Basic Physics

Pressure

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The aim of this package is to provide a short self assessment programme for students who want to understand pressure and some of its elementary applications.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Introduction (Density)

Density (symbol ρ) is defined by

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{V}\,.$$

Example 1 A cubic metre of gold has a *mass* of $m \approx 2 \times 10^4\,\text{kg}$. Hence the density of gold is given by mass over volume:

$$\rho_{\text{gold}} = \frac{m}{V} = \frac{2 \times 10^4\,\text{kg}}{1\,\text{m}^3} = 2 \times 10^4\,\text{kg}\,\text{m}^{-3}\,.$$

Example 2 The *density of water* is $1\,\text{g}\,\text{cm}^{-3}$, i.e., a cubic centimetre of water has a mass of one gram. This can be expressed in **S.I. units** (see the packages on **Units** and on **Changing Units**) as follows. One gram is $10^{-3}\,\text{kg}$ and one centimetre is $10^{-2}\,\text{m}$. Therefore $1\,\text{cm}^3 = (10^{-2})^3 = 10^{-6}\,\text{m}^3$. This implies (see the package on **Powers**):

$$\rho_{\text{water}} = \frac{1\,\text{g}}{1\,\text{cm}^3} = \frac{10^{-3}\,\text{kg}}{10^{-6}\,\text{m}^3} = 10^3\,\text{kg}\,\text{m}^{-3}\,.$$

Exercise 1.

(a) A rectangular metal block with mass $18\,\text{kg}$ has dimensions $0.1\,\text{m} \times 0.15\,\text{m} \times 0.2\,\text{m}$. Calculate the volume of the block and thus its density.

(b) The density of lead is just over $1 \times 10^4\,\text{kg}\,\text{m}^{-3}$. Find the mass of a cylindrical rod of length $\ell = 0.5\,\text{m}$ and radius $r = 0.02\,\text{cm}$. (The volume of a cylinder is $V = \pi r^2 \ell$.)

(c) What is the volume of $4\,\text{kg}$ of water in litres? ($1\,\text{L} = 10^{-3}\,\text{m}^3$.)

(d) The density of mercury is $\rho_{\text{merc}} = 13,600\,\text{kg}\,\text{m}^{-3}$, what is the volume of $4\,\text{kg}$ of mercury?

Quiz The **specific gravity** of a substance is the dimensionless ratio:

$$\text{specific gravity} = \frac{\rho}{\rho_{\text{water}}}$$

where ρ is the density of the substance. Such ratios are just numbers and have no units. Select the specific gravity of mercury.

(a) 136 (b) 0.074 (c) 1.36×10^3 (d) 13.6

2. Pressure

People walking on ice are advised to use snowshoes so as to spread their weight (a force) over a larger area of ice. The force per unit area is called the **pressure**, P:

$$\text{pressure} = \frac{\text{force}}{\text{area}} \,.$$

Snowshoes decrease the pressure exerted on the ice. The *S.I. units of pressure* are newtons per square metre ($\mathrm{N\,m^{-2}}$) which are also called *pascals* (symbol Pa, see the packages on **Forces** and **Units**).

Example 3 If your mass is $70\,\mathrm{kg}$ and the total area of the soles of your feet is $0.2\,\mathrm{m}^2$, what pressure would you exert on the ground?

The force on the ground is your weight: $F = mg = 70 \times 9.8 = 686\,\mathrm{N}$ (we take g, the acceleration due to gravity, to be approximately $9.8\,\mathrm{ms^{-2}}$). Hence to two significant figures the pressure is

$$P = \frac{F}{A} = \frac{70 \times 9.8}{0.2} = 3,400\,\mathrm{Pa} \,.$$

Quiz Find the pressure from a force of $100\,\mathrm{N}$ on an area of $0.25\mathrm{m}^2$?

(a) $400\,\mathrm{Pa}$ (b) $25\,\mathrm{Pa}$ (c) $4,000\,\mathrm{Pa}$ (d) $2.5\,\mathrm{Pa}$

Exercise 2.

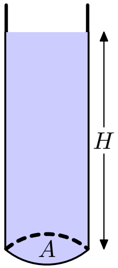
(a) If a pressure gauge measures an increase in $3 \times 10^4\,\mathrm{Pa}$ on an area of $0.07\,\mathrm{m}^2$ what is the increase in the force applied to the area?

(b) Find the pressure produced by a kilogram of lead on a horizontal surface if the area it rests on is $0.02\,\mathrm{m}^2$?

(c) A scuba diver measures an increase in pressure of around $10^5\,\mathrm{Pa}$ upon descending by $10\,\mathrm{m}$, what is the change in force per square centimetre on the diver's body?

To understand the pressures in liquids it is important to know that **liquids are not easily compressible**. This means that the density of liquids does not change when forces are applied to them. (E.g., the density of sea water does not vary much with depth.)

To find the **pressure at the bottom of a column of liquid** of density ρ, height H and area A, consider the diagram below:



Volume of liquid: $V = HA$
Density of liquid, ρ, so its mass: $m = V\rho = HA\rho$

Thus the **pressure exerted by the liquid** is

$$\begin{aligned} P &= \frac{\text{weight}}{\text{area}} = \frac{mg}{A} \\ &= \frac{HA\rho g}{A} = H\rho g \end{aligned}$$

where $g = 9.8\,\text{ms}^{-2}$ is the acceleration due to gravity. Note that the area of the column has cancelled – the pressure only depends on the height of the column.

The column of atmosphere above us also weighs downwards and produces **atmospheric pressure**. Atmospheric pressure on Earth is $101.3\,\mathrm{kPa}$. Most pressure gauges measure the pressure above or below atmospheric pressure. This difference is called **gauge pressure**. The total pressure is called **absolute pressure**.

Example 4 If the density of sea water is $\rho = 1,030\,\mathrm{kg\,m^{-3}}$, what is the pressure at $10\,\mathrm{m}$ below sea level?

From $P = H\rho g$, the pressure at $10\,\mathrm{m}$ is given by

$$P = 10 \times 1030 \times 9.8 = 100,940\,\mathrm{Pa}\,.$$

Thus the **gauge pressure** (i.e., from the sea water) is $P = 101\,\mathrm{kPa}$ which is very close to the pressure exerted by the atmosphere.

The **absolute pressure** $10\,\mathrm{m}$ under the sea is the sum of the gauge pressure and the atmospheric pressure:

$$P_{\mathrm{abs}} = 101 + 103 = 204\,\mathrm{kPa}\,.$$

Exercise 3.

(a) What is the absolute pressure $30\,\mathrm{m}$ below the surface of the sea?

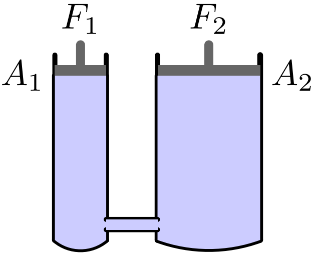
(b) At what depth below the surface of the sea is absolute pressure three times atmospheric pressure?

(c) The density of mercury is $\rho_{\mathrm{merc}} = 13,600\,\mathrm{kg\,m^{-3}}$, what is the gauge pressure under ten metres of mercury?

(d) At what depth of mercury would the absolute pressure be twice atmospheric pressure?

The origin of pressure in fluids: gases, such as our atmosphere, or liquids, such as the oceans, are made up of huge numbers of minute atoms and molecules moving around very quickly. As well as colliding with each other they hit the edges of any solid object (such as your body). These collisions exert a force upon the object and are the source of air pressure or water pressure.

3. Hydraulics



In a **hydraulic lift**, when a force, F_1, is exerted on a piston of area A_1, this produces a force, F_2, on a piston of area A_2, given by:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} .$$

Proof When a force, F_1, is exerted on the left hand piston the extra pressure, ΔP, on the piston on the left is:

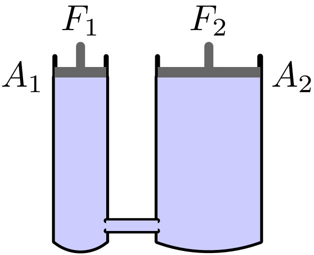
$$\Delta P = \frac{F_1}{A_1}$$

this is transmitted through the liquid to the other piston

$$\Delta P = \frac{F_2}{A_2}$$

which proves the relation.

Example 5 If a force $F_1 = 100\,\mathrm{N}$ is applied on the left in the hydraulic lift shown, where $A_1 = 0.02\,\mathrm{m}^2$ and $A_2 = 0.08\,\mathrm{m}^2$, what force F_2 will be produced?



Since the pressure exerted by the force F_1, on the left, is transmitted by the liquid through to the other piston, we have

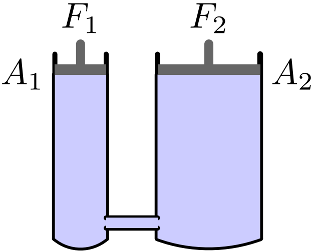
$$\begin{aligned} \frac{F_2}{A_2} &= \frac{F_1}{A_1} \\ \therefore\ F_2 &= \frac{F_1 A_2}{A_1} \\ &= \frac{100 \times 0.08}{0.02} = 100 \times 4 = 400\,N. \end{aligned}$$

Thus the force on the right is four times larger.

Quiz If in a hydraulic lift, one of the areas is three times larger than the other and a force of 15 N is applied to the smaller area, what force will be measured at the larger area?

(a) 15 N (b) 5 N (c) 18 N (d) 45 N

EXERCISE 4. These questions refer to the lift in the diagram.



(a) If $F_1 = 350\,\mathrm{N}$, $A_1 = 0.7\,\mathrm{m}^2$ and $A_2 = 1.2\,\mathrm{m}^2$ what is F_2?

(b) If $F_1 = 210\,\mathrm{N}$, $A_1 = 0.3\,\mathrm{m}^2$ and $F_2 = 500\,\mathrm{N}$ what is A_2?

(c) If $F_1 = 1,000\,\mathrm{N}$, $A_1 = 1\,\mathrm{m}^2$ and $A_2 = 10,000\,\mathrm{cm}^2$ what is F_2?

(d) What must F_1 be to exert a force $F_2 = 10,000\,\mathrm{N}$ if $A_1 = 0.75\,\mathrm{m}^2$ and $A_2 = 3\,\mathrm{m}^2$?

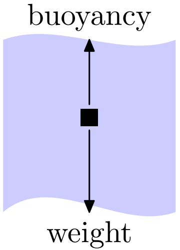
4. Archimedes' Principle

If an object is in a liquid its weight pulls it down, but the pressure of the liquid around the object exerts an upwards force on it. **Archimedes' principle** states that:

> The upthrust or buoyancy force on a submerged object is equal to the weight of the liquid displaced by the object.

There are three alternatives:

1) Weight of displaced liquid is equal to weight of object: the object is in equilibrium and will not move.
2) Weight of displaced liquid is greater than the weight of object: the object will rise.
3) Weight of displaced liquid is less than the weight of object: the object will sink.



Example 6 The mass of the *Titanic* was roughly $40,000\,\text{tonnes}$, how much water did it displace?
Since a tonne is $1,000\,\text{kg}$, the mass of the Titanic, in kilograms, was approximately $m = 40,000 \times 1,000 = 4 \times 10^7\,\text{kg}$. To float, its weight had to be equal to the weight of displaced water, i.e., it needed to displace its own mass of water. The density of water is $\rho_{\text{water}} = 1,000\,\text{kg}\,\text{m}^{-3}$, thus the volume of water it displaced was

$$V = \frac{m}{\rho} = \frac{4 \times 10^7\,\text{kg}}{1,000\,\text{kg}\,\text{m}^{-3}} = 4 \times 10^4\,\text{m}^3\,.$$

Exercise 5.

(a) The mass (or displacement) of the fully loaded Titanic was $53,147\,\text{tonnes}$. How much extra water did it displace?

(b) Find the upthrust on an object of volume $0.02\,\text{m}^3$ submerged in water. What would it be if it were submerged in mercury?

(c) How heavy would this object need to be to sink in mercury? ($\rho_{\text{merc}} = 13,600\,\text{kg}\,\text{m}^{-3}$.)

5. Final Quiz

Begin Quiz

1. What is the volume occupied by a kilogram of gold? (Recall that $\rho_{\text{gold}} = 2 \times 10^4 \text{ kg m}^{-3}$.)

(a) $5 \times 10^{-3} \text{ m}^3$ (b) $2 \times 10^4 \text{ m}^3$ (c) $5 \times 10^{-5} \text{ m}^3$ (d) $5 \times 10^{-4} \text{ m}^3$

2. One atmosphere of pressure is defined as 101.325 kPa. From the answers below, select the closest approximation to the mass of air above a square metre of the earth.

(a) 10^4 kg (b) 10^5 kg (c) 10^2 kg (d) $1,000 \text{ kg}$

3. In a hydraulic lift a force of $5,000 \text{ N}$ is applied to a piston with area of 0.0125 m^2. If the area of the other piston is 0.25 m^2, select the force on the other piston.

(a) 250 N (b) $1,000 \text{ N}$ (c) 10^5 N (d) $25,000 \text{ N}$

End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

The volume V of a rectangular block with mass $m = 18\,\mathrm{kg}$, whose dimensions are $0.1\,\mathrm{m} \times 0.15\,\mathrm{m} \times 0.2\,\mathrm{m}$ is

$$V = 0.1\,\mathrm{m} \times 0.15\,\mathrm{m} \times 0.2\,\mathrm{m} = 0.003\,\mathrm{m}^3 = 3 \times 10^{-3}\mathrm{m}^3\,.$$

Therefore its density ρ is

$$\begin{aligned}\rho &= \frac{m}{V}\\ &= \frac{18\,\mathrm{kg}}{3 \times 10^{-3}\,\mathrm{m}^3}\\ &= \frac{18}{3} \times 10^3\,\mathrm{kg\,m}^{-3} = 6 \times 10^3\,\mathrm{kg\,m}^{-3}\,.\end{aligned}$$

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Exercise 1(b)

A cylindrical rod of length $\ell = 0.5\,\text{m}$ and radius

$$r = 0.02\,\text{cm} = 2 \times 10^{-4}\text{m}$$

has volume

$$V_{\text{cyl}} = \pi r^2 \ell = 3.14 \times 4 \times 10^{-8}\text{m} \times 0.5\,\text{m} = 6.28 \times 10^{-8}\text{m}^3\,.$$

It is made from lead (density is approximately $\rho_{\text{lead}} = 1 \times 10^4\,\text{kg}\,\text{m}^{-3}$). Thus the mass of this rod equals to

$$\begin{aligned} m &= V_{\text{cyl}} \times \rho_{\text{lead}} \\ &= 6.28 \times 10^{-8}\text{m}^3 \times 1 \times 10^4\,\text{kg}\,\text{m}^{-3} \\ &= 6.28 \times 10^{-4}\text{kg}\,. \end{aligned}$$

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Exercise 1(c)

From **Example 2** we know that the density of water is

$$\rho_{\text{water}} = 1\,\text{g}\,\text{cm}^{-3} = \frac{10^{-3}\text{kg}}{10^{-6}\,\text{m}^3} = 10^3\,\text{kg}\,\text{m}^{-3}$$

The volume of $4\,\text{kg}$ of water is thus

$$\begin{aligned} V &= \frac{m}{\rho_{\text{water}}} \\ &= \frac{4\,\text{kg}}{10^3\,\text{kg}\,\text{m}^{-3}} \\ &= 4 \times 10^{-3}\,\text{m}^3. \end{aligned}$$

Since $1\,\text{L} = 10^{-3}\,\text{m}^3$, this volume in litres is

$$V = 4 \times 10^{-3}\,\text{m}^3 = 4 \times 10^{-3} \times 10^3\text{L} = 4\,\text{L}\,.$$

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Exercise 1(d)

The volume of $m = 4\,\mathrm{kg}$ of mercury whose density is

$$\rho_{\mathrm{mercury}} = 13,600\,\mathrm{kg\,m^{-3}} = 1.36 \times 10^{4}\,\mathrm{kg\,m^{-3}}$$

can be calculated as follows

$$\begin{aligned} V &= \frac{m}{\rho_{\mathrm{mercury}}} \\ &= \frac{4\,\mathrm{kg}}{1.36 \times 10^{4}\,\mathrm{kg\,m^{-3}}} \\ &= 2.9 \times 10^{-4}\,\mathrm{m^{3}}. \end{aligned}$$

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Exercise 2(a)

When a pressure gauge measures an increase $\Delta P = 3 \times 10^4\,\mathrm{Pa}$ on an area $A = 0.07\,\mathrm{m}^2$, it means that the increase in the applied force ΔF is given by

$$\Delta P = \frac{\Delta F}{A} \qquad \therefore \quad \Delta F = \Delta P \times \; A\,.$$

Inserting the given data, we find that the increase in force is

$$\begin{aligned} \Delta F &= 3 \times 10^4\,\mathrm{Pa} \times 0.07\,\mathrm{m}^2 \\ &= 0.21 \times 10^4\,\mathrm{Pa} \times \mathrm{m}^2 \\ &= 2.1 \times 10^3\,\mathrm{N}\,. \end{aligned}$$

Recall that $1\,\mathrm{Pa}\,\mathrm{m}^2 = 1\mathrm{N}$

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Exercise 2(b)

A mass $m = 1\,\mathrm{kg}$ of lead which rests on a horizontal surface of area $A = 0.02\,\mathrm{m}^2$ exerts a pressure P due to its weight:

$$
\begin{aligned}
P &= \frac{\text{weight}}{\text{area}} \\
&= \frac{m\,g}{A} \\
&= \frac{1\,\mathrm{kg} \times 9.8\,\mathrm{ms}^{-2}}{0.02\,\mathrm{m}^2} = 490\,\mathrm{Pa}\,.
\end{aligned}
$$

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Exercise 2(c)

When descending by $10\,\mathrm{m}$ the scuba diver measures an increase in pressure $\Delta P = 10^5\,\mathrm{Pa}$. The increase in the force ΔF on an area $A = 1\,\mathrm{cm}^2$ of the diver's body is given by $\Delta F = \Delta P \times A$:

$$\Delta F = \Delta P \times A = 10^5\,\mathrm{Pa} \times 1\,\mathrm{cm}^2 = 10^5\,\mathrm{Pa} \times 10^{-4}\mathrm{m}^2 = 10\,\mathrm{N}\,.$$

Let us show that this force is just the extra weight W of the additional $h = 10\,\mathrm{m}$ water column above the diver.

Its volume is $V = hA = 10\,\mathrm{m} \times 10^{-4}\,\mathrm{m}^2 = 10^{-3}\,\mathrm{m}^3$. From the density of water, the mass of the additional column of water is thus:

$$m = V\rho_{\mathrm{water}} = 10^{-3}\,\mathrm{m}^{-3} \times 10^3\,\mathrm{kg\,m}^{-3} = 1\,\mathrm{kg}\,.$$

Hence the weight of the water is mg where $g = 9.8\,\mathrm{ms}^{-2}$. The weight of $1\,\mathrm{kg}$ is $9.8\,\mathrm{N}$. (The difference between this and the $10\,\mathrm{N}$ result shows that the diver did not measure the pressure change perfectly!)

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Exercise 3(a)

To find the gauge pressure 30 m under the surface of the sea, we use

$$P = H\rho g$$

with $H = 30\,\mathrm{m}$, $\rho = 1,030\,\mathrm{kg\,m^{-3}}$ and $g = 9.8\,\mathrm{m\,s^{-2}}$. This gives

$$P = H = 30\,\mathrm{m} \times 1,030\,\mathrm{kg\,m^{-3}} \times 9.8\,\mathrm{m\,s^{-2}} = 303,000\,\mathrm{Pa}$$

to three significant figures.

The absolute pressure 30 m under the sea is the sum of this gauge pressure and the atmospheric pressure:

$$P_{\mathrm{abs}} = 303,000 + 103,000 = 406\,\mathrm{kPa}\,.$$

We used here the fact that liquids are hard to compress, so the density of sea water does not increase much at greater depths. (For gases the density does change with altitude as gases are easy to compress.)

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Exercise 3(b)

If the absolute pressure $P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atms}}$ below sea level is three times atmospheric pressure

$$P_{\text{abs}} = 3 \times P_{\text{atms}}$$

then

$$P_{\text{gauge}} = 2 \times P_{\text{atms}} \,.$$

The depth H where this occurs can be found expressing the gauge pressure:

$$H\rho g = 2 \times P_{\text{atms}} \,, \qquad \therefore \quad H = \frac{2P_{\text{atms}}}{\rho g}$$

Using the data we have

$$H = \frac{2 \times 101 \times 10^3 \,\text{Pa}}{1.03 \times 10^3 \,\text{kg}\,\text{m}^{-3} \times 9.8\,\text{m}s^{-2}} \approx 20\,\text{m}\,.$$

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Exercise 3(c)

The gauge pressure under $H = 10\,\mathrm{m}$ of mercury is

$$\begin{aligned} P_{\mathrm{merc}} &= H\rho_{\mathrm{merc}}g \\ &= 10\,\mathrm{m} \times 1.36 \times 10^4\,\mathrm{kg\,m^{-3}} \times 9.8\,\mathrm{m}s^{-2} \\ &= 1.33 \times 10^6\,\mathrm{Pa}\,. \end{aligned}$$

This is more than ten times larger than atmospheric pressure!

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Exercise 3(d)

When the absolute pressure of mercury $P_{\rm abs} = P_{\rm gauge} + P_{\rm atms}$ is twice atmospheric pressure

$$P_{\rm abs} = 2 \times P_{\rm atms}$$

the gauge pressure is

$$P_{\rm gauge} = P_{\rm atms} \, .$$

Therefore the corresponding depth H of mercury can be found as:

$$gH\rho_{\rm merc} = P_{\rm atms} \, , \qquad \therefore \quad H = \frac{P_{\rm atms}}{g\rho_{\rm merc}}$$

Recalling the density of mercury $\rho_{\rm merc} = 1.36 \times 10^4 \, {\rm kg \, m^{-3}}$, we obtain

$$H = \frac{101 \times 10^3 \, {\rm Pa}}{1.36 \times 10^4 \, {\rm kg \, m^{-3}} \times 9.8 \, {\rm m} s^{-2}} = 0.76 \, {\rm m} \, .$$

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Exercise 4(a)

If in a hydraulic lift, a force $F_1 = 350\,\mathrm{N}$ is exerted on a piston of area $A_1 = 0.7\,\mathrm{m}^2$ and the area of the second piston is $A_2 = 1.2\,\mathrm{m}^2$ then the resulting force F_2 can be found from

$$\frac{F_2}{A_2} = \frac{F_1}{A_1}$$
$$\therefore\ F_2 = \frac{F_1 \times A_2}{A_1}$$

This yields

$$F_2 = \frac{350\,\mathrm{N} \times 1.2\,\mathrm{m}^2}{0.7\,\mathrm{m}^2} = \frac{350 \times 1.2}{0.7}\,\mathrm{N} = 600\,\mathrm{N}\,.$$

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Exercise 4(b)

If in a hydraulic lift, the force applied to the piston of area $A_1 = 0.3\,\mathrm{m}^2$ is $F_1 = 210\,\mathrm{N}$ and the resulting force is $F_2 = 500\,\mathrm{N}$ then the area A_2 of the second piston can be found as follows

$$\begin{aligned}
\frac{F_2}{A_2} &= \frac{F_1}{A_1} \\
\therefore\ A_2 &= \frac{F_2 \times A_1}{F_1} \\
&= \frac{500\,\mathrm{N} \times 0.3\,\mathrm{m}^2}{210\,\mathrm{N}} = 0.71\,\mathrm{m}^2\,.
\end{aligned}$$

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Exercise 4(c)

We are told that in a hydraulic lift, the force applied to the first piston of area $A_1 = 1\,\mathrm{m}^2$ is $F_1 = 1,000\,\mathrm{N}$ and the area of the second piston is $A_2 = 10,000\,\mathrm{cm}^2$. Since $1\,\mathrm{cm}^2 = 10^{-4}\,\mathrm{m}^2$, we have $A_2 = 1\,\mathrm{m}^2$. Thus the force F_2 produced on the second piston is given by

$$\begin{aligned} \frac{F_2}{A_2} &= \frac{F_1}{A_1} \\ \therefore\ F_2 &= \frac{F_1 \times A_2}{A_1} = \frac{1,000\,\mathrm{N} \times 1\,\mathrm{m}^2}{1\,\mathrm{m}^2} = 10^3\,\mathrm{N}\,. \end{aligned}$$

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Exercise 4(d)

To find the force F_1 which must be applied to a first piston of area $A_1 = 0.75\,\mathrm{m}^2$ in order to produce a force $F_2 = 10,000\,\mathrm{N}$ on a second piston of area $A_2 = 3\,\mathrm{m}^2$, we use the equation

$$\begin{aligned}\frac{F_2}{A_2} &= \frac{F_1}{A_1}\\ \therefore\ F_1 &= \frac{F_2 \times A_1}{A_2}\\ &= \frac{10,000\,\mathrm{N} \times 0.75\,\mathrm{m}^2}{3\,\mathrm{m}^2} = 2,500\,\mathrm{N}\,.\end{aligned}$$

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Exercise 5(a)

Since the mass of a boat is directly related to how much water it displaces, ship designers call the mass of a boat its **displacement**.

The mass of the fully loaded Titanic in kilograms was

$$m_T = 53,147 \times 1,000\text{kg} \approx 5.3 \times 10^7\,\text{kg}\,.$$

The density of water is $\rho_{\text{water}} = 1,000\,\text{kg}\,\text{m}^{-3}$, thus the total volume of water it displaced was

$$V_T = \frac{m}{\rho} = \frac{5.3 \times 10^7\,\text{kg}}{1,000\,\text{kg}\,\text{m}^{-3}} = 5.3 \times 10^4\,\text{m}^3\,.$$

Comparing the total volume V_T with the volume V displaced by the non-loaded Titanic, we see that the extra water displaced was

$$\Delta V = V_T - V = 5.3 \times 10^4\,\text{m}^3 - 4 \times 10^4\,\text{m}^3 = 1.3 \times 10^4\,\text{m}^3\,.$$

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Exercise 5(b)

The buoyancy force on a submerged object of volume $V = 0.02\,\mathrm{m}^3$ is equal to the weight of the liquid displaced by the object, therefore

$$\begin{aligned} F_{\text{buoyancy}} &= m_{\text{water}} \times g = V \times \rho_{\text{water}} \times g \\ &= 0.02\,\mathrm{m}^3 \times 1,000\,\mathrm{kg\,m}^{-3} \times 9.8\mathrm{ms}^{-2} \\ &= 196\,\mathrm{kg\,ms}^{-2} = 200\,\mathrm{N}\,. \end{aligned}$$

If the same object is submerged in mercury then the buoyancy force is

$$\begin{aligned} F_{\text{buoyancy}} &= m_{\text{mercury}} \times g = V \times \rho_{\text{mercury}} \times g \\ &= 0.02\,\mathrm{m}^3 \times 13,600\,\mathrm{kg\,m}^{-3} \times 9.8\mathrm{ms}^{-2} \\ &= 2700\,\mathrm{N}\,. \end{aligned}$$

Both answers are given to two significant figures.

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Exercise 5(c)

An object of volume $V = 0.02\,\mathrm{m}^3$ submerged in mercury sinks when its weight is more than the weight of displaced mercury:

$$m_{\mathrm{object}} \times g > m_{\mathrm{mercury}} \times g\,.$$

The mass of displaced mercury is

$$\begin{aligned} m_{\mathrm{mercury}} &= V \times \rho_{\mathrm{mercury}} \\ &= 0.02\,\mathrm{m}^3 \times 13,600\,\mathrm{kg\,m}^{-3} \\ &= 272\,\mathrm{kg}\,. \end{aligned}$$

Therefore, in order to sink, the mass of an object must be at least

$$m_{\mathrm{object}} > 272\,\mathrm{kg}\,.$$

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Solutions to Quizzes

Solution to Quiz:
From the **Example 1** and Exercise 1d we know that the density of water and mercury are

$$\begin{aligned}\rho_{\text{water}} &= 1\,\text{g}\,\text{cm}^{-3} = \frac{10^{-3}\text{kg}}{10^{-6}\,\text{m}^3} = 10^3\text{kg}\,\text{m}^{-3}\,, \\ \rho_{\text{mercury}} &= 13,600\,\text{kg}\,\text{m}^{-3} = 1.36\,\text{kg}\,\text{m}^{-3} \times 10^4\,.\end{aligned}$$

Therefore the specific gravity of mercury is the dimensionless ratio

$$\frac{\rho_{\text{mercury}}}{\rho_{\text{water}}} = \frac{1.36 \times 10^4\,\text{kg}\,\text{m}^{-3}}{10^3\,\text{kg}\,\text{m}^{-3}} = 13.6\,.$$

End Quiz

Solution to Quiz:

The pressure caused by a force of

$$F = 100\,\mathrm{N}$$

on an area $A = 0.25\,\mathrm{m}^2$ is

$$P = \frac{F}{A} = \frac{100\,\mathrm{N}}{0.25\,\mathrm{m}^2} = 400\ \mathrm{N\,m^{-2}} = 400\,\mathrm{Pa}\,.$$

End Quiz

Solution to Quiz:
If in a hydraulic lift, one of the areas is three times larger than the other, say

$$A_1 = 3 \times A_2$$

and the force applied to the smaller area A_2 is $F_2 = 15\,\mathrm{N}$, then the force F_1 measured at the larger area can be found from

$$\begin{aligned} \frac{F_2}{A_2} &= \frac{F_1}{A_1} \\ \therefore\ F_1 &= \frac{F_2 \times A_1}{A_2} = \frac{F_2 \times 3 \times A_2}{A_2} \\ &= 3 \times F_2 = 3 \times 15\,\mathrm{N} = 45\,\mathrm{N}. \end{aligned}$$

End Quiz