Basic Physics

Introduction to Waves

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The aim of this package is to provide a short self assessment programme for students who want to understand the basics of waves.

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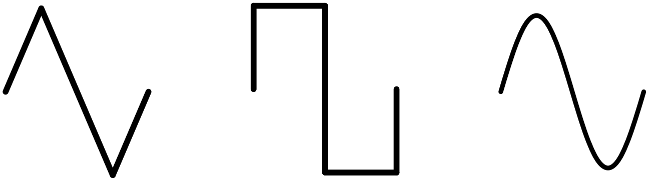
The full range of these packages and some instructions, should they be required, can be obtained from our web page Mathematics Support Materials.

1. Introduction

Waves occur in almost all branches of science. The basic concepts discussed in this introductory package may be applied to all waves including the three shown below:



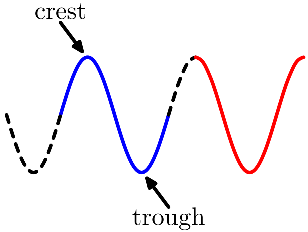
Sawtooth wave *Square wave* *Sine wave*

Waves are typically disturbances in some medium: they include electromagnetic waves (such as light or radio waves); movements of gas molecules (sound waves) and movements in solids (seismic waves).

An understanding of waves is crucial for all these phenomena and much more.

2. Amplitude, Wavelength, Frequency and Period

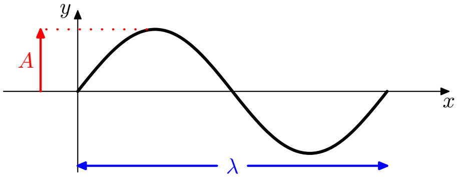
Consider the waves below. The solid red and blue parts are both a full cycle.



For the *blue cycle* the wave oscillates up from its central value to a crest, then down through a trough and back up to its central value.

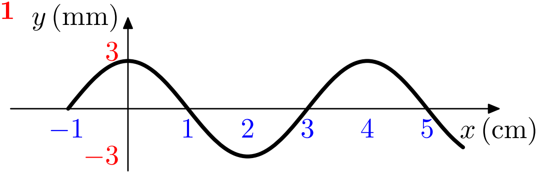
The *red cycle* passes down from a crest to a trough and back up again to its initial peak value.
Let us now consider how to parameterise the shape of a wave.

The figure below shows a snapshot of a wave at a fixed time:



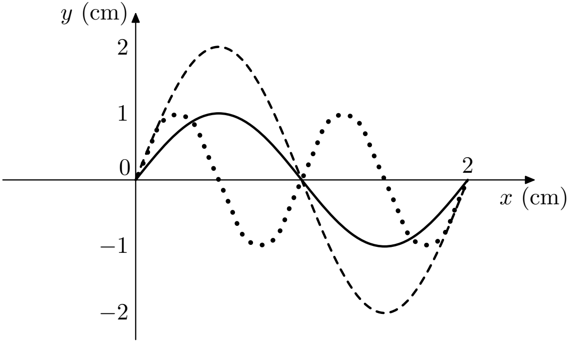
The maximum disturbance, A, from the central value is the **amplitude** of the wave. The length of a full cycle, λ, is the **wavelength**.

Example 1



Here this wave has an amplitude of 3 mm and a wavelength of 4 cm (measured, say, from crest to crest or trough to trough).

Exercise 1. These exercises refer to the three waves shown below:



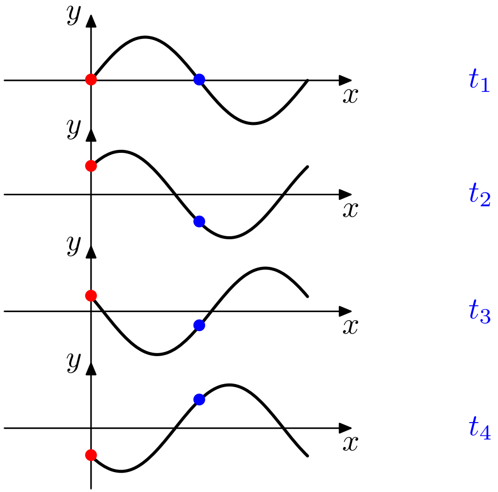
(a) What is the amplitude of the solid wave?

(b) What is the wavelength of the solid wave?

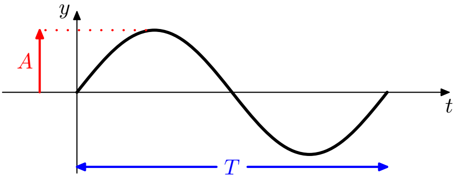
(c) How are the wavelengths of the dashed and dotted waves related?

(d) Which property (if any) has the same value for the dashed and solid waves?

Here a wave is plotted at successive moments in time. This shows how the wave oscillates at a fixed position (for example the red or blue points).



Here a wave at a **fixed position** is plotted against time:



Again A is the **amplitude** of the wave. The time taken for a full cycle T is the **period** of the wave.

The **frequency** ν is the number of cycles per second. It is given by:

$$\nu = \frac{1 \text{ second}}{\text{time for a full cycle}} = \frac{1}{T}$$

This also implies:

$$T = \frac{1}{\nu}$$

Frequency units: 1 cycle per second is also called 1 Hertz (1 Hz).

Example 2 If the frequency of a wave is 4 Hz, what is its period?

$$T = \frac{1}{\nu} = \frac{1}{4} = 0.25\, s.$$

This result shows that four times the period is equal to one second, i.e., the frequency is 4 cycles per second.

Frequency, wavelength and the **speed of a moving wave** are related as follows: during the period of a full cycle, T, the wave travels a wavelength, λ, so its speed, v, is

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T}$$

Since $\nu = 1/T$, this can be rewritten as

$$v = \nu\lambda$$

Thus, for a given speed v, high frequencies correspond to short wavelengths while low frequencies have long wavelengths.

Example 3 If a seismic wave is measured to have a wavelength of 15 m and frequency 100 Hz. We can find its speed as follows:

$$\begin{aligned} v &= \nu\lambda \\ &= 100 \times 15 = 1,500\,\mathrm{ms}^{-1}\,. \end{aligned}$$

EXERCISE 2.

(a) If a sound wave in air has frequency 1,700 Hz, what is its wavelength? (The speed of sound in air is $340\,\mathrm{ms}^{-1}$.)

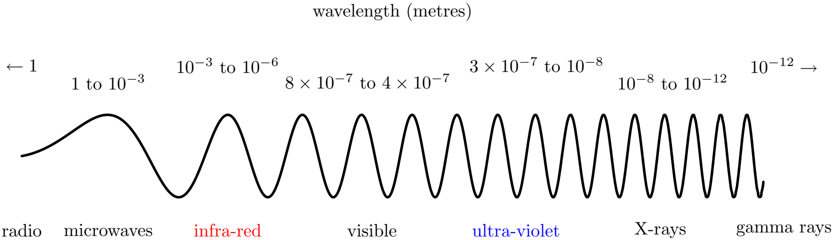
(b) What would the wavelength of the same sound waves be in a metal where their speed was $5{,}100\,\mathrm{ms}^{-1}$?

(c) BBC Radio Devon broadcasts on 103.4 MHz (in FM) and on 855 kHz (AM band). Use the speed of light, $c = 3 \times 10^8\,\mathrm{ms}^{-1}$, to find the wavelengths of each of these sets of radio waves.

Quiz The audible frequency range lies between 20 Hz and 20,000 Hz (20 kHz). What is the maximum period of an audible wave?

(a) $2\,\mathrm{s}$ (b) $20,000\,\mathrm{s}$ (c) $0.05\,\mathrm{s}$ (d) $0.5\,\mathrm{s}$

Exercise 3. These questions refer to the electromagnetic spectrum which is shown (not to scale) below. (Recall that the speed of light is $c = 3 \times 10^8\,\mathrm{ms}^{-1}$.)



The electro-magnetic spectrum

(a) What is the highest frequency of microwave radiation?

(b) What is the lowest frequency of X rays?

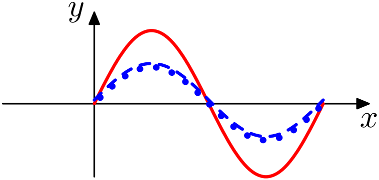
(c) What is the band of frequencies of infra-red radiation?

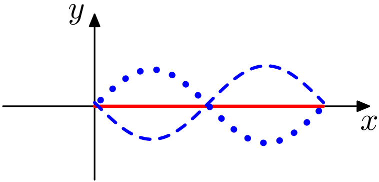
(d) What is the lowest period of visible light?

3. Interference and Phases

When two waves are added together they either reinforce or cancel each other. This important property is called **interference**.

Example 4 Here are examples of adding the blue dotted and dashed waves. The red solid lines are the results of the interference:

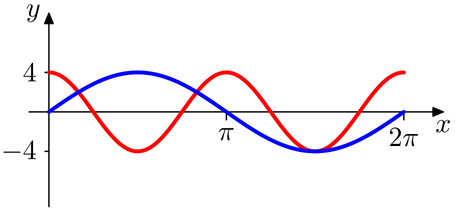




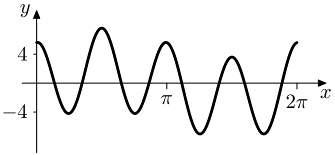
Reinforcement, or **constructive interference**, occurs when crests from the one wave arrive at the same point as the crests from the other wave. (They are said to be *superimposed.*)

Cancellation, called **destructive interference**, takes place when crests from one wave are superimposed on troughs from the other wave.

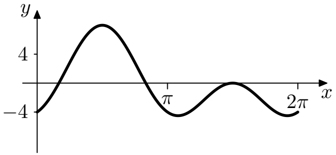
Quiz Select from the answers below the result of superimposing these two waves.



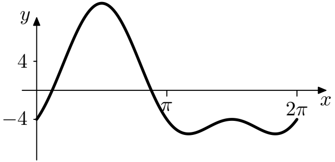
(a)



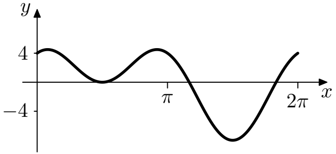
(b)



(c)

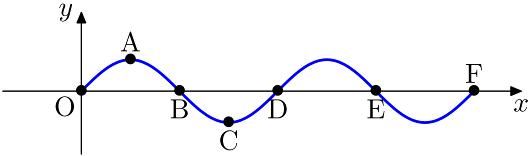


(d)



The **phase difference**, $\Delta\phi$, between two points on a wave is the fraction of a complete cycle between the points. It is measured in degrees (360° being a full cycle) or radians (one cycle is 2π radians).

Example 5 Consider the waves below:

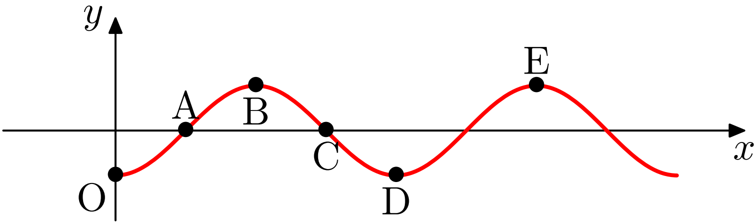


The phase differences *relative to the point* O are:

Point	A	B	C	D	E	F
Degrees	90°	180°	270°	360°	180°	360°
Radians	$\pi/2$	π	$3\pi/2$	2π	π	2π

The points O, D and F are all **in phase** with each other. B is **out of phase** with O as the wave is then moving in the opposed direction. The phase difference between B and C is $\pi/2$ or 90°.

Exercise 4. These exercises refer to the points marked on the wave below.



(a) What is the phase difference between the points O and A?

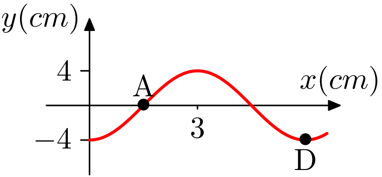
(b) Which point is in phase with D?

(c) What is the phase difference between O and C?

(d) Which point has a phase difference $\Delta\phi = \pi$ with A?

(e) What is the phase difference between the points B and D?

4. Final Quiz



Begin Quiz

1. Select the phase difference (in radians) between A and D above

(a) $\pi/2$ (b) $3\pi/2$ (c) π (d) $3\pi/4$

2. Choose the amplitude of the above wave.

(a) 8 cm (b) 3 cm (c) 4 cm (d) 1.5 cm

3. What is the frequency of a sound wave with $\lambda = 0.17\,\text{m}$?

(a) 200 Hz (b) 57.8 Hz (c) 10^5 Hz (d) 2 kHz

4. Find the wavelength of a wave with $\nu = 0.8\,\text{Hz}$ and $v = 4\,\text{ms}^{-1}$

(a) 5 m (b) 0.5 m (c) 3.2 m (d) 0.2 m

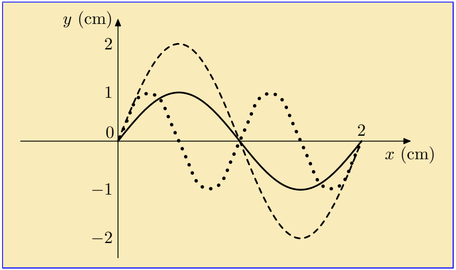
End Quiz Score: Correct

Solutions to Exercises

Exercise 1(a)

The picture shows that the maximum positive disturbance of the solid wave from the x-axis is 1 cm, therefore the amplitude A is

$$A = 1\,\mathrm{cm}\,.$$



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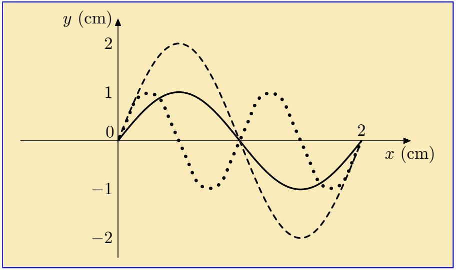
Exercise 1(b)

From the picture we see that the solid wave has a full cycle between two points, with x= 0 cm and x = 2 cm. Its wavelength λ is therefore

$$\lambda = 2\,\text{cm}\,.$$



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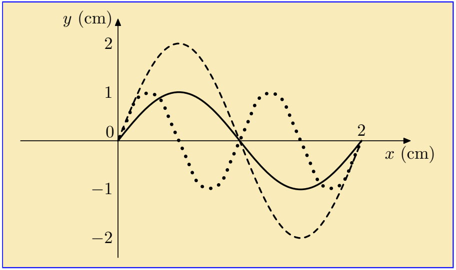
Exercise 1(c)

Comparing the cycles of the dashed and dotted waves shown on the picture, we see that the dashed wave cycle is twice as long as the dotted one.



The wavelengths λ_{dashed} and λ_{dotted} obey the relation

$$\lambda_{\text{dashed}} = 2 \times \lambda_{\text{dotted}} .$$

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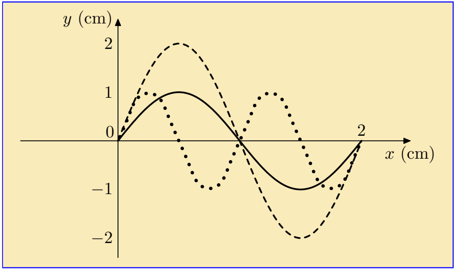
Exercise 1(d)

From the picture, the full cycles of the dashed and solid waves are the same, therefore the wavelengths λ_{dashed} and λ_{solid} are equal

$$\lambda_{\text{dashed}} = \lambda_{\text{solid}} .$$



Note however, that the corresponding amplitudes are different; the amplitude A_{solid} is half of A_{dashed}

$$A_{\text{solid}} = \frac{1}{2} \times A_{\text{dashed}} .$$

Click on the **green** square to return □

Exercise 2(a)

Given the frequency $\nu = 1,700$ Hz, the wavelength λ can be found from the relation

$$v = \nu\lambda\,,$$

where v is the speed of sound in air. Using the known experimental value of the speed of sound in air $v = 340\,\mathrm{ms}^{-1}$, we have

$$340 = 1.700 \times \lambda\,,$$

$$\therefore \lambda = \frac{340\ \mathrm{ms}^{-1}}{1,700\ \mathrm{Hz}} = \frac{2}{10}\,\mathrm{m} = 0.2\,\mathrm{m}\,,$$

The wavelength of a sound wave propagating in air with frequency $\nu = 1,700$ Hz is thus

$$\lambda = 20\,\mathrm{cm}\,.$$

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□

Exercise 2(b)

Consider the same sound wave with frequency $\nu = 1,700$ Hz but now in a metal where its speed is much higher, $v = 5,100\,\mathrm{ms}^{-1}$. From this data and the equation

$$v = \nu\lambda\,,$$

it follows that

$$\begin{aligned} 5,100 &= 1.700 \times \lambda\,, \\ \therefore \lambda &= \frac{5,100\ \mathrm{ms}^{-1}}{1,700\ \mathrm{Hz}} = \frac{51}{17}\,\mathrm{m} = 3\,\mathrm{m}\,. \end{aligned}$$

The wavelength of a sound wave in this metal with frequency $\nu = 1,700$ Hz is thus

$$\lambda = 300\,\mathrm{cm}\,,$$

which is fifteen times larger than the same sound wavelength in air.

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Exercise 2(c) Consider the radio waves with frequencies $\nu = 103.4$ MHz and $\nu = 855$ kHz, propagating with the speed of light, that is approximately $c = 3 \times 10^8\,\mathrm{ms}^{-1}$. The corresponding wavelengths λ can be found from

$$v = \nu\lambda\,.$$

For the FM waves, $\nu = 103.4$ MHz, so

$$3 \times 10^8 = 103.4 \times 10^6 \times \lambda\,,$$

$$\therefore \lambda = \frac{3 \times 10^8\ \mathrm{ms}^{-1}}{103.4 \times 10^6\mathrm{Hz}} = \frac{3}{103.4} \times 10^2\ \mathrm{m} = 0.3\,\mathrm{m}\,,$$

while for the AM band with frequency $\nu = 855$ kHz we have

$$3 \times 10^8 = 855 \times 10^3 \times \lambda\,,$$

$$\therefore \lambda = \frac{3 \times 10^8\ \mathrm{ms}^{-1}}{855 \times 10^3\mathrm{Hz}} = \frac{3}{855} \times 10^5\ \mathrm{m} = 300\,\mathrm{m}\,.$$

Note the relations $1\ \mathrm{MHz}{=}10^6\ \mathrm{Hz}$ and $1\ \mathrm{kHz} = 10^3\,\mathrm{Hz}$.

Click on the **green** square to return □

Exercise 3(a)

The frequency of an electromagnetic wave is given by

$$\nu = \frac{\text{speed of light}}{\text{wavelength}} = \frac{3 \times 10^8 \, \text{ms}^{-1}}{\lambda \, \text{m}} \, ,$$

and noting that the range of the wavelength λ of the microwave radiation is

$$10^{-3} \, \text{m} < \lambda_{microwave} < 1 \, \text{m}$$

the highest frequency of a microwave radiation is

$$\nu_{microwave}^{max} = \frac{3 \times 10^8}{10^{-3}} \, \text{Hz} = 3 \times 10^{11} \, \text{Hz} \, .$$

Click on the **green** square to return □

Exercise 3(b)

The range of λ_X ray wavelengths is

$$10^{-12}\,\mathrm{m} < \lambda_X < 10^{-8}\,\mathrm{m}\,.$$

From

$$\nu = \frac{\text{speed of light}}{\text{wavelength}} = \frac{3\times 10^{8}\,\mathrm{ms}^{-1}}{\lambda\,\mathrm{m}}\,,$$

one can calculate the lowest X ray frequency

$$\nu_x^{min} = \frac{3\times 10^{8}}{10^{-8}}\,\mathrm{Hz} = 3\times 10^{16}\,\mathrm{Hz}\,.$$

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Exercise 3(c)

From

$$\nu = \frac{\text{speed of light}}{\text{wavelength}} = \frac{3 \times 10^8 \,\text{ms}^{-1}}{\lambda \,\text{m}},$$

and noting that the wavelength interval for the infra-red radiation is

$$10^{-6}\,\text{m} < \lambda_{IR} < 10^{-3}\,\text{m}.$$

one finds the band of frequencies of infra-red radiation

$$3 \times 10^{11}\,\text{Hz} < \lambda_{IR} < 3 \times 10^{14}\,\text{Hz}.$$

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Exercise 3(d)

The visible light wavelength interval is

$$4 \times 10^{-7}\,\mathrm{m} < \lambda_{visible} < 8 \times 10^{-7}\,\mathrm{m}\,.$$

The formula for the electromagnetic wave period $T = \dfrac{1}{\nu}$ in terms of the speed of light and its wavelength λ is

$$T = \frac{\text{wavelength}}{\text{speed of light}} = \frac{\lambda\,\mathrm{m}}{3 \times 10^{8}\,\mathrm{ms}^{-1}}\,,$$

thus the lowest period of the visible is

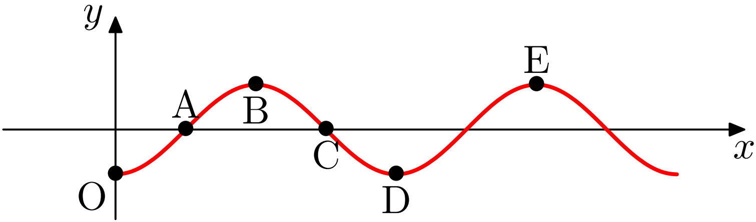
$$T^{min}_{visible} = \frac{10^{-8}}{3 \times 10^{8}}\,\mathrm{s} = 3 \times 10^{16}\,\mathrm{s}\,.$$

Click on the **green** square to return □

Exercise 4(a)

The point O lags the point A by a quarter of a full cycle and therefore the phase difference between these points is

$$\Delta\phi_{\text{OA}} = \frac{1}{4} \times 2\pi = \frac{\pi}{2}\,.$$



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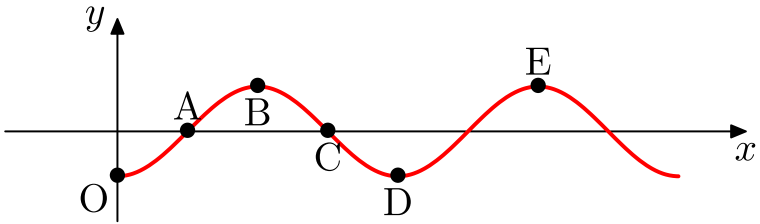
Exercise 4(b)

From the graph, we see that the point O is in phase with point D.



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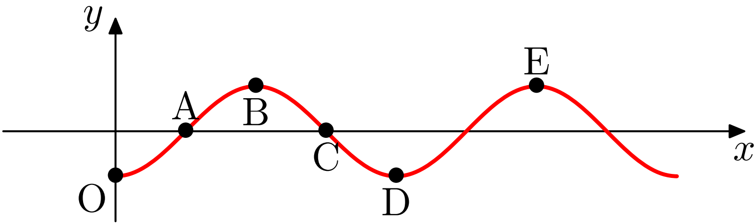
Exercise 4(c)

From the picture below we see that the point C leads the point O by three quarters of a cycle. Therefore the phase difference between these points is

$$\Delta\phi_{\rm OC} = \frac{3}{4} \times 2\pi = \frac{3}{2}\,\pi\,.$$



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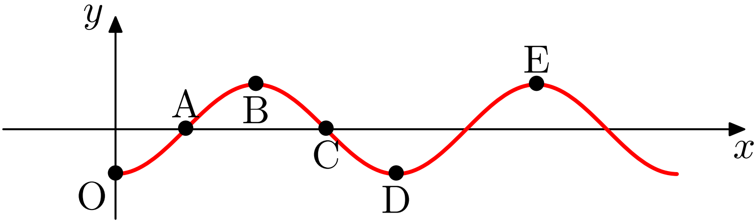
Exercise 4(d)

The picture below shows that the point A lags the point C by half of the whole cycle. So the phase difference between the points A and C is

$$\Delta\phi_{\text{AC}} = \frac{1}{2} \times 2\pi = \pi\,.$$



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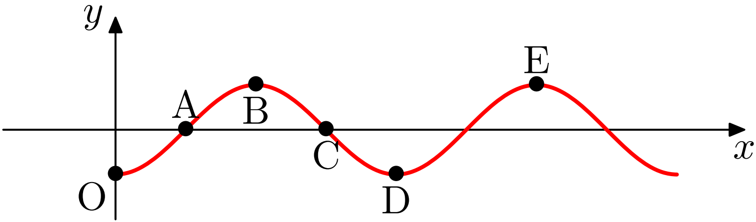
Exercise 4(e)

From the picture below we see that the point B lags the point C by a quarter of the whole cycle while the point D leads the point C a quarter cycle. Therefore the phase difference between the points B and D is

$$\Delta\phi_{\text{BD}} = \frac{1}{2}\,\pi + \frac{1}{2}\,\pi = \pi\,.$$



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Solutions to Quizzes

Solution to Quiz:

Frequency ν and period T of the wave are inversely related

$$\nu = \frac{1}{T}\,.$$

Therefore if the audible frequency range is

$$20\,\mathrm{Hz} < \nu < 20,000\,\mathrm{Hz}$$

then the period lies between (note the reversal of the inequalities !)

$$\frac{1}{20,000}\,\mathrm{s} < T < \frac{1}{20}\,\mathrm{s}\,.$$

So, the maximum period of an audible wave is

$$T_{max} = \frac{1}{20}\,\mathrm{s} = 0.05\,\mathrm{s}\,,$$

End Quiz

Solution to Quiz:
To select the correct result of superposing the two given waves, consider the interference at different points x. From the picture at point $x = 0$ the disturbance from the x-axis of the red wave is $y = 4$, while the disturbance of the blue wave is $y = 0$. The resulting wave disturbance y_{sup} at this point is thus

$$y_{sup}(x = 0) = 4 + 0 = 4\,.$$

The pictures **(b)** and **(c)** are therefore incorrect.

Similarly we see that constructive interference occurs at $x = 3\pi/2$:

$$y_{sup}(x = \frac{3}{2}\pi) = -4 - 4 = -8\,.$$

Comparing the waves of the remaining pictures **(a)** and **(d)** we see that only picture **(d)** is correct.

End Quiz