so

$$\begin{aligned}\frac{3}{4} + \frac{4}{5} &= \frac{3 \times 5}{4 \times 5} + \frac{4 \times 4}{5 \times 4} \\ &= \frac{15}{20} + \frac{16}{20} \\ &= \frac{15 + 16}{20} = \frac{31}{20}.\end{aligned}$$

This is another *improper fraction* which may be written as  $1\frac{11}{20}$ .

Click on green square to return



**Exercise 2(d)**

The *least common denominator* of the two fractions is 12 so

$$\begin{aligned}\frac{4}{3} - \frac{1}{4} &= \frac{4 \times 4}{3 \times 4} - \frac{1 \times 3}{4 \times 3} \\ &= \frac{16}{12} - \frac{3}{12} \\ &= \frac{16 - 3}{12} = \frac{13}{12}.\end{aligned}$$

This is another *improper fraction* which is equal to  $1\frac{1}{12}$ ..

Click on green square to return



**Exercise 2(e)**

The *least common denominator* of the two fractions is 6 so

$$\begin{aligned}\frac{5}{6} - \frac{3}{2} &= \frac{5}{6} - \frac{3 \times 3}{2 \times 3} \\ &= \frac{5}{6} - \frac{9}{6} \\ &= \frac{5 - 9}{6} = -\frac{4}{6} \\ &= -\frac{2 \times 2}{3 \times 2} = -\frac{2}{3}.\end{aligned}$$

where the common factor 2 has been cancelled to obtain the final answer.

Click on green square to return



**Exercise 2(f)**

The *least common denominator* of the two fractions is 30 so

$$\begin{aligned}\frac{5}{6} - \frac{3}{10} &= \frac{5 \times 5}{6 \times 5} - \frac{3 \times 3}{10 \times 3} \\ &= \frac{25}{30} - \frac{9}{30} \\ &= \frac{25 - 9}{30} = \frac{16}{30} \\ &= \frac{8 \times 2}{15 \times 2} = \frac{8}{15},\end{aligned}$$

where the final answer is obtained after cancellation of the common factor 2.

Click on green square to return



**Exercise 3(a)** Using [exercise 2](#) part **(a)** we have

$$\begin{aligned}\left(\frac{1}{3} + \frac{1}{8}\right) \times \frac{2}{11} &= \frac{11}{24} \times \frac{2}{11} \\ &= \frac{2}{24} = \frac{1}{12},\end{aligned}$$

where the common factors of 11 and then 2 have been cancelled.

[Click on green square to return](#)



**Exercise 3(b)**

Using the result of [exercise 2](#) part **(b)** we have

$$\begin{aligned}\left(\frac{5}{6} + \frac{1}{2}\right) \div \frac{1}{3} &= \frac{4}{3} \div \frac{1}{3} \\ &= \frac{4}{3} \times \frac{3}{1} \\ &= \frac{4}{1} = 4,\end{aligned}$$

after recalling that [division](#) by  $1/3$  is the same as [multiplication](#) by  $3/1=3$ , and cancelling the common factor 3.

[Click on green square to return](#)



**Exercise 3(c)** From [exercise 2](#) part **(c)**, the **numerator** is  $31/20$ . The **denominator** is evaluated below.

$$\begin{aligned}\frac{1}{4} - \frac{1}{5} &= \frac{1 \times 5}{4 \times 5} - \frac{1 \times 4}{5 \times 4} \\ &= \frac{5}{20} - \frac{4}{20} = \frac{1}{20}.\end{aligned}$$

Thus

$$\begin{aligned}\frac{\left(\frac{3}{4} + \frac{4}{5}\right)}{\left(\frac{1}{4} - \frac{1}{5}\right)} &= \frac{\left(\frac{31}{20}\right)}{\left(\frac{1}{20}\right)} \\ &= \left(\frac{31}{20}\right) \times \left(\frac{20}{1}\right) \\ &= 31,\end{aligned}$$

after cancellation of the common factor 20.

[Click on green square to return](#)



**Exercise 3(d)** In this case, from [exercise 2](#) part (c), the denominator is  $13/12$ . Evaluating the numerator,

$$\begin{aligned}\frac{5}{6} - \frac{3}{2} &= \frac{5}{6} - \frac{9}{6} \\ &= -\frac{4}{6} = -\frac{2}{3}.\end{aligned}$$

after cancelling the common factor 2. Thus

$$\begin{aligned}\frac{\left(\frac{5}{6} - \frac{3}{2}\right)}{\left(\frac{4}{3} - \frac{1}{4}\right)} &= -\frac{\left(\frac{2}{3}\right)}{\left(\frac{13}{12}\right)} = -\left(\frac{2}{3}\right) \times \left(\frac{12}{13}\right) \\ &= -\left(\frac{2}{3}\right) \times \left(\frac{4 \times 3}{13}\right) \\ &= -\left(\frac{2}{1}\right) \times \left(\frac{4 \times 1}{13}\right) = -\frac{8}{13},\end{aligned}$$

after cancellation of the common factor 3.

[Click on green square to return](#)



## Solutions to Quizzes

**Solution to Quiz:** Putting each of them in the form of a fraction with denominator 24,

$$\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24},$$

$$\frac{2}{3} = \frac{2 \times 8}{3 \times 8} = \frac{16}{24},$$

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}.$$

These are obviously in ascending order. Repeating this with the others will show that this is the only set in ascending order.

End Quiz

**Solution to Quiz:**

The *least common denominator* of the three fractions is 12, so

$$\begin{aligned}\frac{3}{4} - \frac{2}{3} + \frac{1}{6} &= \frac{3 \times 3}{4 \times 3} - \frac{2 \times 4}{3 \times 4} + \frac{1 \times 2}{6 \times 2} \\ &= \frac{9}{12} - \frac{8}{12} + \frac{2}{12} \\ &= \frac{9 - 8 + 2}{12} = \frac{11 - 8}{12} \\ &= \frac{3}{12} = \frac{1 \times 3}{4 \times 3} = \frac{1}{4}.\end{aligned}$$

End Quiz

**Solution to Quiz:** The *least common denominator* of the two fractions is 15. Writing both fractions with this denominator gives

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15},$$
$$\frac{4}{5} = \frac{4 \times 3}{5 \times 3} = \frac{12}{15}.$$

The fraction mid-way between 10/15 and 12/15 is 11/15.

End Quiz