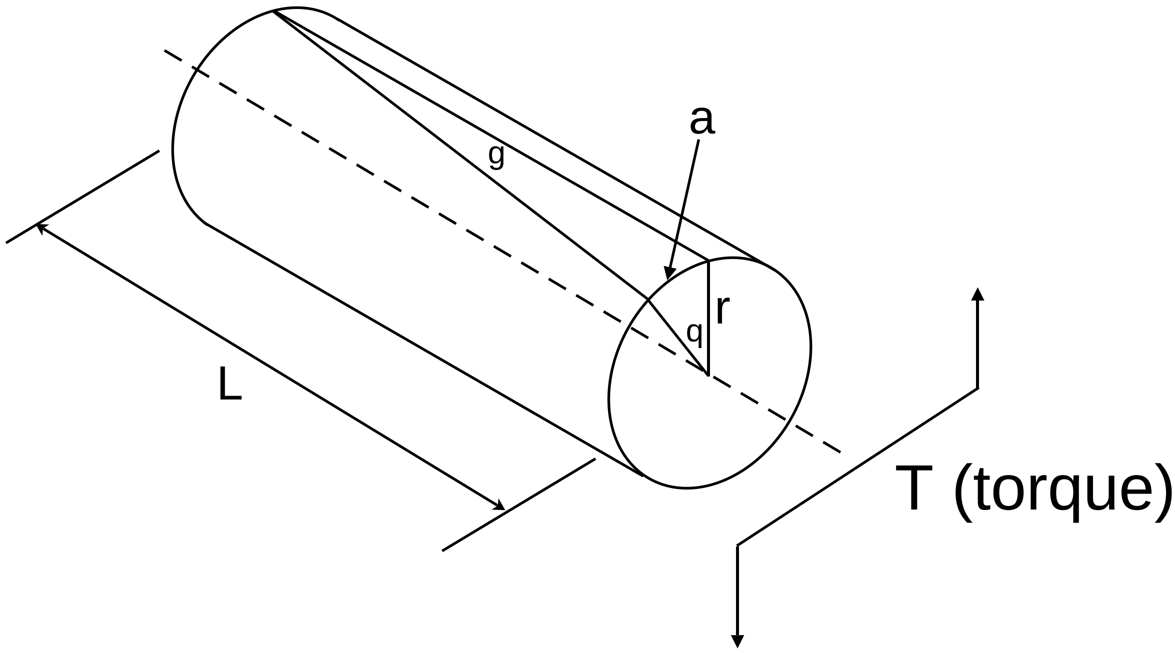
7. CIRCULAR SHAFTS in TORSION

The stresses which arise from torsion are shear stresses



arc length a = Lg = rq

$$g = \frac{rq}{L}$$

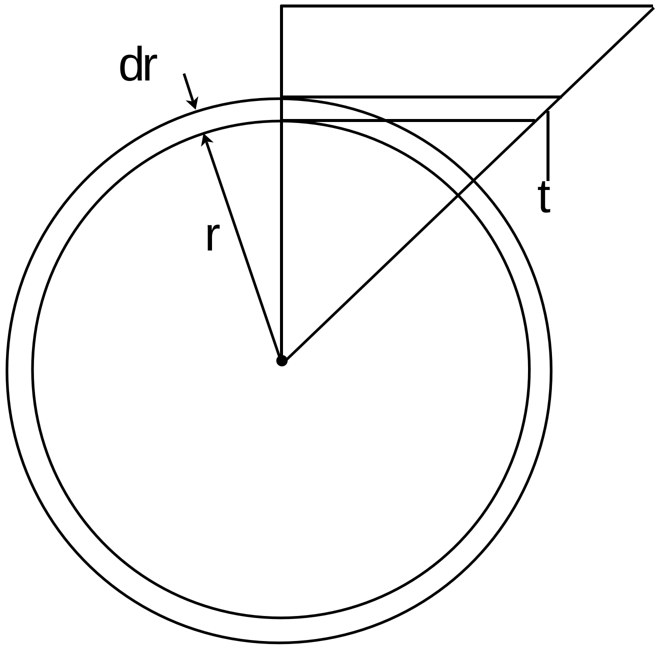
but $\quad t = G\,g$

$$t = \frac{G\,r\,q}{L}$$

or

$$\boxed{\frac{t}{r} = \frac{G\,q}{L}}$$

The net effect of the internal stresses caused by the externally applied torque (T) must be equal and opposite to it.



Force = stress x area = $t \times dr \times 2pr$

Torque = force x distance

$$dT = t \times dr \times 2pr \times r$$

To find the total torque we have to integrate from the centre to full radius (D/2).

$$T = \int_0^{\frac{D}{2}} t \times dr \times 2pr^2$$

but $$t = \frac{G\,r\,q}{L}$$

$$T = \int_0^{\frac{D}{2}} \frac{G\,q\,2pr^3 dr}{L}$$

$$T = \frac{G\,q}{L} \underbrace{\int_0^{\frac{D}{2}} 2pr^3 dr}$$

This is a geometric property of the cross-section known as the polar moment of inertia.
It is an important section property given symbol **J** .

We can summarise the formulae for torsion as below:

$$\frac{T}{\mathbf{J}} = \frac{Gq}{L} = \frac{t}{r}$$

This is known as the Torsion Equation.

Note that for a circular section $\mathbf{J} = \frac{pD^4}{32}$

For a tube (hollow circular section) $\mathbf{J} = \frac{p(D^4 - d^4)}{32}$

T = external Torque applied (Nm).

J = Polar moment of inertia (m^4)

G = Shear modulus of the material under bending.(N/m^2)

L = length of shaft under torsion (m) being twisted through an angle of q (radians).

t = shear stress level in the shaft (N/m^2) at radius **r** (m)

Care - when using the Torsion Equation make sure you use <u>standard SI units</u>
T often given in kNm [$=10^3$Nm]
J often given in cm^4 [$= (10^{-2}m)^4 = 10^{-8}m^4$]
G often given in GN/m^2 [$= 10^9$ N/m^2]
q often given in degrees [= p /180 radians]
t often given in MN/m^2 [$=10^6$ N/m^2]
r may be in mm or cm [10^{-3} or 10^{-2} m]