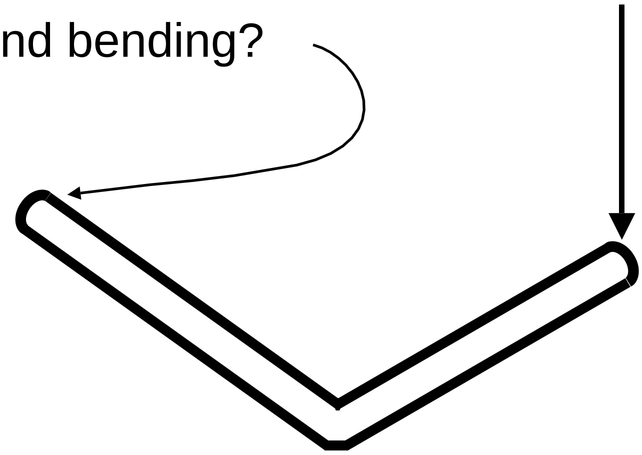
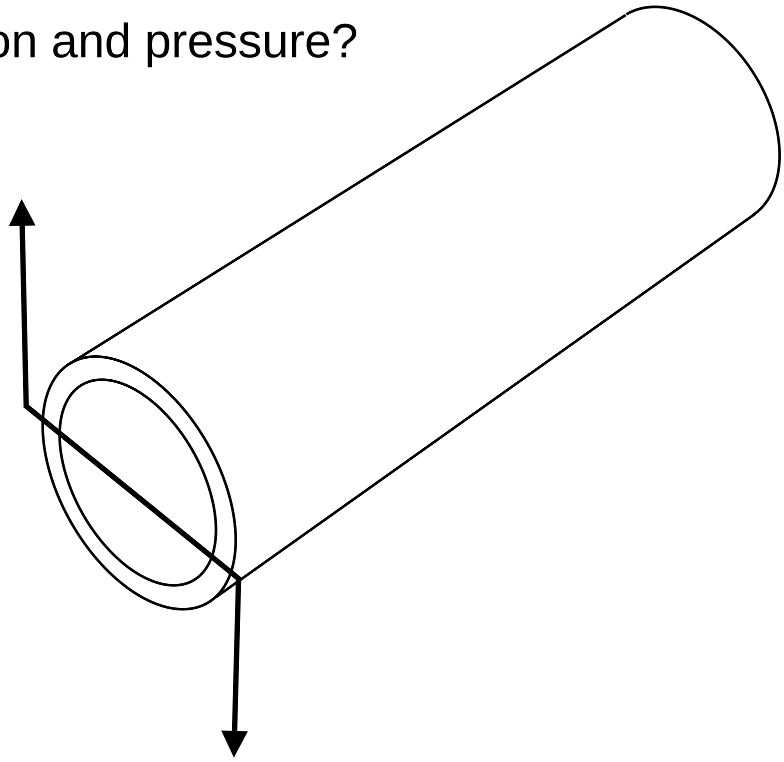
9. COMBINED STRESSES

We have studied a number of separate situations (tension, compression, torsion, bending, pressure in cylinders and spheres), but what if two or more of these situations occurred together?

e.g torsion and bending?

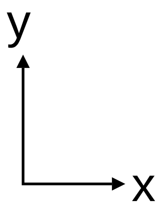


e.g torsion and pressure?

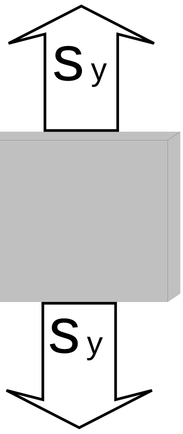


In order to find the combined effect we have to look at an element of material at particular locations, where both effects determine the stresses. We calculate the stresses as though they occurred separately, and then combine them to find the overall effect expressed as **principal stresses.**

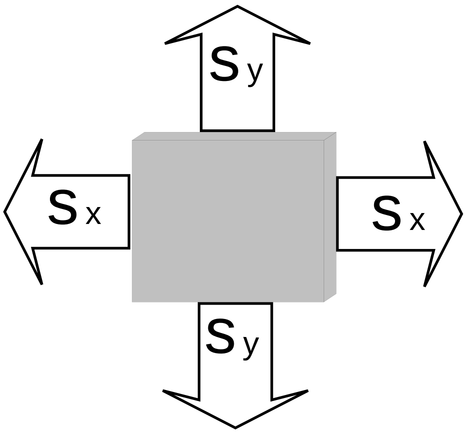
One principal
stress S_x
No shear stress



e.g pure tension or compression

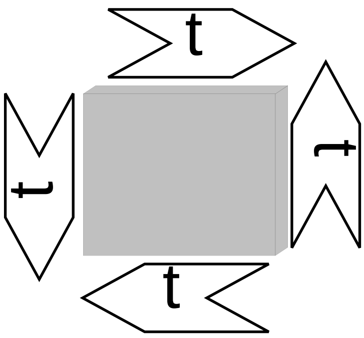


One principal
stress S_y
No shear stress



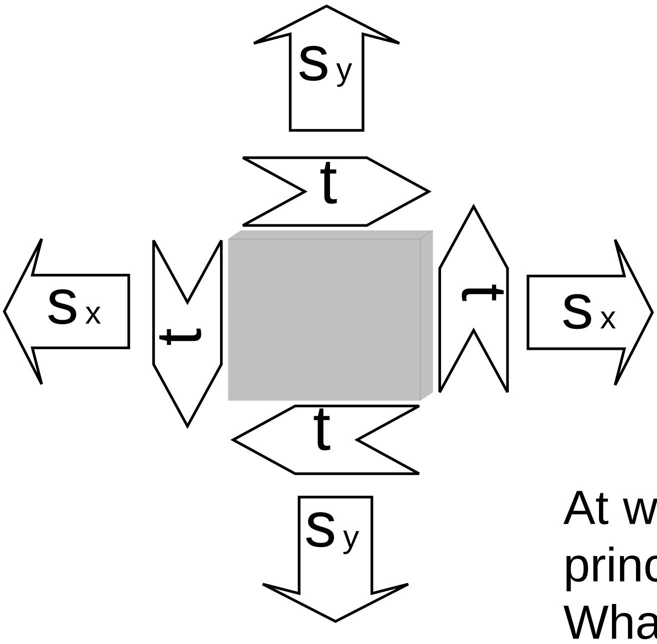
Two principal
stresses S_x & S_y
No shear stress

e.g stresses in cylinder or sphere



Element in pure shear:
Principal stresses are at 45°, one compressive the other tensile.

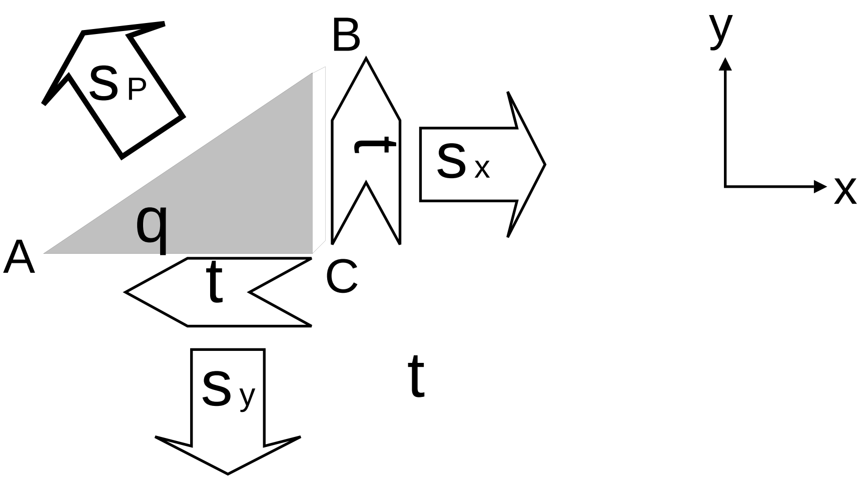
e.g pure torsion



At what angle will the principal stresses lie?
What will the values be?

e.g combined torsion and pressure

To find the principal stresses and their angle we need to consider the equilibrium of a triangular element where the sloping side is one of the principal planes
(i.e. - no shear stress acts on it.)



For equilibrium : SFx=0 & SFy=0

$$SF_x = s_x BC - t AC - s_P AB \sin q = 0$$
$$SF_y = s_P AB \cos q - s_y AC + t BC = 0$$

but

if AB = 1, then BC = sin q, and AC = cos q

then the above equations become:

$$s_x \sin q - t \cos q - s_P \sin q = 0 \ldots\ldots\ldots\ldots (i)$$
$$s_P \cos q - s_y \cos q + t \sin q = 0 \ldots\ldots\ldots\ldots (ii)$$

These represent two simultaneous equations which can be solved to find s_P *and* q.

If we go through the necessary algebra we find that:

$$2\theta = \tan^{-1}\left[\frac{\tau}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}\right]$$

$$\sigma_P = \left(\frac{\sigma_x + \sigma_y}{2}\right) \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

We can more easily interpret these equations as those describing a circle drawn on axes of: -

Direct (x and y) stresses (horizontal);

and Shear stress (vertical):

Its centre is at the average value of σ_x and σ_y and its radius is the hypotenuse of a right angled triangle of sides $\frac{(\sigma_x - \sigma_y)}{2}$ and τ.

This construction is known as Mohr's Circle

Mohr's circle