PPLATO PROMOTING PHYSICS LEARNING & TEACHING OPPORTUNITIES

Basic Mathematics

Introduction to Vectors

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The aim of this document is to provide a short, self assessment programme for students who wish to acquire a basic understanding of vectors.

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The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Vectors (Introduction)

A vector is a combination of three things:

- a positive number called its *magnitude*,
- a *direction* in space,
- a *sense* making more precise the idea of direction.

Typically a vector is illustrated as a directed straight line.

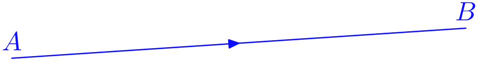


Diagram 1

The vector in the above diagram would be written as $\vec{AB}$ with:

- the direction of the arrow, from the point A to the point B, indicating the *sense* of the vector,
- the *magnitude* of $\vec{AB}$ given by the length of AB.

The *magnitude* of $\vec{AB}$ is written $|\vec{AB}|$.

There are very many physical quantities which are best described as vectors; velocity, acceleration and force are all *vector* quantities.

Two vectors are *equal* if they have the same *magnitude*, the same *direction* (i.e. they are *parallel*) and the same *sense*.

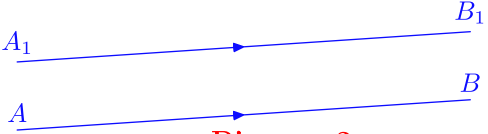


Diagram 2

In **diagram 2** the vectors $\overrightarrow{AB}$ and $\overrightarrow{A_1B_1}$ are equal, i.e. $\overrightarrow{AB}=\overrightarrow{A_1B_1}$. If two vectors have the same length, are parallel but have *opposite* senses then one is the negative of the other.

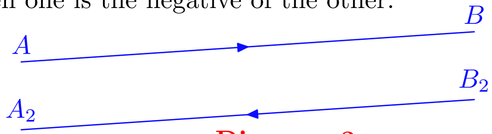


Diagram 3

In **diagram 3** the vectors $\overrightarrow{AB}$ and $\overrightarrow{B_2A_2}$ are of equal length, are parallel but are *opposite* in sense, so $\overrightarrow{AB}=-\overrightarrow{B_2A_2}$.

Quiz

Diagram 4 shows a parallelogram. Which of the following equations is the correct one?

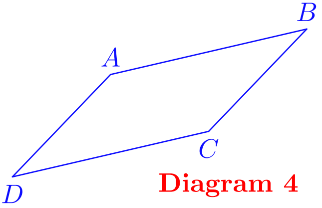


Diagram 4

(a) $\vec{DA} = \vec{BC}$, (b) $\vec{AD} = -\vec{CB}$, (c) $\vec{AD} = \vec{CB}$, (d) $\vec{DA} = -\vec{CB}$.

If two vectors are parallel, have the same sense but different magnitudes then one vector is a *scalar* (i.e. numeric) multiple of the other.

In **diagram 5** the vector $\vec{AB}$ is parallel to $\vec{A_3B_3}$, has the same sense but is twice as long, so $\vec{AB} = 2\ \vec{A_3B_3}$.

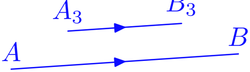


Diagram 5

In general *multiplying a vector by a positive number* λ *gives a vector parallel to the original vector, with the same sense but with magnitude* λ *times that of the original*. If λ is *negative* then the sense is reversed. Thus from **diagram 5** for example, $\vec{A_3B_3} = -\frac{1}{2}\ \vec{BA}$.

2. Addition of Vectors

In **diagram 6** the three vectors given by $\vec{AB}$,$\vec{BC}$, and $\vec{AC}$, make up the sides of a triangle as shown. Referring to this diagram, the law of addition for vectors, which is usually known as the *triangle law of addition*, is

$$\vec{AB} + \vec{BC} = \vec{AC} \ .$$

The vector $\vec{AC}$ is called the *resultant vector*.

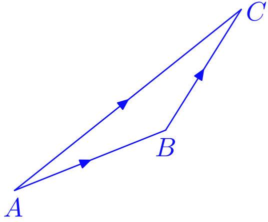


Diagram 6

Physical quantities which can be described as vectors satisfy such a law. One such example is the action of forces. If two forces are represented by the vectors $\vec{AB}$ and $\vec{BC}$ then the effect of applying *both of these forces together* is equivalent to a single force, the *resultant force*, represented by the vector $\vec{AC}$.

One further vector is required, the *zero vector*. This has *no direction* and *zero magnitude*. It will be written as $\mathbf{0}$.

Example 1 (**The mid-points theorem**)

Let ABC be a triangle and let D be the midpoint of AC and E be the midpoint of BC. Prove that DE is parallel to AB and half its length i.e. $|AB| = 2|DE|$.

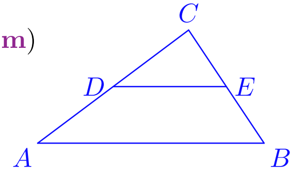


Diagram 7

Proof

Since D is the midpoint of $\vec{AC}$, it follows that $\vec{AC} = 2\,\vec{DC}$. Similarly $\vec{CB} = 2\,\vec{CE}$. Then

$$\begin{aligned}\vec{AC} + \vec{CB} &= 2\,\vec{DC} + 2\,\vec{CE} \\ &= 2(\vec{DC} + \vec{CE})\,.\end{aligned}$$

Now $\quad \vec{AC} + \vec{CB} = \vec{AB} \quad$ and $\quad \vec{DC} + \vec{CE} = \vec{DE}$.

Substituting these into the equation above gives $\vec{AB} = 2\,\vec{DE}$.

Since these are vectors, AB must be parallel to DE and the length of AB is twice that of DE, i.e. $|\vec{AB}| = 2|\vec{DE}|$.

3. Component Form of Vectors

The diagram shows a vector $\vec{OC}$ at an angle to the x axis. Take **i** to be a vector of length 1 (called a *unit vector*) parallel to the x axis and in the positive direction, and **j** to be a vector of length 1 (another *unit vector*) parallel to the y axis and in the positive direction.

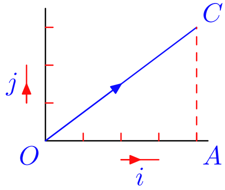


Diagram 8

From **diagram 8**, $\vec{OC} = \vec{OA} + \vec{AC}$. The vector $\vec{OA}$ is parallel to the vector **i** and four times its length so $\vec{OA} = 4\mathbf{i}$. Similarly $\vec{AC} = 3\mathbf{j}$. Thus the vector $\vec{OC}$ may be written as

$$\vec{OC} = 4\mathbf{i} + 3\mathbf{j}\,.$$

This is known as the *2-dimensional component form* of the vector. In general every vector can be written in component form. This package will consider only 2-dimensional vectors.

Exercise 1. From **diagram 9**, write down the component form of the following vectors: (Click on the green letters for solutions.)

(a) $\overrightarrow{OA}$, (b) $\overrightarrow{OB}$,

(c) $\overrightarrow{OC}$, (d) $\overrightarrow{OD}$,

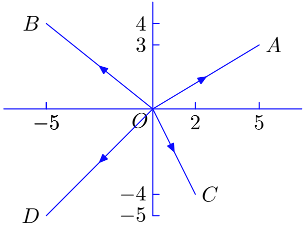


Diagram 9

In this package, the following properties of vectors are used.

- To add two or more vectors in component form, add the corresponding components.
- To multiply a vector in component form by a scalar, multiply each of the components by the scalar.
- If a vector in component form is $a\mathbf{i} + b\mathbf{j}$ then its magnitude is $\sqrt{a^2 + b^2}$. (*Pythagoras' theorem*)

Example 3

If $\vec{AB} = 2\mathbf{i} + 2\mathbf{j}$ and $\vec{BC} = \mathbf{i} + 2\mathbf{j}$, prove that the magnitude of $\vec{AC}$ is 5.

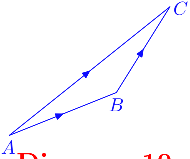


Diagram 10

Proof

The three vectors form three sides of a triangle (see **diagram 10** which is NOT to scale) so

$$\begin{aligned}\vec{AC} = \vec{AB} + \vec{BC} &= (2\mathbf{i} + 2\mathbf{j}) + (1\mathbf{i} + 2\mathbf{j})\\ &= (2\mathbf{i} + 1\mathbf{i}) + (2\mathbf{j} + 2\mathbf{j}) = 3\mathbf{i} + 4\mathbf{j}\,.\end{aligned}$$

Thus $|\ \vec{AC}\ | = \sqrt{3^2 + 4^2} = 5$.

NB Vectors are often printed as boldface lower case letters such as $\mathbf{a}$.

Exercise 2. If $\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{c} = \mathbf{i} - 2\mathbf{j}$, calculate:

(a) $\mathbf{a} + \mathbf{b}$, (b) $\mathbf{b} + \mathbf{c}$, (c) $\mathbf{a} + \mathbf{b} + \mathbf{c}$,

(d) $\mathbf{a} + 2\mathbf{b}$, (e) $2\mathbf{b} - 3\mathbf{a}$, (f) $|\mathbf{a}|$,

(g) $|\mathbf{a} + \mathbf{b}|$, (h) $|\mathbf{a}| + |\mathbf{b}|$, (i) $|2\mathbf{a} - \mathbf{b}|$,

Example 4 Two vectors are $\vec{AB} = \mathbf{i} + \mathbf{j}$ and $\vec{CD} = 2\mathbf{i} + 3\mathbf{j}$. Find
(a) The value of λ such that $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $\mathbf{i}$,
(b) The value of λ such that $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $4\mathbf{i} + 3\mathbf{j}$.

Solution First find $\lambda\,\vec{AB} + \vec{CD}$ in component form.

$$\begin{aligned}\lambda\,\vec{AB} + \vec{CD} &= \lambda(\mathbf{i} + \mathbf{j}) + (2\mathbf{i} + 3\mathbf{j})\\ &= (\lambda\mathbf{i} + \lambda\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j})\\ &= (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j}\,.\end{aligned}$$

(a) If $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $\mathbf{i}$ then the $\mathbf{j}$ component must be zero, i.e. $\lambda + 3 = 0$. Thus $\lambda = -3$ and we have $-3\,\vec{AB} + \vec{CD} = -\mathbf{i}$.

(b) If $\lambda\,\vec{AB} + \vec{CD}$ is parallel to $4\mathbf{i} + 3\mathbf{j}$ then there is a number κ such that

$$\begin{aligned}(\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j} &= \kappa(4\mathbf{i} + 3\mathbf{j})\\ \therefore (\lambda + 2)\mathbf{i} + (\lambda + 3)\mathbf{j} &= 4\kappa\mathbf{i} + 3k\mathbf{j}\\ \text{so} \quad \lambda + 2 = 4\kappa \quad &\text{and} \quad \lambda + 3 = 3\kappa\,.\end{aligned}$$

Then
$$\begin{aligned}\frac{\lambda+2}{\lambda+3} &= \frac{4\kappa}{3\kappa}=\frac{4}{3}\\ \therefore 3(\lambda+2) &= 4(\lambda+3)\\ 3\lambda+6 &= 4\lambda+12\\ 6-12 &= 4\lambda-3\lambda\\ \text{i.e.}\qquad \lambda &= -6\,,\end{aligned}$$

and the vector is $-6(\mathbf{i}+\mathbf{j})+(2\mathbf{i}+3\mathbf{j})=-4\mathbf{i}-3\mathbf{j}=-(4\mathbf{i}+3\mathbf{j})$.

Quiz If $\mathbf{a}=2\mathbf{i}+3\mathbf{j}$, $\mathbf{b}=-3\mathbf{i}+2\mathbf{j}$ and $\mathbf{c}=2\mathbf{i}-\mathbf{j}$, which of the following vectors is parallel to the resultant of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, i.e. $\mathbf{a}+\mathbf{b}+\mathbf{c}$?

(a) $-2\mathbf{i}-6\mathbf{j}$, (b) $2\mathbf{i}-6\mathbf{j}$, (c) $2\mathbf{i}+8\mathbf{j}$, (d) $2\mathbf{i}-8\mathbf{j}$.

Quiz If $\mathbf{a}=\mathbf{i}+\mathbf{j}$ and $\mathbf{b}=\mathbf{i}-\mathbf{j}$, for which of the following values of λ is the vector $\lambda\mathbf{a}+\mathbf{b}$ parallel to $\mathbf{c}=2\mathbf{i}-3\mathbf{j}$?

(a) $\lambda=\frac{1}{5}$, (b) $\lambda=-\frac{1}{5}$, (c) $\lambda=5$, (d) $\lambda=-5$.

4. Quiz on Vectors

Choose the correct option for each of the following.

Begin Quiz

1. If $\mathbf{a} = -2\mathbf{i} + 4\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j}$, $\mathbf{c} = 4\mathbf{i} + 5\mathbf{j}$ then $\mathbf{a} + \mathbf{b} + \mathbf{c}$ is

(a) $-5\mathbf{i} - 7\mathbf{j}$, (b) $5\mathbf{i} - 7\mathbf{j}$, (c) $-5\mathbf{i} + 7\mathbf{j}$, (d) $5\mathbf{i} + 7\mathbf{j}$.

2. If $\mathbf{u} = -2\mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$, $\mathbf{w} = 4\mathbf{i} + 6\mathbf{j}$ then $|\mathbf{u} + \mathbf{v} + \mathbf{w}|$ is

(a) 5, (b) 13, (c) 4, (d) 15.

3. If $\mathbf{u} = -\mathbf{i}+3\mathbf{j}$ and $\mathbf{v} = \mathbf{i}+2\mathbf{j}$, then $\lambda\mathbf{u}+\mathbf{v}$ is parallel to $\mathbf{w} = -\mathbf{i}+4\mathbf{j}$ if λ is

(a) -6, (b) 6, (c) -5, (d) 5.

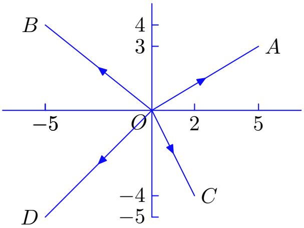
End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

For the vector $\vec{OA}$ shown on the diagram the component in the direction given by the unit vector $\mathbf{i}$ is 5 and the component in the direction $\mathbf{j}$ is 3. Therefore the 2-dimensional vector $\vec{OA}$ is, in component form, written as

$$\vec{OA} = 5\mathbf{i} + 3\mathbf{j}\,.$$



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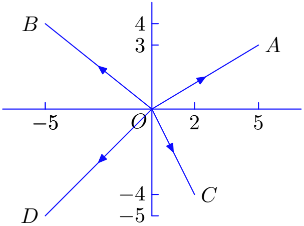
Exercise 1(b)

The vector $\vec{OB}$ shown on the diagram has the component -5 in the $\mathbf{i}$ direction while the component in the $\mathbf{j}$ direction is 4. Thus the 2-dimensional vector $\vec{OB}$ in component form is written as

$$\vec{OB} = -5\mathbf{i} + 4\mathbf{j}\,.$$



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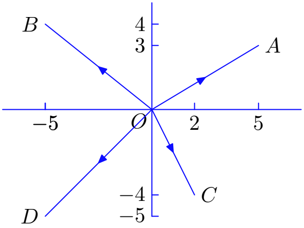
Exercise 1(c)

For the vector $\overrightarrow{OC}$ shown on the diagram the component in the direction given by the unit vector $\mathbf{i}$ is 2 while the component in the direction given by $\mathbf{j}$ is -4. Therefore the component form of the 2-dimensional vector $\overrightarrow{OC}$ is

$$\overrightarrow{OC} = 2\mathbf{i} - 4\mathbf{j}\,.$$



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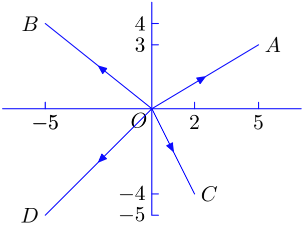
Exercise 1(d)

For the vector $\overrightarrow{OD}$ shown on the diagram the component in the direction given by the unit vector $\mathbf{i}$ is -5 and the component in the direction given by $\mathbf{j}$ is also -5. The component form of the 2-dimensional vector $\overrightarrow{OD}$ is therefore

$$\overrightarrow{OC} = -5\mathbf{i} - 5\mathbf{j}\,.$$



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Exercise 2(a)
The sum of the two vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$$

is found by summing up the corresponding components of each vector. Thus

$$\mathbf{a} + \mathbf{b} = (-\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) = (-1 + 2)\mathbf{i} + (3 + 3)\mathbf{j} = \mathbf{i} + 6\mathbf{j}\,.$$

Click on the green square to return

□

Exercise 2(b)
The sum of the two vectors

$$\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{c} = \mathbf{i} - 2\mathbf{j}$$

is found by adding the corresponding components of each vector. Thus

$$\mathbf{b} + \mathbf{c} = (2\mathbf{i} + 3\mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) = (2 + 1)\mathbf{i} + (3 - 2)\mathbf{j} = 3\mathbf{i} + \mathbf{j}\,.$$

Click on the green square to return

□

Exercise 2(c)
To find the sum of the three vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}\,, \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{c} = \mathbf{i} - 2\mathbf{j}\,,$$

add the corresponding components of each vector. The resulting vector is thus

$$\begin{aligned} \mathbf{a} + \mathbf{b} + \mathbf{c} &= (-\mathbf{i} + 3\mathbf{j}) + (2\mathbf{i} + 3\mathbf{j}) + (\mathbf{i} - 2\mathbf{j}) \\ &= (-1 + 2 + 1)\mathbf{i} + (3 + 3 - 2)\mathbf{j} = 2\mathbf{i} + 4\mathbf{j}\,. \end{aligned}$$

Click on the green square to return

□

Exercise 2(d)
To find the sum $\mathbf{a} + 2\mathbf{b}$ with

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}\,,$$

first find the vector $2\mathbf{b}$:

$$2\,\mathbf{b} = 2 \times (2\mathbf{i} + 3\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j}\,.$$

The vector $\mathbf{a} + 2\mathbf{b}$ is now found by adding the corresponding components of each vector. The resulting vector is thus

$$\begin{aligned} \mathbf{a} + 2\mathbf{b} &= (-\mathbf{i} + 3\mathbf{j}) + (4\mathbf{i} + 6\mathbf{j}) \\ &= (-1 + 4)\mathbf{i} + (3 + 6)\mathbf{j} = 3\mathbf{i} + 9\mathbf{j}\,. \end{aligned}$$

Click on the green square to return

□

Exercise 2(e)
To find the vector $\mathbf{2b-3a}$ with

$$\mathbf{a = -i + 3j} \qquad \text{and} \qquad \mathbf{b = 2i + 3j}\,,$$

first find the vectors $\mathbf{2b}$ and $\mathbf{3a}$:

$$\begin{aligned} 2\,\mathbf{b} &= 2 \times (2\mathbf{i} + 3\mathbf{j}) = 4\mathbf{i} + 6\mathbf{j}\,, \\ 3\,\mathbf{a} &= 3 \times (-\mathbf{i} + 3\mathbf{j}) = -3\mathbf{i} + 9\mathbf{j}\,, \end{aligned}$$

The vector $\mathbf{2b-3a}$ is now easily found by subtracting the components of these vectors:

$$\begin{aligned} \mathbf{2b - 3a} &= (4\mathbf{i} + 6\mathbf{j}) - (-3\mathbf{i} + 9\mathbf{j}) \\ &= (4+3)\mathbf{i} + (6-9)\mathbf{j} = \mathbf{7i - 3j}\,. \end{aligned}$$

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Exercise 2(f)
The magnitude of the vector

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$$

is given by

$$|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{1+9} = \sqrt{10}\,.$$

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Exercise 2(g)

To find the magnitude of the vector $\mathbf{a}+\mathbf{b}$, first find the sum of the two vectors

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j} \qquad \text{and} \qquad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j}\,.$$

The resulting vector is

$$\mathbf{a}+\mathbf{b} \quad = \quad (-\mathbf{i}+3\mathbf{j}) + (2\mathbf{i}+3\mathbf{j}) = (-1+2)\mathbf{i} + (3+3)\mathbf{j} = \mathbf{i} + 6\mathbf{j}\,.$$

The magnitude of this vector is given by

$$|\mathbf{a}+\mathbf{b}| = \sqrt{1^2+6^2} = \sqrt{37}\,.$$

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Exercise 2(h)
To find $|\mathbf{a}| + |\mathbf{b}|$, first find the magnitude of each of the vectors $\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$.
The magnitude of the vector $\mathbf{a}$ is

$$|\mathbf{a}| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}\,.$$

The magnitude of the vector $\mathbf{b}$ is

$$|\mathbf{b}| = \sqrt{2^2 + 3^2} = \sqrt{13}\,.$$

Therefore

$$|\mathbf{a}| + |\mathbf{b}| = \sqrt{10} + \sqrt{13}\,.$$

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Exercise 2(i)

To find $|\mathbf{2a} - \mathbf{b}|$, first find $\mathbf{2a} - \mathbf{b}$. The vector $\mathbf{a}$ in component form is given as

$$\mathbf{a} = -\mathbf{i} + 3\mathbf{j}$$

so the component form of the vector $2\mathbf{a}$ is

$$2\mathbf{a} = 2 \times (-1)\mathbf{i} + 2 \times 3\mathbf{j} = -2\mathbf{i} + 6\mathbf{j}\,.$$

The difference between $2\mathbf{a}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j}$ is the vector

$$2\mathbf{a} - \mathbf{b} = (-2\mathbf{i} + 6\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) = (-2-2)\mathbf{i} + (6-3)\mathbf{j} = -4\mathbf{i} + 3\mathbf{j}\,.$$

The magnitude of the resulting vector $2\mathbf{a} - \mathbf{b}$ is therefore

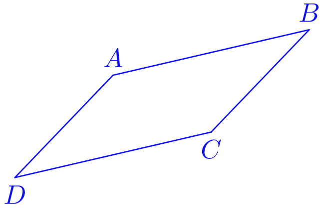
$$|2\mathbf{a} - \mathbf{b}| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5\,.$$

Click on the green square to return

Solutions to Quizzes

Solution to Quiz:

According to the diagram shown opposite the magnitudes of the vectors $\vec{AD}$ and $\vec{CB}$ are equal, but the direction of the vector $\vec{AD}$ is from the point A to the point D, while the direction of the vector $\vec{CB}$ is opposite, from the point B to the point C. Therefore $\vec{AD} = -\vec{CB}$.



If checked, the other solutions will be found to be false. End Quiz

Solution to Quiz:

In order to determine which of the vectors is parallel to the resultant of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, the resultant must first be calculated.

The resultant of the three vectors

$$\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}\,, \qquad \mathbf{b} = -3\mathbf{i} + 2\mathbf{j} \qquad \text{and} \qquad \mathbf{c} = 2\mathbf{i} - \mathbf{j}\,.$$

is

$$\begin{aligned} \mathbf{a} + \mathbf{b} + \mathbf{c} &= (2\mathbf{i} + 3\mathbf{j}) + (-3\mathbf{i} + 2\mathbf{j}) + (2\mathbf{i} - \mathbf{j}) \\ &= (2 - 3 + 2)\mathbf{i} + (3 + 2 - 1)\mathbf{j} = \mathbf{i} + 4\mathbf{j}\,. \end{aligned}$$

Next note that the vector $2\mathbf{i}+8\mathbf{j}$ given in the answer **(c)** can be written as

$$2\mathbf{i} + 8\mathbf{j} = 2 \times (\mathbf{i} + 4\mathbf{j}) = 2(\mathbf{a} + \mathbf{b} + \mathbf{c})\,,$$

so the resultant is parallel to the vector $2\mathbf{i} + 8\mathbf{j}$. End Quiz

Solution to Quiz: To find the value of λ for which $\lambda\mathbf{a} + \mathbf{b}$ parallel to $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$, first calculate the former. If $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{i} - \mathbf{j}$ then

$$\lambda\mathbf{a} + \mathbf{b} = \lambda(\mathbf{i} + \mathbf{j}) + (\mathbf{i} - \mathbf{j}) = (\lambda + 1)\mathbf{i} + (\lambda - 1)\mathbf{j}\,.$$

If this vector is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$ then there is a number k such that

$$(\lambda + 1)\mathbf{i} + (\lambda - 1)\mathbf{j} = k(2\mathbf{i} - 3\mathbf{j})\,.$$

This holds when $\quad \lambda + 1 = 2k \quad$ and $\quad \lambda - 1 = -3k\,.$

Multiply the first equation by 3

$$3\lambda + 3 = 6k\,,$$

and the second one by 2

$$2\lambda - 2 = -6k\,.$$

Now add the left and right sides of these equations to obtain:

$$5\lambda + 1 = 0\,, \qquad \text{thus} \qquad \lambda = -\frac{1}{5}\,.$$

End Quiz