Centre for Teaching Mathematics News

Issue 4 Autumn 2001

www.tech.plym.ac.uk/maths/CTMHOME/CTM.HTML

Welcome

Welcome to the Autumn edition of the CTM News. We publish this newsletter every term and distribute it to schools, colleges and interested people. If you are reading somebody else's copy please contact the Centre secretary to be added to the mailing list. The newsletter will contain information on the staff and activities of the CTM. Each issue will contain a teaching resource which might be a graphic calculator activity, a problem solving activity or a practical mechanics problem. This issue contains the first of a series of games from around the world. Games are a great way of encouraging problem solving skills, logical thinking and visual thinking – all good skills for mathematicians.

The Centre for Teaching Mathematics

The CTM is an inter-faculty group of mathematics educators based at the University of Plymouth within the Mathematics Department and the Education Faculty at Exmouth plus associate members.

The aims of the Centre are:

Creative Resources and Research

Training for Teachers

Mathematics Enrichment for Pupils

Contacting Us

Members of the CTM can be contacted via the Secretary:

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Teacher Training News

Following the very successful training courses run last summer we have been busy preparing for Summer 2002. I know it seems a long way off, but if you get your request in now to those who hold the purse strings, you may be ahead of all the others!

The course we are running are:

Decision and Discrete Mathematics:

Teaching and Learning Decision and Discrete Mathematics 1

Teaching and Learning Decision and Discrete Mathematics 2

Graphic Calculators:

Teaching and Learning Mathematics with a Graphic Calculator for A level and International Baccalaureate

Using Graphic Calculators at Key Stage 3 of The National Mathematics Strategy

Dynamic Geometry:

Dynamic Geometry for KS3 and KS4

International Baccalaureate:

Supporting IB Mathematics.

Mechanics:

Mechanics at A level.

Science:

Data logging and hand held technology for KS4 Science

Details of all these courses will be mailed to your school in January but information and booking forms are already available on our website

www.tech.plym.ac.uk/maths/CTMHOME/training_courses.htm

In addition to these courses we are available to run any courses in your school or college at any time and tailor it to meet your specific needs. You may wish to invite teachers from other schools to participate. For further details please contact Jenny Sharp.
jsharp@plymouth.ac.uk

Staff Profile: Ted Graham

Ted Graham has been a member of the Centre for Teaching Mathematics since its formation in 1990, but has worked in the University since 1988. His first contact was when he started his PhD in 1988, working on the use of video and misconceptions in mechanics.

Ted had obtained a degree in mathematics from Imperial College in 1981 and gone on to train as a teacher at Chelsea College, gaining his PGCE in 1982. He then worked as a mathematics teacher at Tavistock College in Devon and part-time at HMP Dartmoor before moving into research. Since completing his PhD, Ted has worked in the University, lecturing, working on a number of projects, continued his own research and supervised other research students.

Research Interests

Ted's main focus of research has been in the area of student understanding of mechanics. He has continued to pursue this since completing his PhD and supervised the work that Stuart Rowlands did to obtain his PhD. The work has moved from researching misconceptions in a variety of aspects of mechanics to developing strategies to overcome misconceptions. The use of concept questions particularly within a framework of parallel questions has been the most significant strand to this strategy. However the use of computer packages and graphics calculators has also played a part. The current emphasis of his work is in the use of motion detectors connected to graphics calculators to overcome misconceptions associated with kinematics graphs.

Ted's other main interest is in the use of new technology in the teaching of mathematics. He has done work on the use of the computer algebra package DERIVE. Currently Ted is supervising two research students working in this area. He is working with Brian Walker on an evaluation of the of desk top video conferencing provide one to one tutorial support. This is an exciting project that is making the use of the latest technology and is promising to provide some very interesting results. Paulette Smith has just started work with Ted. Her work is on exploring the potential to use graphics calculators with children in Key Stage 2. It is hoped that this project will break new ground and provide both interesting results and some quality resources for teachers to use. Our new research student has just started to work with Ted on a graphic calculator project.

Ted is associated with two journals. He is an assistant editor of the International Journal of Computer Algebra in Mathematics Education and a member of the Advisory Group of the journal Teaching Mathematics and its Applications.

Teaching Interests

Ted is involved in four main areas of teaching in the University. One is the Foundation Pathways programme, for which he is the admissions tutor and leads modules on mechanics and mathematical modelling. The second is on the Mathematics with Education degree. Here Ted runs the second year placement module and supervises final year projects. For several years Ted has taught mathematics to civil engineering and building students. Finally Ted faces a new challenge this year as he takes on a new module to provide the mathematics input for the geology students.

Projects and Other Interests

Ted has been heavily involved in the Mathematics Enhancement Project run by David Burghes at the Centre for Innovation in Mathematics Teaching. More details on this project can be found on the CIMT website, but Ted has played a key role in writing new texts for the project schools to use and acting as a consultant to a number of schools as the project was first implemented. Ted has written 9 text books to support the project, 3 for Key Stage 4 and most recently a series of 6 to cover Key Stage 3. This has been a massive task that has provided many challenges. The books, along with teacher support materials can be found on the MEP website. (www.intermep.org)

Ted has also authored a number of other books, many dealing with mechanics for A-level. Other titles have been intended to support students taking A-level mathematics or have been designed to encourage the use of new technology. Ted is also extensively involved in A-level examining. He has been a chief examiner in mechanics since 1994 and was extensively involved in the development of one of the new specifications.

egraham@plymouth.ac.uk

Continuing Professional Development for Teachers

The Centre offers mathematics teachers opportunities for continuing professional development in several ways. Our programme of summer courses focus on curriculum developments such as the key stage 3 mathematics initiative and Decision Mathematics at A level. Research leading to Masters and Doctorate higher degrees are available in part-time and full-time format.

The DfES funds several initiatives as part of its continuing professional development for teachers.

The **Best Practice Research Scholarships** have been set up "to enable teachers to undertake classroom-based and sharply focused small-scale studies in priority areas, and to apply and disseminate their findings. Using research processes to investigate classroom practice is a good way of increasing understanding about how to raise standards of teaching and learning." (DfES, 2001)

The **Sabbatical Scheme for Experienced Teachers in Challenging Schools** is designed to create opportunities for experieced teachers working in challenging schools to undertake a significant period of development to enhance their own learning and effectiveness, and bring subsequent benefits to their pupils and to their school.

If you, or a colleague, are interested in one of these schemes and would like to collaborate with an University then the Centre for Teaching Mathematics would be pleased to provide support in the preparation of a submission and supervision for the research or development project.
For further details of these schemes please visit the DfES web site: www.dfee.gov.uk/teachers/cpd .
For an informal discussion of how you might collaborate with the Centre please contact John Berry.

jberry@plymouth.ac.uk

What grade is needed to study for a Mathematics Degree?

This is a common question that we are asked by teachers and sixth formers when visiting schools and when students visit the university. There has been considerable debate over recent years about the decline in level and standards of examinations in Mathematics, both at GCSE and at Advanced level. So if students know less mathematics now than say five years ago should we expect higher grades at A level before accepting them onto our mathematics degree at Plymouth?

To answer this question we have carried out a simple study of the relationship between entry qualifications and degree classification for students joining our Mathematics degree during the past five years. A summary of the data is shown in the table below:

	1st	2:1	2:2	3rd	pass	DipHE	
A	4	1	4	1	1	0	11
B	2	6	5	1	0	2	16
C	1	7	10	1	3	5	27
D	2	4	6	2	5	1	20
E	1	0	4	1	1	0	7
Access	0	0	0	0	0	0	0
Other	0	0	0	0	0	0	0
	10	18	29	6	10	8	81

The number of possible categories of grade or degree needs considerably more data to be confident in using data categories as presented. So it is necessary to concatenate the data. Using the following amalgamations: A/B, C, D/E grades; 1^{st}/2:1, 2:2/3^{rd}, pass/dip leads to a three by three arrangement and sufficient expected counts to feel it is possible to rely on the statistical analysis. There is insufficient evidence to support the hypothesis that A level grade and degree outcome are related.

This conclusion is not new. There have been several studies that show that good A level grades do not necessarily lead to good degree classifications. Mathematics takes on a different form on degree programmes; there is a breadth and depth that builds on the algebra and calculus developed in school but there are also new topics and applications that provide new opportunities. There are not the pressures of the teach/test school environment where mathematics is often seen as a body of knowledge that must be learned for an examination. Most universities have adapted their first year modules to take account of the changes in school mathematics over the past ten years. At Plymouth, during the first term, we review much of the A2 mathematics curriculum to ensure a solid foundation for further study. At the same time we develop the rigour and elements of proof that are the cornerstone of pure mathematics. There are also opportunities to explore applications of mathematics.

Thus the answer to the question "what grade is needed to study for a Mathematics Degree?" is ideally a grade A-C, but if a student gains a grade D or E then they could still consider joining a mathematics degree. The most important ingredient is an interest in mathematics.

The benefits of studying for a mathematics degree are enormous. Mathematics graduates are highly sought after in business, commerce and industry and five years after graduation are, on average, among the highest paid graduates. Students who choose a mathematics degree programme are able to keep their career options open unlike those who opt for more vocational degree routes.

We hope that as a mathematician you will encourage your mathematics students to consider joining a mathematics degree programme. For further details of studying Mathematics and/or Statistics at Plymouth and career opportunities in these disciplines please contact the Centre.

John Berry and Roger Fentem

Coming soon

In the early 1990's The Centre for Teaching Mathematics published a series of books – the Exploring... Series. They covered Pure Mathematics, Mechanics and Statistics and provided a bank of problems and investigations which challenged the student to think more deeply about the concepts and methods addressed in A level Mathematics. The A level investigation in this issue and the one in the last issue came from Exploring Pure Mathematics. The books are now out of print but many teachers have requested copies after seeing the students work on some of the problems at our VIth form days. We are in the process of adapting these books for our web pages where they will be available free of charge as PDF files.
www.tech.plym.ac.uk/maths/CTMHOME/resources.htm

Diary of a Work Experience Student

I chose to work at the Centre for Teaching Mathematics for my work experience because I thought it would be an interesting insight into the way in which a university department is run, and also because I hoped to learn more about the next step after GCSE's regarding maths as a career.

During the week I was set the task of programming Graphics Calculators, which I found and interesting challenge. I was helped though by having a prior knowledge of some BASIC programming, even with this however I found it difficult. Because of the difficulty of the task, finishing each program gave me a feeling of great satisfaction. As well as doing this task I also sat in on lectures, visited various people in different departments and toured the university, seeing all the different facilities for the students and staff.

I learnt a lot about the structure of a university and the differences between Universities and school, which are too numerous to mention. The major thing I learnt in the week however was how to program. Although I only acquired a basic knowledge this will be useful for numerous careers in the computer and technology industries. I think that my work experience gave me a good idea of the world of work (as it was supposed to) but I think I learnt a lot from the people around me and the skills that I gained whilst there will be helpful in later life.

I would recommend the CTM and the university in general as a place for work experience because there is a wide range of skills to be learnt and perfected, knowledge to be gained that will be needed in later life and it is a good way to experience life at university before you arrive there for your first day.

Daniel Stubbs
Year 11 at Hele's School, Plymouth

If your school wishes to place a student at the Centre for their work experience please contact Ted Graham for further details.

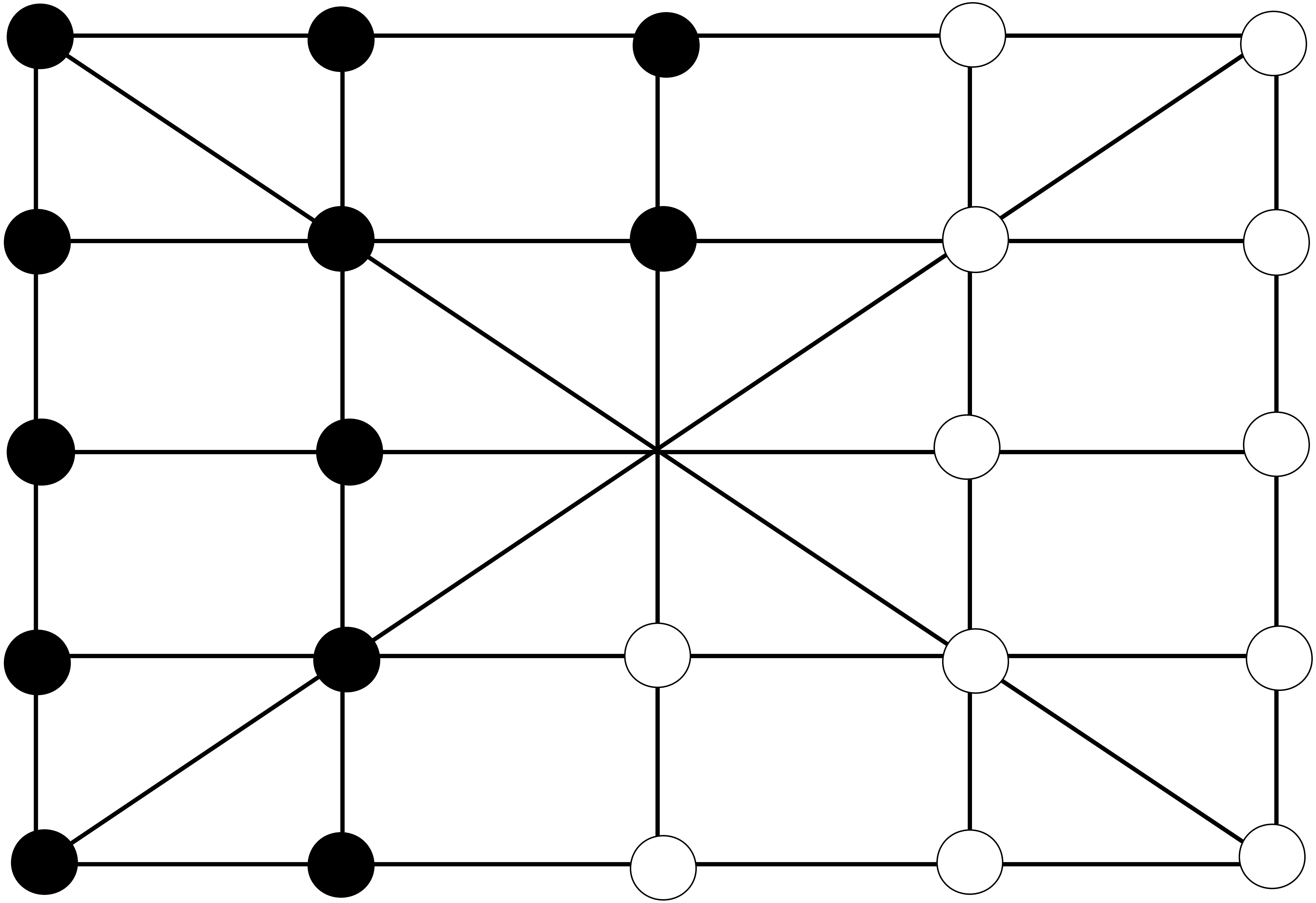
Alquerque
A Spanish Board Game

Alquerque is known to date back at least as far as 1400BC, since boards have been found cut into the roofing slabs of the temple at Kurna in Egypt. A game called *Quirkat* is mentioned in an Arabic work of the 10th Century AD. The earliest set of rules is found in the *Libro de Acedrex, Dados e Tablas*, a magnificently illuminated manuscript compiled between 1251 and 1282 by order of the King of Leon and Castile, Alfonso X. The game's Spanish name, derived from 'El-quirkat', was *Alquerque.*

Alquerque Rules

- One player has 12 white pieces, the other 12 black pieces laid out on the nodes of the board (except the centre).
- Decide who will move first. Each player, in his turn, moves one of his pieces from its current location to another point.
- A piece may move along one of the marked lines to an adjacent unoccupied point.
- Alternatively, if an adjacent point (along one of the marked lines) is occupied by an opponent's piece but the point beyond that (in a straight line) is empty, the player may capture his opponent's piece by jumping over it to the unoccupied point.
- If, after the jump is completed, another of the opponent's pieces is now available for capture, that piece may also be captured even if the second jump is along a different line to the first.
- If a player is able to capture an opponent's piece during his move, he must do so. If he does not, his opponent may, at the start of his own turn, take the piece that could have made a capture. (This is in addition to the player's normal move.)
- Play continues until one player has lost all his pieces.

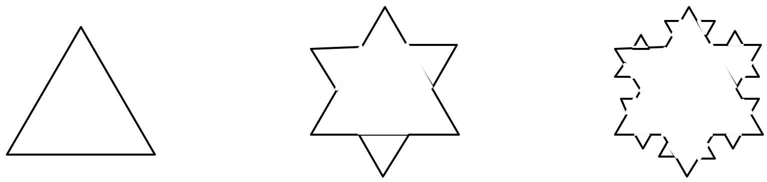
Game board overleaf (enlarge to A3).............



Snowflakes

The following procedure describes the formulation of the snowflake curve.
Take an equilateral triangle and on each side, at its midpoint, construct an equilateral triangle whose side is one third the length of the original triangle.

Now repeat this process, constructing an equilateral triangle on each straight line segment of the resulting figure.
If the process was repeated indefinitely, investigate the length of the resulting outline and the area it would enclose.



Hints and Nudges

First it would be advisable to generate a snowflake curve yourself , and to note how the curve is formed.

1. How many new straight line segments are generated from each straight line segment at each stage of the formation process?
2. What is the length of the new segments at each stage of the formation process? Suppose initially that the side of the original equilateral triangle is one unit
3. How is the area of each new equilateral triangle related to the area of the original equilateral triangle?

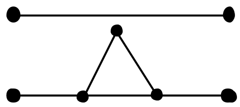
Teachers notes on Snowflakes

In order to determine the length of the snowflake curve and the area it would enclose, it is necessary to determine:

1. the number of straight line segments
2. the length of each straight line segment
3. the area of each new equilateral triangle added to the snowflake at each stage.

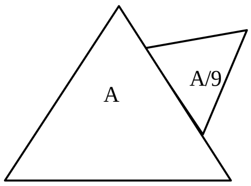
After some experimentation it becomes evident that

1. each straight line segment generates 4 new segments at each stage.



2. the length of each new segment is 1/3 of the length of the segment from which it is formed

3. each new equilateral triangle added to the snowflake is 1/9 of the area of the triangle to which it is added.



To determine the length of the curve
Initially there are 3 segments of length l. Applying the procedure once produces 4×3 segments of length $\frac{l}{3}$, twice produces 4×4×3 segments of length $\frac{1}{3}\times\frac{l}{3}$ etc. After applying the procedure n times there are $4^n\times 3$ segments of length $\left(\frac{1}{3}\right)^n\times l$ i.e. the length of the snowflake curve is given by $3\times\left(\frac{4}{3}\right)^n l$.

Thus if the process was repeated indefinitely, i.e. $n\to\infty$ then the length of the curve increases without limit.

To determine the area enclosed by the snowflake
Each time the procedure is applied the area increases by the number of new triangles added. One new triangle is added for each segment. Thus the nth time the procedure is applied the increase in area is given by

$$\left(4^{n-1}\times 3\right)\times\left(\frac{1}{9^n}A\right)$$

and the total area is:

$$A+3\times\left(\frac{1}{9}A\right)+(4\times 3)\times\left(\frac{1}{9^2}A\right)+$$

$$(4^2\times 3)\times\left(\frac{1}{9^3}A\right)+\ldots+(4^{n-1}\times 3)\times\left(\frac{1}{9^n}A\right)$$

$$=A\left(1+\frac{3}{9}\left(1+\frac{4}{9}+\left(\frac{4}{9}\right)^2+\ldots+\left(\frac{4}{9}\right)^{n-1}\right)\right)$$

Thus the area is determined by summing a geometric series of common ration 4/9. If the process is applied indefinitely i.e. as $n\to\infty$, the area converges to

$$A\left[1+\frac{3}{9}\times\frac{1}{\left(1-\frac{4}{9}\right)}\right]=1.6A$$

In conclusion, although the length of the snowflake increases without limit, the area it encloses never exceeds 1.6 times the area of the original triangle.