



Indefinite Integration

R Horan & M Lavelle

The aim of this package is to provide a short self assessment programme for students who want to be able to calculate basic indefinite integrals.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page [Mathematics Support Materials](#).

1. Anti-Derivatives

If $f = \frac{dF}{dx}$, we call F the **anti-derivative** (or **indefinite integral**) of f .

Example 1 If $f(x) = x$, we can find its anti-derivative by realising that for $F(x) = \frac{1}{2}x^2$

$$\frac{dF}{dx} = \frac{d}{dx}\left(\frac{1}{2}x^2\right) = \frac{1}{2} \times 2x = x = f(x)$$

Thus $F(x) = \frac{1}{2}x^2$ is an anti-derivative of $f(x) = x$.

However, if C is a constant:

$$\frac{d}{dx}\left(\frac{1}{2}x^2 + C\right) = \frac{1}{2} \times 2x = x$$

since the derivative of a constant is zero. The **general anti-derivative** of x is thus $\frac{1}{2}x^2 + C$ where C can be *any* constant.

Note that you should **always check** an anti-derivative F by differentiating it and seeing that you recover f .

Quiz Using $\frac{d(x^n)}{dx} = nx^{n-1}$, select an anti-derivative of x^6

- (a) $6x^5$ (b) $\frac{1}{5}x^5$ (c) $\frac{1}{7}x^7$ (d) $\frac{1}{6}x^7$

In general the anti-derivative or integral of x^n is:

$$\text{If } f(x) = x^n, \text{ then } F(x) = \frac{1}{n+1}x^{n+1} \text{ for } n \neq -1$$

N.B. this rule does not apply to $1/x = x^{-1}$. Since the derivative of $\ln(x)$ is $1/x$, the anti-derivative of $1/x$ is $\ln(x)$ – see later.

Also **note** that since $1 = x^0$, the rule says that the anti-derivative of 1 is x . This is correct since the derivative of x is 1.

We will now introduce **two important properties of integrals**, which follow from the corresponding rules for derivatives.

If **a is any constant** and $F(x)$ is the anti-derivative of $f(x)$, then

$$\frac{d}{dx}(aF(x)) = a\frac{d}{dx}F(x) = af(x).$$

Thus

$aF(x)$ is the anti-derivative of $af(x)$

Quiz Use this property to select the **general anti-derivative** of $3x^{\frac{1}{2}}$ from the choices below.

- (a) $2x^{\frac{3}{2}} + C$ (b) $\frac{3}{2}x^{-\frac{1}{2}} + C$ (c) $\frac{9}{2}x^{\frac{3}{2}} + C$ (d) $6\sqrt{x} + C$

If $\frac{dF}{dx} = f(x)$ and $\frac{dG}{dx} = g(x)$, from the sum rule of differentiation

$$\frac{d}{dx}(F + G) = \frac{d}{dx}F + \frac{d}{dx}G = f(x) + g(x).$$

(See the package on the **product and quotient rules**.) This leads to the **sum rule for integration**:

If $F(x)$ is the anti-derivative of $f(x)$ and $G(x)$ is the anti-derivative of $g(x)$, then $F(x) + G(x)$ is the anti-derivative of $f(x) + g(x)$.

Only one arbitrary constant C is needed in the anti-derivative of the sum of two (or more) functions.

Quiz Use this property to find the general **anti-derivative** of $3x^2 - 2x^3$.

(a) C (b) $x^3 - \frac{1}{2}x^4 + C$ (c) $\frac{3}{2}x^3 - \frac{2}{3}x^4 + C$ (d) $x^3 + \frac{2}{3}x + C$

We now introduce the integral notation to represent anti-derivatives.

2. Indefinite Integral Notation

The notation for an anti-derivative or indefinite integral is:

$$\text{if } \frac{dF}{dx} = f(x), \quad \text{then } \int f(x) dx = F(x) + C$$

Here \int is called the **integral sign**, while dx is called the **measure** and C is called the **integration constant**. We read this as “the integral of f of x with respect to x ” or “the integral of f of x dx ”.

In other words $\int f(x) dx$ means the **general anti-derivative of $f(x)$** *including* an integration constant.

Example 2 To calculate the integral $\int x^4 dx$, we recall that the anti-derivative of x^n for $n \neq -1$ is $x^{n+1}/(n+1)$. Here $n = 4$, so we have

$$\int x^4 dx = \frac{x^{4+1}}{4+1} + C = \frac{x^5}{5} + C$$

Quiz Select the correct result for the indefinite integral $\int \frac{1}{\sqrt{x}} dx$

- (a) $-\frac{1}{2}x^{-\frac{3}{2}} + C$ (b) $2\sqrt{x} + C$ (c) $\frac{1}{2}x^{\frac{1}{2}} + C$ (d) $\frac{2}{\sqrt{x^2}} + C$

The previous **rules** for anti-derivatives may be expressed in integral notation as follows.

The integral of a function multiplied by any constant a is:

$$\int a f(x) dx = a \int f(x) dx$$

The sum rule for integration states that:

$$\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$$

To be able to integrate a greater number of functions, it is convenient first to recall the **derivatives of some simple functions**:

y	$\sin(ax)$	$\cos(ax)$	e^{ax}	$\ln(x)$
$\frac{dy}{dx}$	$a \cos(ax)$	$-a \sin(ax)$	$a e^{ax}$	$\frac{1}{x}$

EXERCISE 1. From the above table of derivatives calculate the indefinite integrals of the following functions: (click on the **green** letters for the solutions)

(a) $\sin(ax)$,

(b) $\cos(ax)$,

(c) e^{ax} ,

(d) $\frac{1}{x}$

These results give the following table of indefinite integrals (the integration constants are omitted for reasons of space):

$y(x)$	x^n ($n \neq -1$)	$\sin(ax)$	$\cos(ax)$	e^{ax}	$\frac{1}{x}$
$\int y(x)dx$	$\frac{1}{n+1}x^{n+1}$	$-\frac{1}{a}\cos(ax)$	$\frac{1}{a}\sin(ax)$	$\frac{1}{a}e^{ax}$	$\ln(x)$

EXERCISE 2. From the above table, calculate the following integrals: (click on the **green** letters for the solutions)

(a) $\int x^7 dx$,

(b) $\int 2 \sin(3x) dx$,

(c) $\int 4 \cos(2x) dx$,

(d) $\int 15 e^{-5s} ds$,

(e) $\int \frac{3}{w} dw$,

(f) $\int (e^s + e^{-s}) ds$.

Quiz Select the indefinite integral of $4 \sin(5x) + 5 \cos(3x)$.

- (a) $20 \cos(5x) - 15 \sin(3x) + C$ (b) $4 \sin\left(\frac{5x^2}{2}\right) + 5 \cos\left(\frac{3x^2}{2}\right) + C$
(c) $-\frac{2}{3} \cos(5x) + \frac{5}{4} \sin(3x) + C$ (d) $-\frac{4}{5} \cos(5x) + \frac{5}{3} \sin(3x) + C$

EXERCISE 3. It may be shown that

$$\frac{d}{dx} [x(\ln(x) - 1)] = \ln(x).$$

(See the package on the **product and quotient rules** of differentiation.) From this result and the properties reviewed in the package on **logarithms** calculate the following integrals: (click on the **green** letters for the solutions)

- (a) $\int \ln(x) dx$, (b) $\int \ln(2x) dx$,
(c) $\int \ln(x^3) dx$, (d) $\int \ln(3x^2) dx$.

Hint expressions like $\ln(2)$ are constants!

3. Fixing Integration Constants

Example 3 Consider a rocket whose velocity in metres per second at time t seconds after launch is $v = bt^2$ where $b = 3 \text{ ms}^{-3}$. If at time $t = 2$ s the rocket is at a position $x = 30$ m away from the launch position, we can calculate its position at time t s as follows.

Velocity is the derivative of position with respect to time: $v = \frac{dx}{dt}$, so it follows that x is the integral of v ($= bt^2 \text{ ms}^{-1}$):

$$x = \int 3t^2 dt = 3 \times \frac{1}{3}t^3 + C = t^3 + C$$

The information that $x = 30$ m at $t = 2$ s, can be substituted into the above equation to find the value of C :

$$30 = 2^3 + C$$

$$30 = 8 + C$$

$$\text{i.e., } 22 = C.$$

Thus at time t s, the rocket is at $x = t^3 + 22$ m from the launch site.

Quiz If $y = \int 3x dx$ and at $x = 2$, it is measured that $y = 4$, calculate the integration constant.

- (a) $C = 2$ (b) $C = 4$ (c) $C = -2$ (d) $C = 10$

Quiz Find the position of an object at time $t = 4$ s if its velocity is $v = \alpha t + \beta \text{ ms}^{-1}$ for $\alpha = 2 \text{ ms}^{-2}$ and $\beta = 1 \text{ ms}^{-1}$ and its position at $t = 1$ s was $x = 2$ m.

- (a) 12 m (b) 24 m (c) 0 m (d) 20 m

Quiz Acceleration a is the rate of change of velocity v with respect to time t , i.e., $a = \frac{dv}{dt}$.

If a ball is thrown upwards on the Earth, its acceleration is constant and approximately $a = -10 \text{ ms}^{-2}$. If its initial velocity was 3 ms^{-1} , when does the ball stop moving upwards (i.e., at what time is its velocity zero)?

- (a) 0.3 s (b) 1 s (c) 0.7 s (d) 0.5 s

4. Final Quiz

Begin Quiz Choose the solutions from the options given.

- Which of the following is an anti-derivative with respect to x of $f(x) = 2 \cos(3x)$?
(a) $2x \cos(3x)$ (b) $-6 \sin(3x)$ (c) $\frac{2}{3} \sin(3x)$ (d) $\frac{2}{3} \sin(\frac{3}{2}x^2)$
- What is the integral with respect to x of $f(x) = 11 \exp(10x)$?
(a) $\frac{11}{10} \exp(10x) + C$ (b) $11 \exp(5x^2) + C$
(c) $\exp(11x) + C$ (d) $110 \exp(10x) + C$
- If the speed of an object is given by $v = bt^{-\frac{1}{2}} \text{ ms}^{-1}$ for $b = 1 \text{ ms}^{-\frac{1}{2}}$, what is its position x at time $t = 9 \text{ s}$ if the object was at $x = 3 \text{ m}$ at $t = 1 \text{ s}$?
(a) $x = 7 \text{ m}$ (b) $x = 11 \text{ m}$ (c) $x = 4 \text{ m}$ (d) $x = 0 \text{ m}$

End Quiz

Solutions to Exercises

Exercise 1(a) To calculate the indefinite integral $\int \sin(ax) dx$ let us use the table of derivatives to find the function whose derivative is $\sin(ax)$.

From the table one can see that if $y = \cos(ax)$, then its derivative with respect to x is

$$\frac{d}{dx} (\cos(ax)) = -a \sin(ax), \quad \text{so} \quad \frac{d}{dx} \left(-\frac{1}{a} \cos(ax) \right) = \sin(ax).$$

Thus one can conclude

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + C.$$

Click on the **green** square to return



Exercise 1(b) We have to find the indefinite integral of $\cos(ax)$. From the table of derivatives we have

$$\frac{d}{dx}(\sin(ax)) = a \cos(ax), \quad \text{so} \quad \frac{d}{dx} \left(\frac{1}{a} \sin(ax) \right) = \cos(ax).$$

This implies

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax) + C.$$

Click on the **green** square to return



Exercise 1(c) We have to find the integral of e^{ax} . From the table of derivatives

$$\frac{d}{dx}(e^{ax}) = a e^{ax}, \quad \text{so} \quad \frac{d}{dx} \left(\frac{1}{a} e^{ax} \right) = e^{ax}.$$

Thus the indefinite integral of e^{ax} is

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C.$$

Click on the **green** square to return



Exercise 1(d) We need to find the function whose derivative is $\frac{1}{x}$. From the table of derivatives we see that the derivative of $\ln(x)$ with respect to x is

$$\frac{d}{dx} (\ln(x)) = \frac{1}{x}.$$

This implies that

$$\int \frac{1}{x} dx = \ln(x) + C.$$

Click on the **green** square to return



Exercise 2(a) We want to calculate $\int x^7 dx$. From the table of indefinite integrals, for any $n \neq -1$,

$$\int x^n dx = \frac{1}{n+1} x^{n+1}.$$

In the case of $n = 7 (\neq -1)$,

$$\begin{aligned} \int x^7 dx &= \frac{1}{7+1} \times x^{7+1} + C \\ &= \frac{1}{8} x^8 + C. \end{aligned}$$

Checking this:

$$\frac{d}{dx} \left(\frac{1}{8} x^8 + C \right) = \frac{1}{8} \frac{d}{dx} x^8 = \frac{1}{8} \times 8 x^7 = x^7.$$

Click on the **green** square to return



Exercise 2(b) To calculate the integral $\int 2 \sin(3x) dx$ we use the formula

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax).$$

In our case $a = 3$. Thus we have

$$\begin{aligned} \int 2 \sin(3x) dx &= 2 \int \sin(3x) dx = 2 \times \left(-\frac{1}{3} \cos(3x) \right) + C \\ &= -\frac{2}{3} \cos(3x) + C. \end{aligned}$$

Checking:

$$\frac{d}{dx} \left(-\frac{2}{3} \cos(3x) + C \right) = -\frac{2}{3} \frac{d}{dx} \cos(3x) = -\frac{2}{3} \times (-3 \sin(3x)) = 2 \sin(3x).$$

Click on the **green** square to return



Exercise 2(c) To calculate the integral $\int 4 \cos(2x) dx$ we use the formula

$$\int \cos(ax) dx = \frac{1}{a} \sin(ax),$$

with $a = 2$. This yields

$$\begin{aligned} \int 4 \cos(2x) dx &= 4 \int \cos(2x) dx \\ &= 4 \times \left(\frac{1}{2} \sin(2x) \right) \\ &= 2 \sin(2x) + C. \end{aligned}$$

It may be checked that

$$\frac{d}{dx} (2 \sin(2x) + C) = 2 \frac{d}{dx} \sin(2x) = 2 \times (2 \cos(2x)) = 4 \cos(2x).$$

Click on the **green** square to return



Exercise 2(d) To find the integral $\int 15 e^{-5s} ds$ we use the formula

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

with $a = -5$. This gives

$$\begin{aligned}\int 15e^{-5s} ds &= 15 \int e^{-5s} ds \\ &= 15 \times \left(-\frac{1}{5} e^{-5s} \right) \\ &= -3 e^{-5s} + C,\end{aligned}$$

and indeed

$$\frac{d}{ds} (-3 e^{-5s} + C) = -3 \frac{d}{ds} e^{-5s} = -3 \times (-5e^{-5s}) = 15e^{-5s}.$$

Click on the **green** square to return



Exercise 2(e) To find the integral $\int \frac{3}{w} dw$ we use the formula

$$\int \frac{1}{x} dx = \ln(x).$$

Thus we have

$$\begin{aligned} \int \frac{3}{w} dw &= \int 3 \times \frac{1}{w} dw &= 3 \int \frac{1}{w} dw \\ & &= 3 \ln(w) + C. \end{aligned}$$

This can be checked as follows

$$\frac{d}{dw} (3 \ln(w) + C) = 3 \frac{d}{dw} \ln(w) = 3 \times \frac{1}{w} = \frac{3}{w}.$$

Click on the **green** square to return



Exercise 2(f) To find the integral $\int(e^s + e^{-s}) ds$ we use the **sum rule for integrals**, rewriting it as the sum of two integrals

$$\int(e^s + e^{-s}) ds = \int e^s ds + \int e^{-s} ds$$

and then use

$$\int e^{ax} dx = \frac{1}{a} e^{ax}.$$

Take $a = 1$ in the first integral and $a = -1$ in the second integral. This implies

$$\begin{aligned} \int(e^s + e^{-s}) ds &= \int e^s ds + \int e^{-s} ds \\ &= e^s + \left(\frac{1}{-1}\right) e^{-s} + C \\ &= e^s - e^{-s} + C. \end{aligned}$$

Click on the **green** square to return



Exercise 3(a) To calculate the indefinite integral $\int \ln(x) dx$ we have to find the function whose derivative is $\ln(x)$. We are given

$$\frac{d}{dx} [x(\ln(x) - 1)] = \ln(x).$$

This implies

$$\int \ln(x) dx = x [\ln(x) - 1] + C.$$

This can be checked by differentiating $x [\ln(x) - 1] + C$ using the **product rule**. (See the package on the **product and quotient rules** of differentiation.)

Click on the **green** square to return



Exercise 3(b) To calculate the indefinite integral $\int \ln(2x) dx$ we recall the following property of **logarithms**

$$\ln(ax) = \ln(a) + \ln(x)$$

and then use the integral $\int \ln(x) dx = x [\ln(x) - 1] + C$ calculated in Exercise 3(a). This gives

$$\begin{aligned} \int \ln(2x) dx &= \int (\ln(2) + \ln(x)) dx \\ &= \ln(2) \times \int 1 dx + \int \ln(x) dx \\ &= x \ln(2) + x (\ln(x) - 1) + C \\ &= x (\ln(2) + \ln(x) - 1) + C \\ &= x (\ln(2x) - 1) + C. \end{aligned}$$

In the last line we used $\ln(2) + \ln(x) = \ln(2x)$.

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Exercise 3(c) To calculate the indefinite integral $\int \ln(x^3) dx$ we first recall from the package on **logarithms** that

$$\ln(x^n) = n \ln(x)$$

and the integral

$$\int \ln(x) dx = x [\ln(x) - 1] + C$$

calculated in Exercise 3(a). This all gives

$$\begin{aligned} \int \ln(x^3) dx &= \int (3 \ln(x)) dx \\ &= 3 \times \int \ln(x) dx \\ &= 3x (\ln(x) - 1) + C. \end{aligned}$$

Click on the **green** square to return



Exercise 3(d) Using the rules from the package on **logarithms**, $\ln(3x^2)$ may be simplified

$$\ln(3x^2) = \ln(3) + \ln(x^2) = \ln(3) + 2\ln(x).$$

Thus

$$\begin{aligned}\int \ln(3x^2) dx &= \int (\ln(3) + 2\ln(x)) dx \\ &= \ln(3) \times \int 1 dx + 2 \times \int \ln(x) dx \\ &= \ln(3)x + 2x [\ln(x) - 1] + C \\ &= x [\ln(3) + 2\ln(x) - 2] + C \\ &= x [\ln(3x^2) - 2] + C,\end{aligned}$$

where the final expression for $\ln(3x^2)$ is obtained by using the rules of logarithms.

Click on the **green** square to return



Solutions to Quizzes

Solution to Quiz: To find an anti-derivative of x^6 first calculate the derivative of $F(x) = \frac{1}{7}x^7$. Using the basic formula

$$\frac{d}{dx}x^n = nx^{n-1}$$

with $n = 7$

$$\frac{dF}{dx} = \frac{d}{dx} \left(\frac{1}{7}x^7 \right) \quad (1)$$

$$= \frac{1}{7} \frac{d}{dx} (x^7) \quad (2)$$

$$= \frac{1}{7} \times 7x^{7-1} \quad (3)$$

$$= x^6. \quad (4)$$

This result shows that the function $F(x) = \frac{1}{7}x^7 + C$ is the general anti-derivative of $f(x) = x^6$. End Quiz

Solution to Quiz: To find the general anti-derivative of $3x^{\frac{1}{2}}$, recall that for constant a the anti-derivative of $af(x)$ is $aF(x)$, where $F(x)$ is the anti-derivative of $f(x)$.

Thus the anti-derivative of $3x^{\frac{1}{2}}$ is $3 \times$ (the anti-derivative of $x^{\frac{1}{2}}$).

To calculate the anti-derivative of $x^{\frac{1}{2}}$ we recall the anti-derivative of $f(x) = x^n$ is $F(x) = \frac{1}{n+1}x^{n+1}$ for $n \neq -1$. In our case $n = \frac{1}{2}$ and therefore this result can be used. The anti-derivative of $x^{\frac{1}{2}}$ is thus

$$\frac{1}{\frac{1}{2} + 1} x^{(\frac{1}{2}+1)} = \frac{1}{3/2} x^{3/2} = 1 \times \frac{2}{3} x^{\frac{3}{2}} = \frac{2}{3} x^{\frac{3}{2}}.$$

Thus the general anti-derivative of $3x^{\frac{1}{2}}$ is $3 \times \frac{2}{3} x^{\frac{3}{2}} + C = 2x^{\frac{3}{2}} + C$.

This result may be checked by differentiating $F(x) = 2x^{3/2} + C$.

End Quiz

Solution to Quiz: To find the general anti-derivative of $3x^2 - 2x^3$, we use the **sum rule** for anti-derivatives. The anti-derivative of $3x^2 - 2x^3$ is (anti-derivative of $3x^2$) - (anti-derivative of $2x^3$). Since the anti-derivative of $f(x) = x^n$ is $F(x) = \frac{1}{n+1}x^{n+1}$ for $n \neq -1$, for $n = 2$:

$$\text{anti-derivative of } x^2 = \frac{1}{2+1}x^{2+1} = \frac{1}{3}x^3.$$

Thus the anti-derivative of $3x^2$ is

$$3 \times (\text{anti-derivative of } x^2) = 3 \times \frac{1}{3}x^3 = x^3.$$

Similarly the anti-derivative of $2x^3$ is

$$2 \times (\text{anti-derivative of } x^3) = 2 \times \frac{1}{3+1}x^{3+1} = \frac{1}{2}x^4.$$

Putting these results together we find that the general anti-derivative of $3x^2 - 2x^3$ is

$$F(x) = x^3 - \frac{1}{2}x^4 + C,$$

which may be confirmed by differentiation.

End Quiz

Solution to Quiz: To calculate the indefinite integral

$$\int \frac{1}{\sqrt{x}} dx = \int \frac{1}{x^{1/2}} dx = \int x^{-1/2} dx$$

we recall the basic result, that the anti-derivative of $f(x) = x^n$ is $F(x) = \frac{1}{n+1}x^{n+1}$ for $n \neq -1$. In this case $n = -\frac{1}{2}$ and so

$$\begin{aligned} \int x^{-1/2} dx &= \frac{1}{-\frac{1}{2} + 1} x^{(-\frac{1}{2} + 1)} + C = \frac{1}{\frac{1}{2}} x^{\frac{1}{2}} + C \\ &= 1 \times \frac{2}{1} x^{\frac{1}{2}} + C = 2x^{\frac{1}{2}} + C \\ &= 2\sqrt{x} + C, \end{aligned}$$

where we recall that dividing by a fraction is equivalent to multiplying by its inverse (see the package on **fractions**). End Quiz

Solution to Quiz: To evaluate $\int(4 \sin(5x) + 5 \cos(3x)) dx$ we use the **sum rule** for indefinite integrals to rewrite the integral as the sum of two integrals. Using

$$\int \sin(ax) dx = -\frac{1}{a} \cos(ax) \quad \text{and} \quad \int \cos(ax) dx = \frac{1}{a} \sin(ax)$$

we get

$$\begin{aligned} \int(4 \sin(5x) + 5 \cos(3x)) dx &= 4 \int \sin(5x) dx + 5 \int \cos(3x) dx \\ &= 4 \times \left(-\frac{1}{5}\right) \cos(5x) + 5 \times \frac{1}{3} \sin(3x) + C \\ &= -\frac{4}{5} \cos(5x) + \frac{5}{3} \sin(3x) + C. \end{aligned}$$

This can be checked by differentiation.

End Quiz

Solution to Quiz: If $y = \int 3x \, dx$ and at $x = 2$, $y = 4$ then

$$\begin{aligned}y &= \int 3x \, dx &= 3 \times \int x \, dx \\ & &= 3 \times \frac{1}{2} x^{1+1} + C \\ & &= \frac{3}{2} x^2 + C\end{aligned}$$

is the general solution. Substituting $x = 2$ and $y = 4$ into the above equation, the value of C is obtained

$$\begin{aligned}4 &= \frac{3}{2} \times (2)^2 + C \\ 4 &= 6 + C \\ \text{i.e., } C &= -2.\end{aligned}$$

Therefore, for all x , $y = \frac{3}{2}x^2 - 2$.

End Quiz

Solution to Quiz:

We are told that $v = \alpha t + \beta$ with $\alpha = 2\text{ms}^{-2}$, $\beta = 1\text{ms}^{-1}$ and at $t = 1\text{s}$, $x = 2\text{m}$. Since x is the integral of v :

$$x = \int v dt = \int (2t + 1) dt = 2 \times \int t dt + \int 1 dt = t^2 + t + C.$$

The position at time $t = 1\text{s}$ was $x = 2\text{m}$ so these values may be substituted into the above equation to find C :

$$2 = 1^2 + 1 + C$$

$$2 = 2 + C$$

$$\text{i.e., } 0 = C.$$

Therefore, for all t , $x = t^2 + t + 0 = t^2 + t$. At $t = 4\text{s}$,

$$x = (4)^2 + 4 = 16 + 4 = 20 \text{ m.}$$

End Quiz

Solution to Quiz: We are given $a = \frac{dv}{dt} = -10\text{ms}^{-2}$ and initial velocity $v = 3\text{ms}^{-1}$, and want to find when the velocity is zero. Since $a = \frac{dv}{dt}$, velocity is the integral of acceleration, $v = \int a dt$. The acceleration of the ball is constant, $a = -10\text{ms}^{-2}$, so that

$$v = \int (-10) dt = -10 \times \int dt = -10t + C.$$

At $t = 0$, $v = 3\text{ms}^{-1}$, so these values may be substituted into the above equation to find the constant C :

$$\begin{aligned}3 &= -10 \times 0 + C \\3 &= C.\end{aligned}$$

Thus $v = -10t + 3$ for this problem. Therefore if $v = 0$

$$\begin{aligned}0 &= -10t + 3 \\10t &= 3, \quad t = 3/10.\end{aligned}$$

The ball stops moving upwards at 0.3 s.

End Quiz