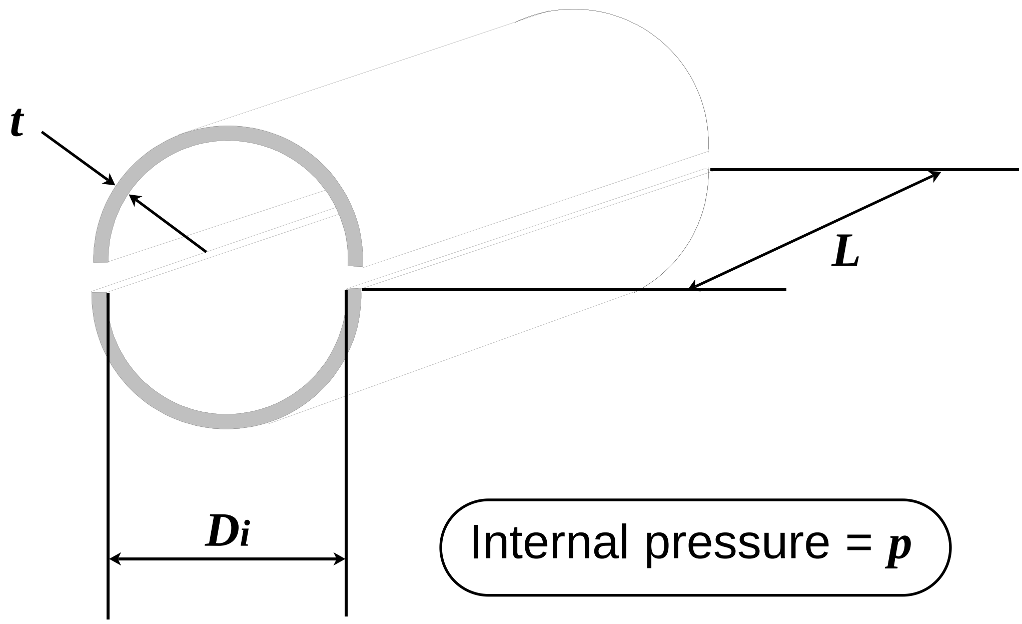
8. THIN WALLED CYLINDERS and SPHERES

Many pressure vessels are of cylindrical shape and may be described as ‘thin-walled’
ie. the wall thickness is typically less than 1/20th of the diameter

When a cylindrical vessel is pressurised two kinds of stress are caused : **circumferential** and **axial**

Circumferential Stress



The force tending to push the two halves apart is given by :

$$F = p\, D_i\, L$$

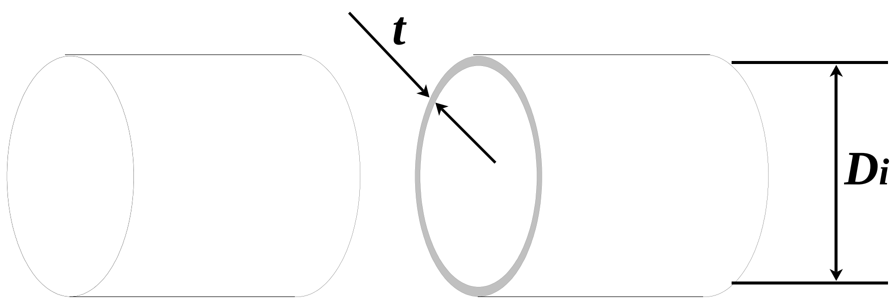
The cross-section area of material which sustains this force is given by:

$$A = 2\, t\, L$$

Therefore the circumferential stress is given by:

$$\sigma_{circ} = \frac{F}{A} = \frac{p\, D_i}{2\, t}$$

Axial (or longitudinal) stress



Internal pressure = p

The force tending to push the two halves apart is given by :

$$F = p\,\frac{\pi D_i^2}{4}$$

The cross-section area of material which sustains this force is given by:

$$A = \pi\, D_i\, t$$

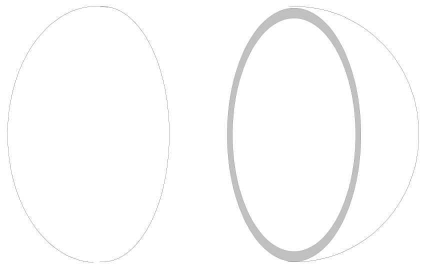
Therefore the axial stress is given by:

$$\sigma_{axial} = \frac{F}{A} = \frac{p\,D_i}{4\,t}$$

NB

$$\sigma_{axial} = \tfrac{1}{2}\,\sigma_{circ}$$

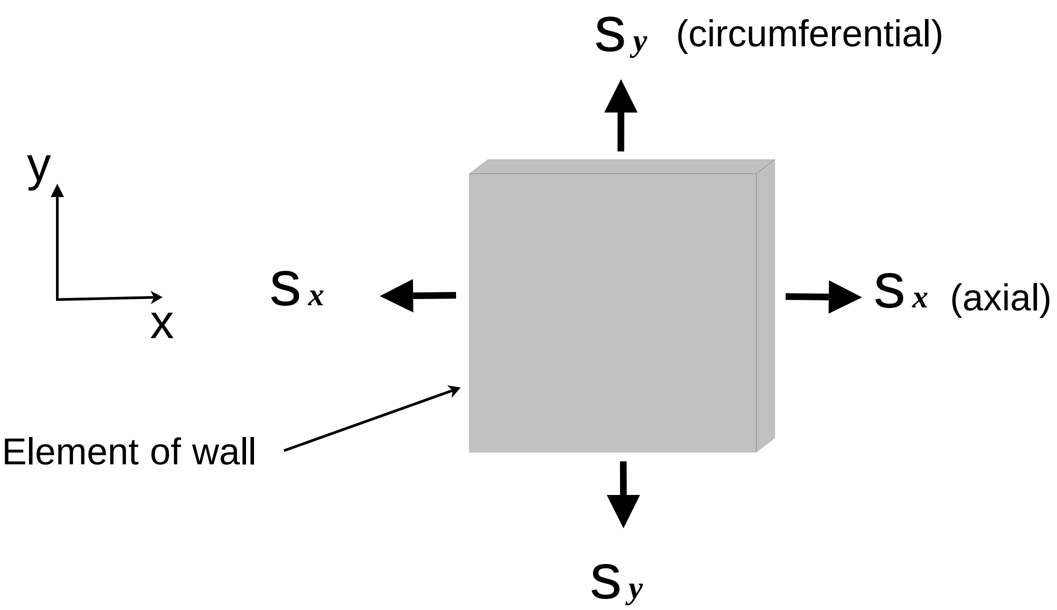
Spherical vessels



The situation is the same as for the axial stress in a cylinder. Since it doesn't matter which way a sphere is cut, the stress is the same in all directions

$$S_{circ} = \frac{F}{A} = \frac{p\,D_i}{2\,t}$$

The general stress situation:

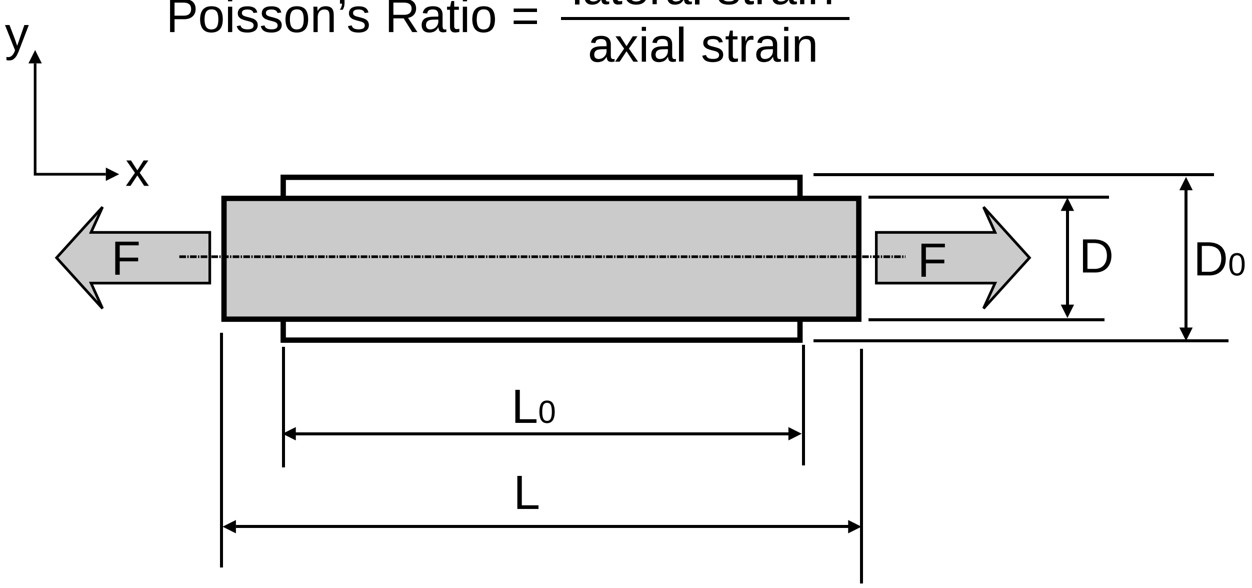


Lateral Effects

In both cases (cylinder and sphere) there are stresses in both the x and y directions.

When a material is stretched in one direction it will contract in the other direction i.e. laterally. (e.g rubber)
This effect is specified by Poisson’s Ratio (n)

$$\text{Poisson's Ratio} = \frac{\text{lateral strain}}{\text{axial strain}}$$



$$\text{Axial strain } (e_x) = \frac{L - L_0}{L_0}$$

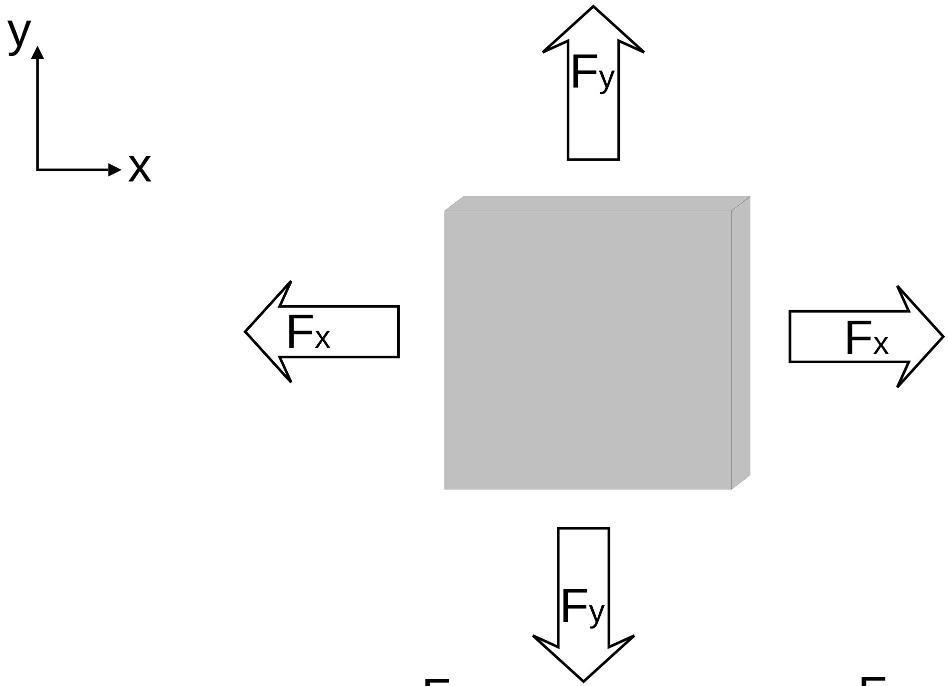
$$\text{Lateral strain } (e_y) = \frac{D - D_0}{D_0}$$ (N.B. e_y is negative)

$$\text{Poisson's ratio } (n) = -\frac{e_y}{e_x}$$

or

$$e_y = -n e_x$$

The situation shown on the previous page is comparatively simple in that we have <u>uni-axial</u> loading,
i.e. simple tension or compression.
If the material is also loaded in the lateral direction <u>both</u> the <u>loading</u> and the <u>Poisson's ratio effect</u> determine the strain.



$$\sigma_x = \frac{F_x}{A_x} \quad \& \quad \sigma_y = \frac{F_y}{A_y} \qquad \boxed{\varepsilon = \frac{\sigma}{E}}$$

In the x-direction there is the axial strain due to $F_x = \frac{\sigma_x}{E}$

there is also the lateral strain due to $F_y = -\nu\varepsilon_y$

but $\varepsilon_y = \frac{\sigma_y}{E}$

the resultant strain in the x-direction is therefore given by:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

We can apply exactly the same argument in the y-direction

To summarise:

With bi-axial stress the strain in each direction is determined by the loading and the Poisson's ratio effect:

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

It can be shown that if a material's volume remains constant when it is stressed Poisson's ratio = 0.5.
In reality the volume of a material does change and Poisson's ratio is less than 0.5. A typical value is ~ 0.3

It can also be shown that the volumetric strain is the sum of the linear strains, i.e.

$$\varepsilon_{vol} = \varepsilon_x + \varepsilon_y + \varepsilon_z$$