



Trigonometry 1

R Horan & M Lavelle

The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic understanding of some trigonometric functions.

Copyright © 2004 rhoran@plymouth.ac.uk , mlavelle@plymouth.ac.uk

Last Revision Date: March 4, 2005

Version 1.0

Table of Contents

1. Trigonometry 1 (Introduction)
 2. Using the Sine Function
 3. Using the Cosine Function
 4. Using the Tangent Function
 5. Quiz on Trigonometric Functions
- Solutions to Exercises
- Solutions to Quizzes

1. Trigonometry (Introduction)

In the right angled triangle shown in **diagram 1**, O is the side *opposite* the angle θ , A is the side *adjacent* to the angle θ and H , the side opposite the right angle, is the *hypotenuse* of the triangle. The three trigonometric functions dealt with in this package are the *sine*, *cosine* and *tangent* functions:

$$\sin \theta = \frac{O}{H}$$

$$\tan \theta = \frac{O}{A}$$

$$\cos \theta = \frac{A}{H}$$

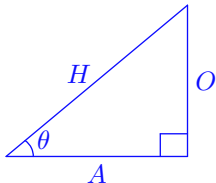


Diagram 1

One useful observation is the following relationship between the three functions:

$$\frac{\sin \theta}{\cos \theta} = \frac{O/H}{A/H} = \frac{O}{A} = \tan \theta.$$

2. Using the Sine Function

Example 1

Find the length of the side marked x in **diagram 2**.

Solution Using the sine function

$$\sin 22.5^\circ = \frac{O}{H} = \frac{x}{10}$$

$$\therefore 10 \sin 22.5^\circ = 10 \times 0.383 = x$$

$$\text{i.e. } x = 3.83 \text{ cm}$$

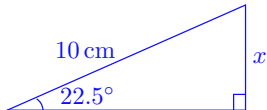


Diagram 2

where the value of $\sin 22.5^\circ$ was found using a calculator.

Example 2

Find the value of the angle denoted by θ in **diagram 3**.

Solution Using the sine function

$$\sin \theta = \frac{O}{H} = \frac{6}{15} = 0.4.$$

Using a calculator will show that

$$\theta \approx 23.58^\circ.$$

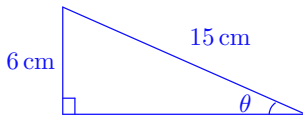


Diagram 3

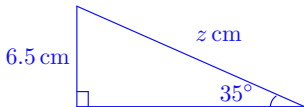
Example 3

Use the sine function to find z from **diagram 4**.

Solution

Using the sine function again:

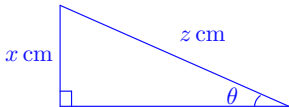
$$\begin{aligned}\sin 35^\circ &= \frac{O}{H} = \frac{6.5}{z} \\ \therefore z \times \sin 35^\circ &= z \times 0.574 = 6.5 \\ z &= \frac{6.5}{0.574} = 11.3 \text{ cm}\end{aligned}$$

**Diagram 4**

EXERCISE 1. In the exercises below, two of the three values of x, z, θ , referring to **diagram 5**, are given. Find the value of the missing one. (Click on the green letters for solutions.)

(a) $x = 5, z = 10,$ (b) $x = 4, \theta = 47^\circ,$

(c) $x = 10, \theta = 50^\circ.$

**Diagram 5**

3. Using the Cosine Function

Example 4

Use the cosine function to find the value of y in **diagram 6**.

Solution

$$\cos 30^\circ = \frac{A}{H} = \frac{y}{15}$$

$$\therefore 15 \times \cos 30^\circ = 15 \times 0.866 = y$$

$$\text{i.e. } y = 13.0 \text{ cm}$$

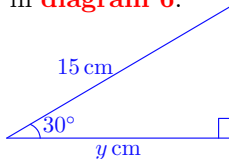


Diagram 6

Example 5

Use the cosine function to find the value of z in **diagram 7**.

Solution

$$\cos 40^\circ = \frac{A}{H} = \frac{25}{z}$$

$$\therefore z \times \cos 40^\circ = z \times 0.766 = 25$$

$$\begin{aligned} \text{i.e. } z &= \frac{25}{0.766} \\ &= 32.6 \text{ cm} \end{aligned}$$

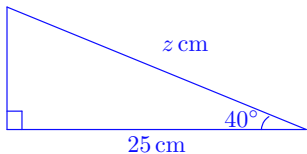


Diagram 7

EXERCISE 2. In the exercises below, two of the three values of y, z, θ , referring to **diagram 8**, are given. Find the value of the missing one. (Click on the green letters for solutions.)

(a) $y = 5, z = 10,$

(b) $y = 4, \theta = 47^\circ,$

(c) $z = 12, \theta = 50^\circ.$

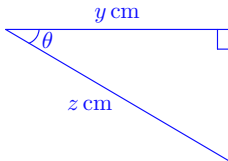


Diagram 8

Quiz Referring to **diagram 9**, which is the value of y ?

(a) 18.0,

(b) 26.9 ,

(c) 38.4,

(d) 12.6.

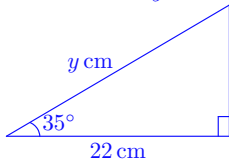


Diagram 9

4. Using the Tangent Function

Example 6

Use the tangent function to find the value of y in **diagram 10**.

Solution

$$\tan 60^\circ = \frac{O}{A} = \frac{y}{5}$$

$$\therefore 5 \times \tan 60^\circ = 5 \times 1.732 = y$$

$$\text{i.e. } y = 8.7 \text{ cm}$$

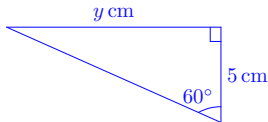


Diagram 10

Example 7

In **diagram 11**, use the tangent function to find the value of x .

Solution

$$\tan 40^\circ = \frac{O}{A} = \frac{17}{x}$$

$$\therefore x \times \tan 40^\circ = x \times 0.839 = 17$$

$$\text{i.e. } x = \frac{17}{0.839} = 20.3$$

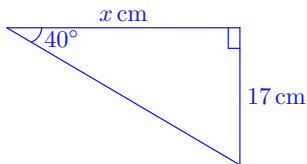


Diagram 11

EXERCISE 3. In the exercises below, two of the three values of x, y, θ , referring to **diagram 12**, are given. Find the value of the missing one. (Click on the green letters for solutions.)

(a) $x = 5, y = 10,$

(b) $y = 4, \theta = 25^\circ,$

(c) $x = 12, \theta = 56^\circ.$

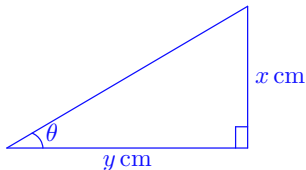


Diagram 12

Quiz Referring to **diagram 13**, which is the value of y ?

(a) 33.6,

(b) 54.2,

(c) 67.2,

(d) 38.7.

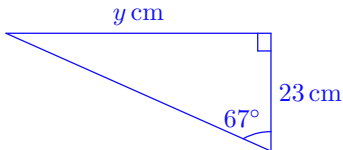


Diagram 13

5. Quiz on Trigonometric Functions

The following questions all refer to **diagram 14**.

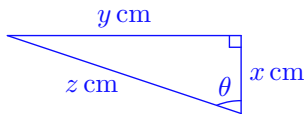


Diagram 14

Begin Quiz

1. If $y = 13$ cm and $z = 22$ cm, find θ .

- (a) 18° , (b) 43° (c) 35° , (d) 36° .

2. If $y = 30$ cm and $\theta = 25^\circ$, find x .

- (a) 69 cm, (b) 64 cm, (c) 38 cm, (d) 16 cm.

3. If $x = 15$ cm and $\theta = 25^\circ$, find z .

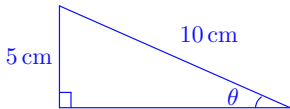
- (a) 24 cm, (b) 17 cm, (c) 25 cm, (d) 14 cm.

End Quiz

Solutions to Exercises

Exercise 1(a)

To find the value of the angle θ in the diagram, we use the sine function:



$$\sin \theta = \frac{5}{10} = 0.5.$$

Using a calculator will show that

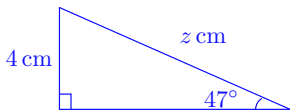
$$\theta = 30^\circ.$$

Click on the green square to return



Exercise 1(b)

To find z , the length of the *hypotenuse* of the triangle, we use the sine function:



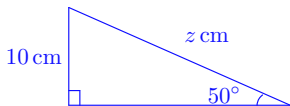
$$\begin{aligned}\sin 47^\circ &= \frac{4}{z} \\ \therefore z \times \sin 47^\circ &= z \times 0.732 = 4 \\ z &= \frac{4}{0.732} = 5.5 \text{ cm},\end{aligned}$$

where the value of $\sin 47^\circ \approx 0.732$ was found using a calculator.
Click on the green square to return



Exercise 1(c)

To find z , the length of the *hypotenuse* of the triangle, we use the sine function:



$$\begin{aligned}\sin 50^\circ &= \frac{10}{z} \\ \therefore z \times \sin 50^\circ &= z \times 0.766 = 10 \\ z &= \frac{10}{0.766} = 13.1 \text{ cm} .\end{aligned}$$

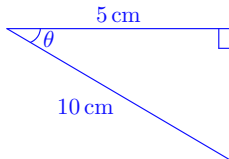
Here the value of $\sin 50^\circ \approx 0.766$ was found using a calculator.

[Click on the green square to return](#)



Exercise 2(a)

To find the value of the angle θ in the triangle given here, use the cosine function:



$$\cos \theta = \frac{5}{10} = \frac{1}{2}.$$

Using a calculator one finds that

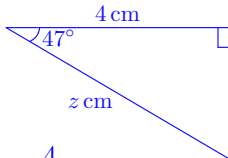
$$\theta = 60^\circ.$$

Click on the green square to return



Exercise 2(b)

To find z , the length of the *hypotenuse* of the triangle given in the picture, we use the cosine function:



$$\cos 47^\circ = \frac{4}{z}$$

$$\therefore z \times \cos 47^\circ = z \times 0.682 = 4$$

$$\begin{aligned} \text{i.e. } z &= \frac{4}{0.682} \\ &= 5.9 \text{ cm} \end{aligned}$$

Click on the green square to return



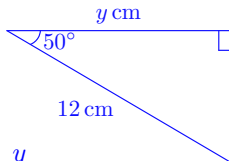
Exercise 2(c)

To find y , the length of the side *adjacent* to the angle $\theta = 50^\circ$ in the triangle given here, we use the cosine function:

$$\cos 50^\circ = \frac{y}{12}$$

$$\therefore 12 \times \cos 50^\circ = 12 \times 0.643 = y$$

$$\text{i.e. } y = 7.7 \text{ cm}$$

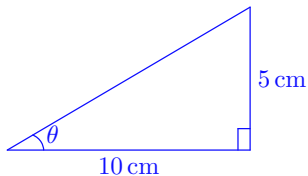


Click on the green square to return



Exercise 3(a)

To find the value of the angle θ of the triangle given in the picture, we use the tangent function:



$$\tan \theta = \frac{5}{10} = 0.5$$

Using a calculator will show that

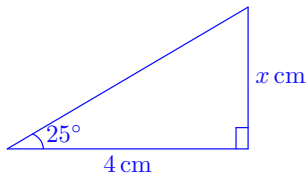
$$\theta \approx 27^\circ.$$

Click on the green square to return



Exercise 3(b)

To find x , the length of the side *opposite* the angle $\theta = 25^\circ$ of the triangle given in the picture, we use the tangent function:



$$\tan 25^\circ = \frac{x}{4}$$

$$\therefore 4 \times \tan 25^\circ = 4 \times 0.466 = x$$

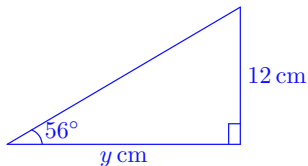
$$\text{i.e. } x = 1.9 \text{ cm}$$

Click on the green square to return



Exercise 3(c)

To find y , the length of the side *adjacent* to the angle $\theta = 25^\circ$ of the triangle given in the picture, we use the tangent function:



$$\tan 56^\circ = \frac{12}{y}$$

$$\therefore y \times \tan 56^\circ = y \times 1.483 = 12$$

$$\text{i.e. } y = \frac{12}{1.483} = 8.1 \text{ cm}$$

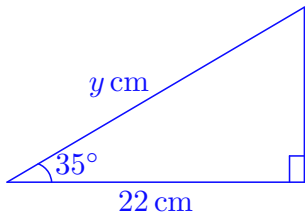
Click on the green square to return



Solutions to Quizzes

Solution to Quiz:

To find y , the length of the *hypotenuse* of the triangle given in the picture, we use the cosine function:



$$\cos 35^\circ = \frac{22}{y}$$

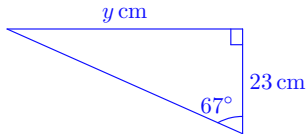
$$\therefore y \times \cos 35^\circ = y \times 0.819 = 22$$

$$\text{i.e. } y = \frac{22}{0.819} = 26.9 \text{ cm}$$

End Quiz

Solution to Quiz:

To find y , the length of the side *opposite* to the angle 67° of the triangle given in the picture, we use the tangent function:



$$\tan 67^\circ = \frac{y}{23}$$

$$\therefore 23 \times \tan 67^\circ = 23 \times 2.356 = y$$

$$\text{i.e. } y = 54.2 \text{ cm}$$

End Quiz