4. FORCES in PIN JOINTED STRUCTURES

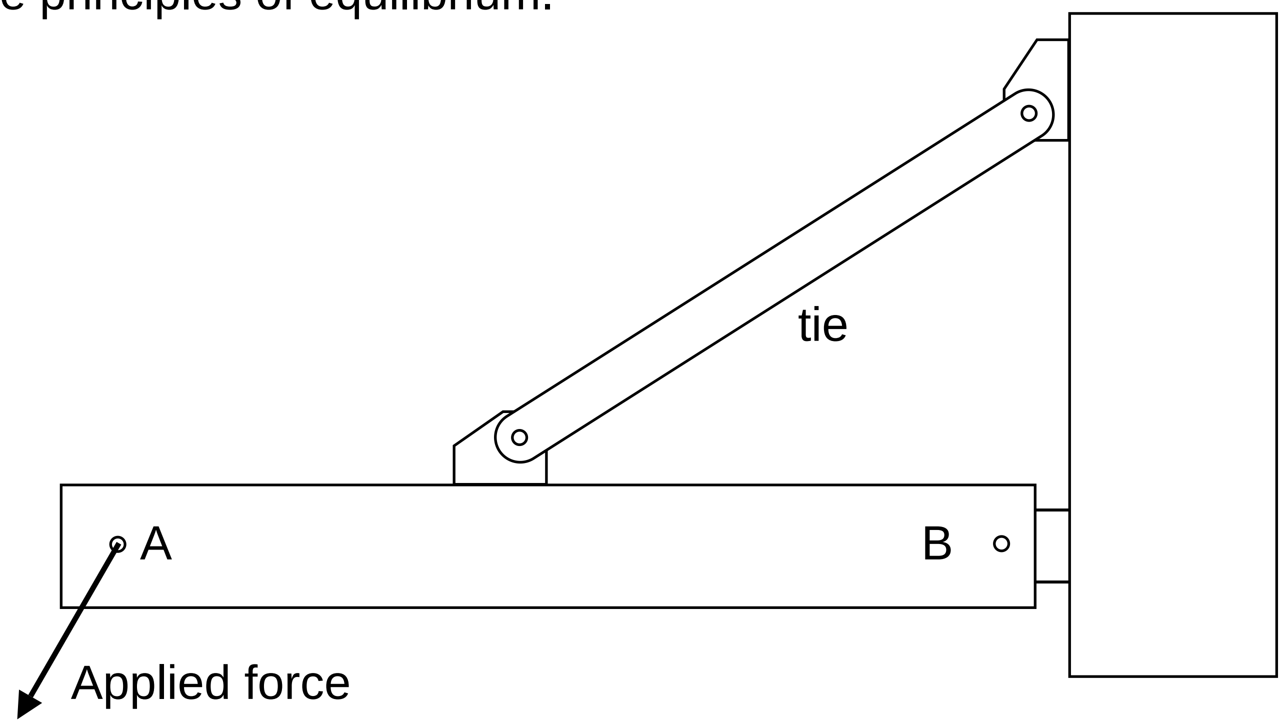
Pin jointed structures are often used because they are simple to design, relatively inexpensive to make, easy to construct, and easy to modify.

They can be ‘fixed’ structures such as frames, or they can be structures that move, more normally referred to as mechanisms.

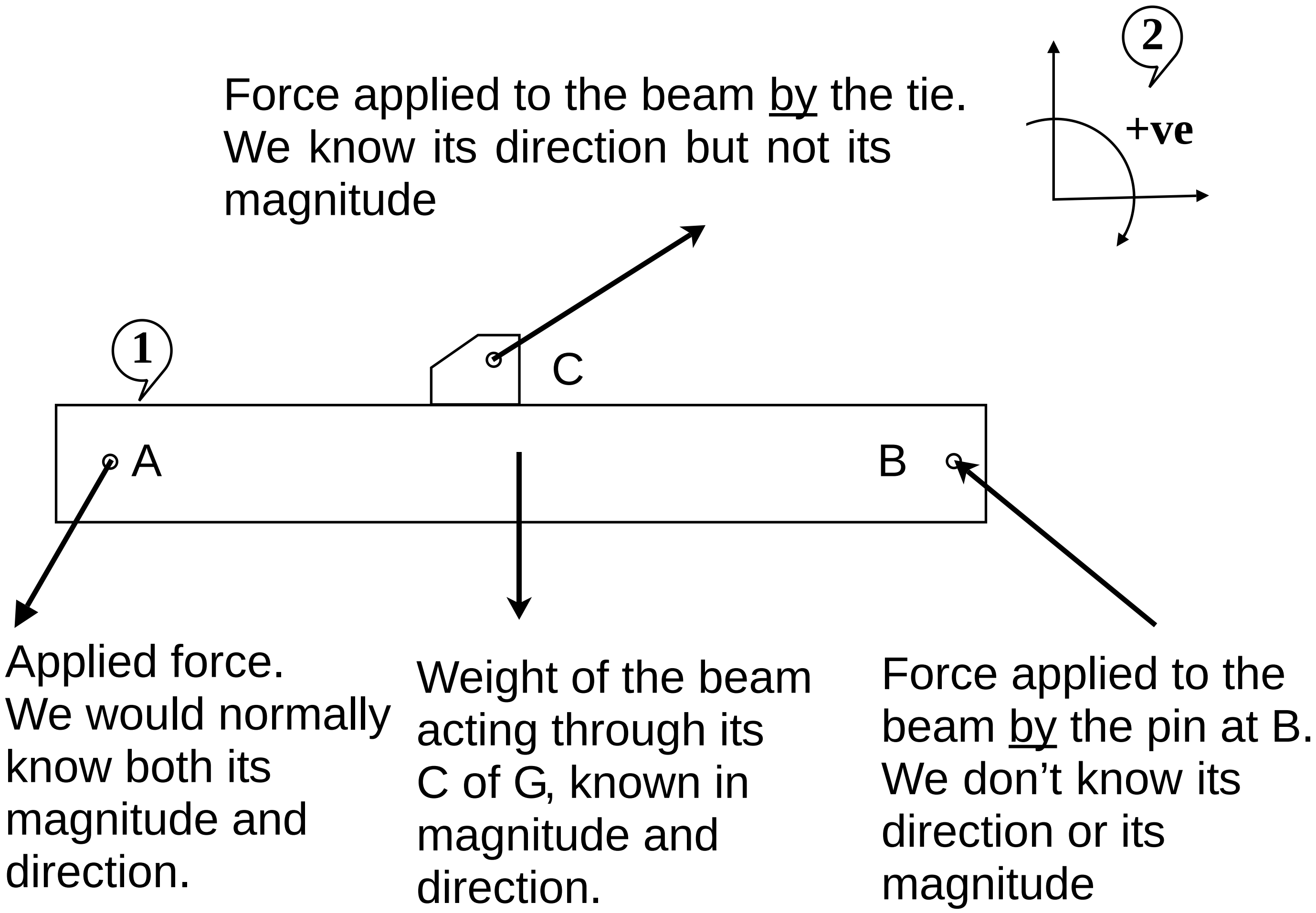
We have already noted that if an interaction is pin jointed, there is no friction at the pin. The pin can only transmit a force and has no ability to resist rotation.
In practice pin jointed structures often use bolts which are tightened and therefore they can resist rotation to a certain extent. However, because this cannot be ‘guaranteed’, (the bolt may work loose), any rotational resistance is usually ignored for the purposes of design and analysis.

In the structure we looked at earlier the tie is clearly in tension (if we cut it the ends would pull apart).

Because the pin joints do not resist rotation, we can easily find the forces on the pins using the principles of equilibrium.

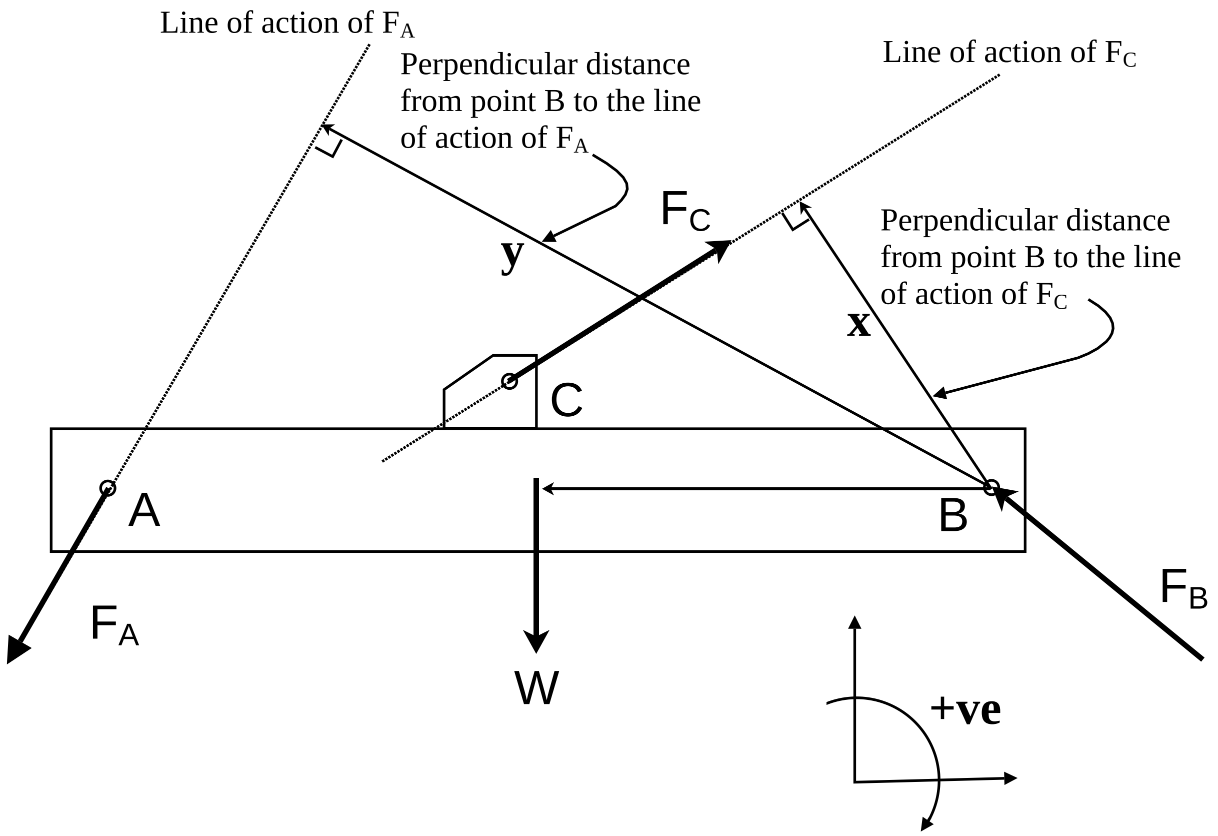


Let us draw the FBD of the beam AB. 3



Since the beam is in equilibrium we know that the vector sum of the forces must be zero, <u>and</u> the sum of the moments about any point must also be zero.

We can now apply the procedure to find the unknown forces. 4



Since we don't know the force at B in magnitude or direction, let us choose a point on its line of action about which to take moments. The only point we know it passes through is B.
So we can take moments about B.

The moment of force F_A about B = $- F_A y$

The moment of force F_C about B = $+ F_C x$

The moment of the weight about B = $- W \frac{AB}{2}$

Therefore the sum of the moments about B = $- F_A y + F_C x - W \frac{AB}{2}$

Since the sum of the moments about B must equal zero we can write:

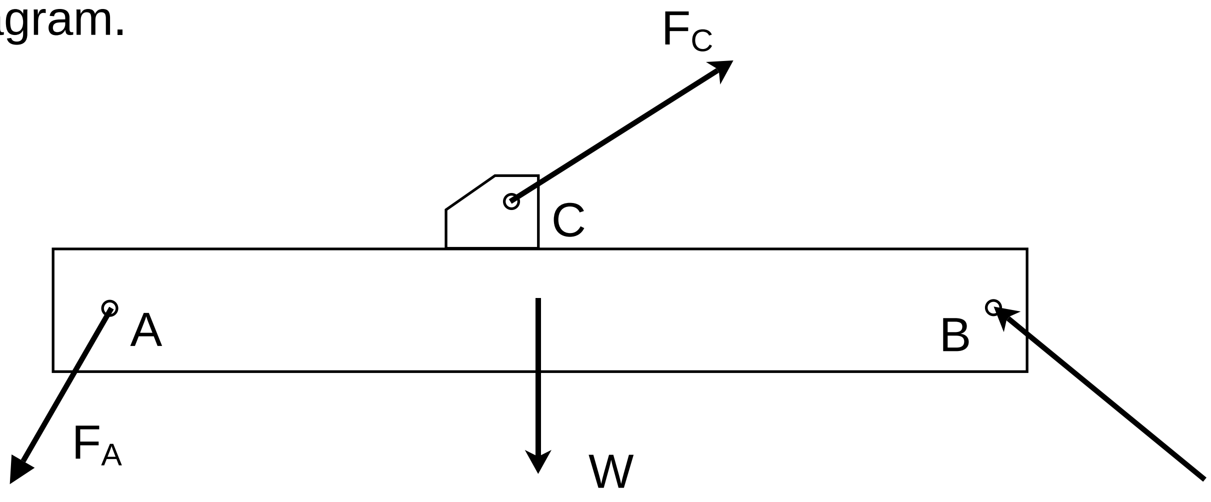
$$- F_A y + F_C x - W \frac{AB}{2} = 0$$

Since F_C is the only unknown we can solve it to find F_C.

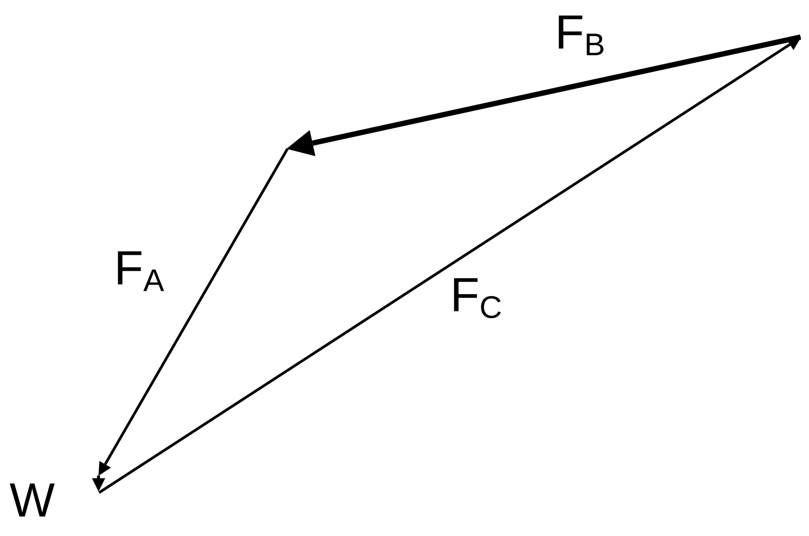
Assuming F_A = 100kN, W = 1kN, and the diagram above is drawn to a scale of 1cm = 1m find F_C

We still need to find F_B.

To find F_B we can use the condition that the vector sum of the forces must equal zero. i.e. if we draw the vector diagram, it must close.
Since we now know all the forces except F_B in magnitude and direction, F_B must be the force required to close the vector diagram.

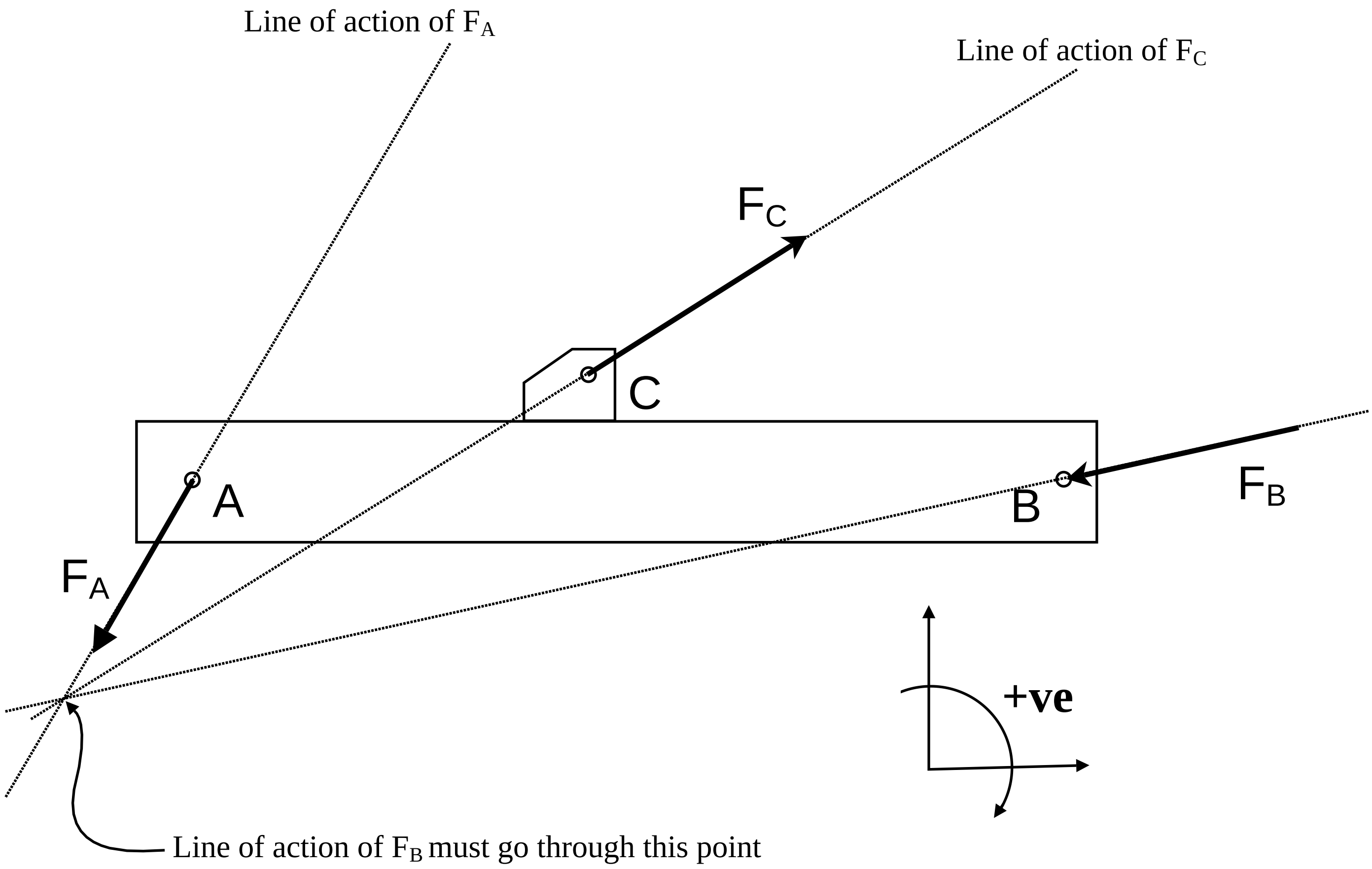


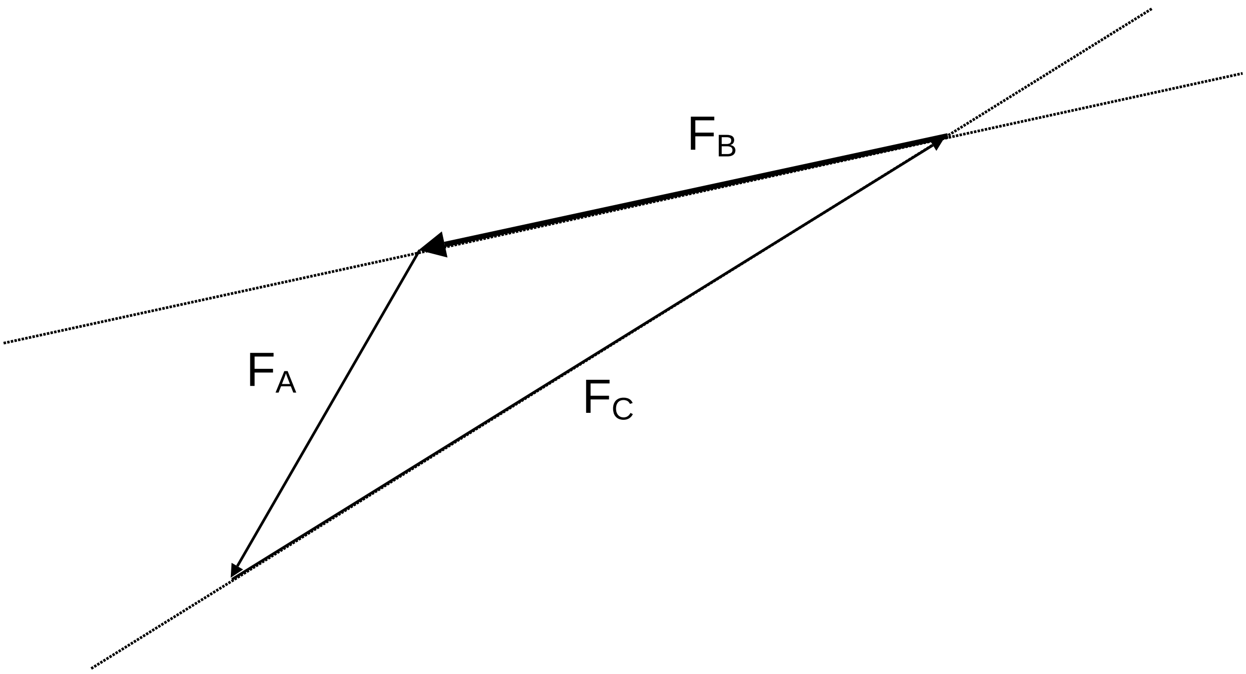
Use a scale of 20kN = 1cm to draw the vector diagram



Note that because the weight of the beam is so small compared to the loads it hardly makes any difference to the tension in the tie. We often ignore the weight of members for this reason.

If we had ignored the weight of the beam, there would have only been three forces acting on it. We know that if there are only three forces acting on a body their lines of action must all pass through a common point. We can use this to find the direction of force F_C, and solve the problem without having to take moments.





STRESS and STRAIN
in TENSION, COMPRESSION and SHEAR

So far we have considered the effects of forces on bodies and components, in terms of how they arise and how they act and interact with each other.
We have taken no account of the size of the components or how the forces affect the material from which they may be made.

In the previous section we determined (by applying the principles of equilibrium) that the tensile force in the tie CD was 221 kN.

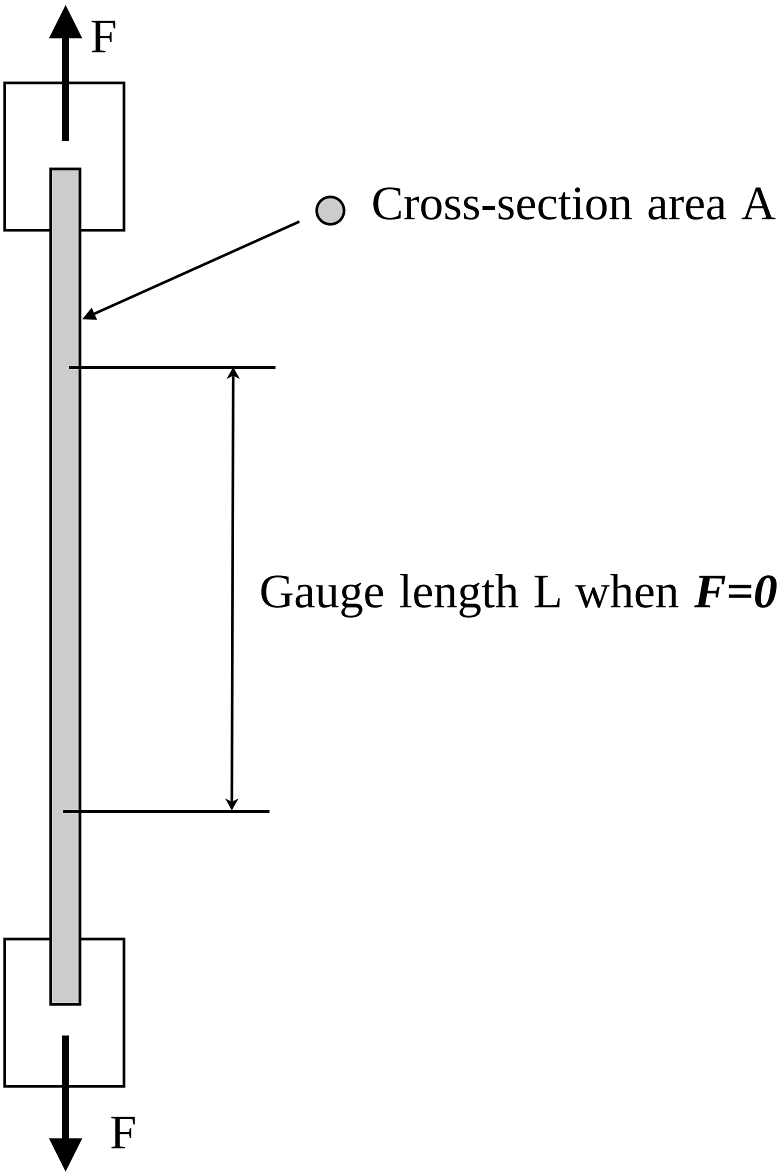
In general, we know that if we apply a high enough force to a component it will bend or break. We may wish to achieve this, or we may wish to prevent it.
e.g. When we manufacture something from a material we may wish to form it, bend it or cut it, whereas in use we normally don't want it to bend or fracture.

In the case of the tie for the previous structure what would you choose to make it from out of the following list:-
20 mm diameter hemp rope
3.0 mm diameter stainless steel wire
35 mm square section rubber
30 mm x 30 mm x 10 gauge RHS steel
50 mm x 50 mm hard wood?

Clearly we need to know something about the material as well as the force in order that the tie is strong enough, doesn't stretch too much, and has such other characteristics as the situation may require.

Properties of a material in tension

In order to compare one material with another we often carry out a simple tensile test on a specimen of uniform cross-section.



Because the pull force required to break the specimen depends upon the diameter we need to measure stress.

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Cross-section area}}$$

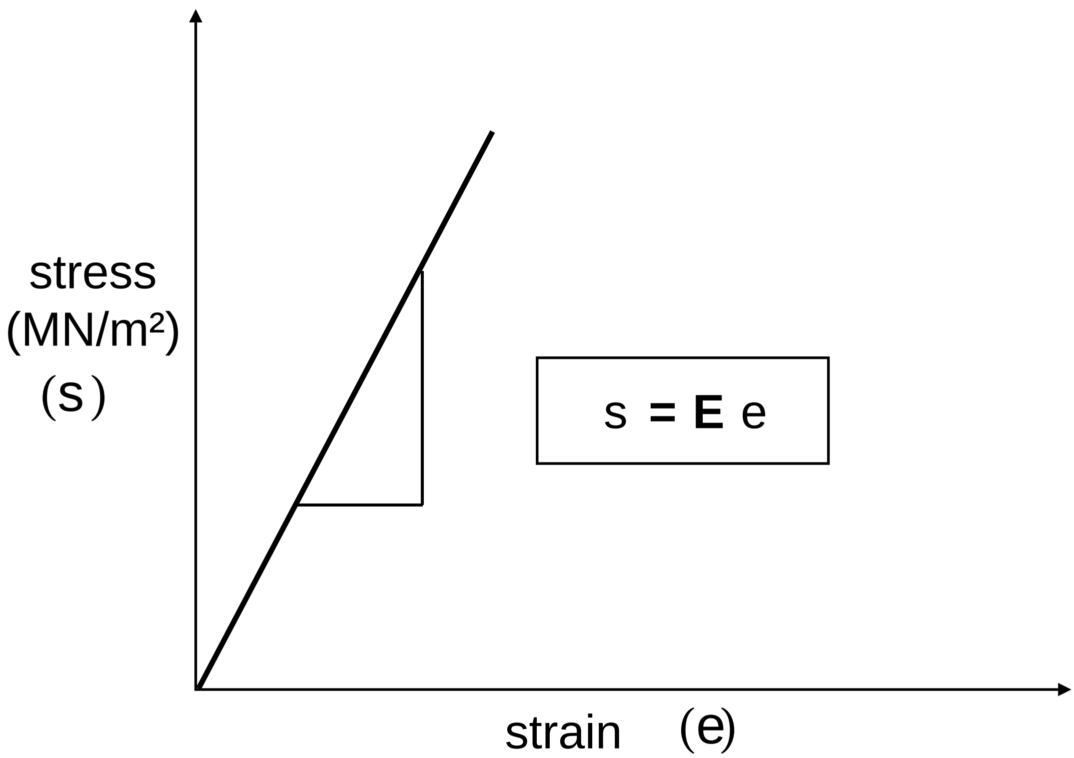
$$s = \frac{\mathbf{F}}{\mathbf{A}}$$

Under tension the material will stretch - however the amount of stretch for a given load will depend on the length of the specimen. We therefore need to measure <u>strain</u>.

$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

$$e = \frac{d\mathbf{L}}{\mathbf{L}}$$

For many materials at stresses well below those at which they will break there is a linear relationship between stress and strain.

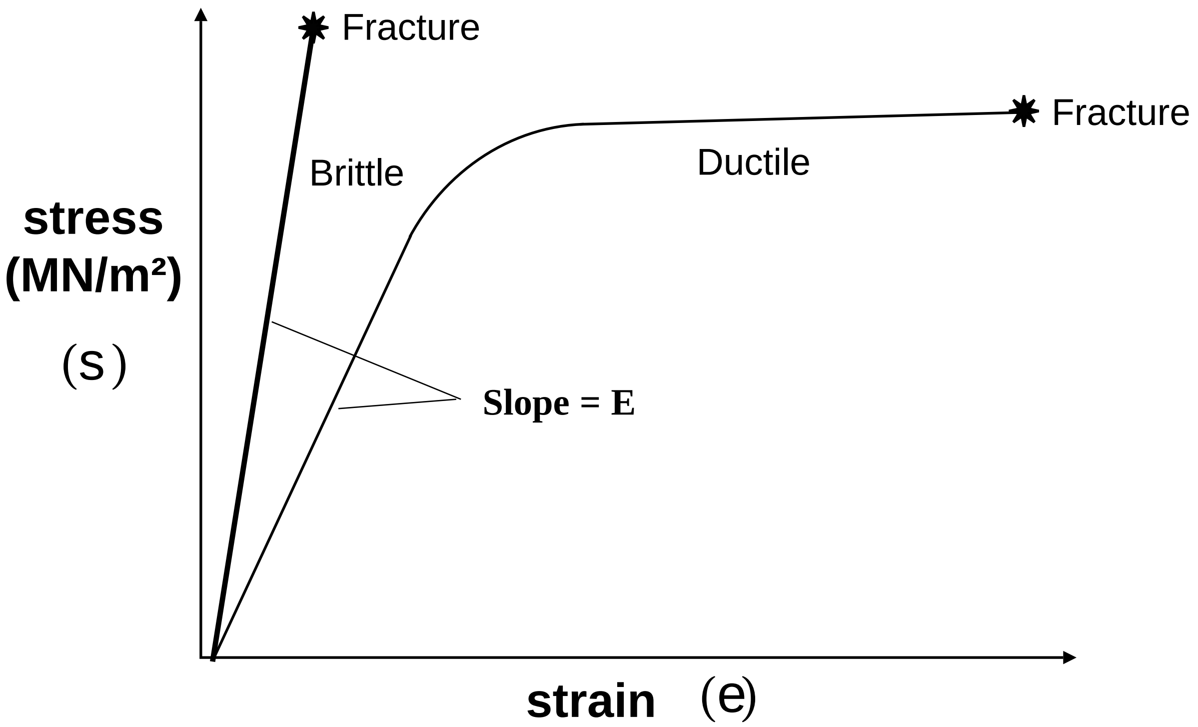


E is known as the <u>Elastic modulus</u> of the material or Young’s modulus - Its units are the same as for stress.

Material strength

The strength of a material (in tension) is defined as the stress at which (or just before) it breaks! This stress is called its Ultimate Tensile Stress (UTS).
Very often the area of the specimen contracts as it stretches but the UTS is normally based on the original (unstressed) cross-section area.

A simple tensile test allows us to characterise materials in respect of:
Mode of failure - e.g. brittle or ductile
Pattern of failure
Elastic modulus



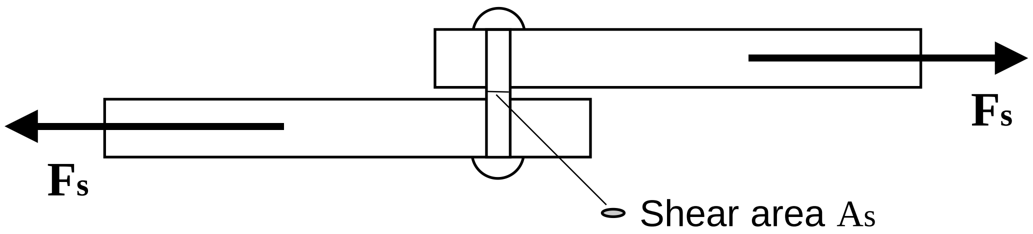
Types of Stresses

Where the force is normal to the cross-section area we obtain what is known as a Direct Stress. With a tension force we obtain tensile stresses with a compression force we obtain compressive stresses.

However, direct pulling or pushing on a material is not the only way of breaking it - usually when we want to cut, or shear, a piece of material we apply a cutting or shearing action.

This gives rise to what is called a shear stress.
Rivets, bolts and pins are often loaded in this way.

Shear stresses occur on an area which is 'parallel' to the force.



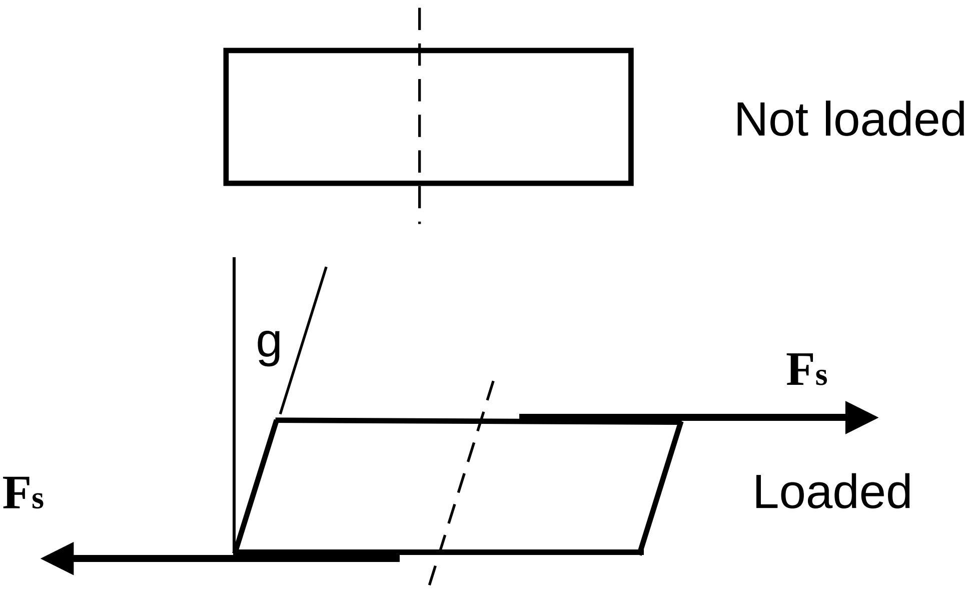
$$\text{Shear stress} = \frac{\text{Shear force}}{\text{Shear area}}$$

$$t = \frac{F_s}{A_s}$$

Shear strain

Loading a material in shear distorts it differently from when loading it directly.

If we focus in on our rivet we can see:



The angle of 'lean' g, is defined as the shear strain.
The greater the shear force the greater the angle g.

For many materials at shear stresses well below those at which they will break there is a linear relationship between shear stress and shear strain.

$$t = \mathbf{G}\, g$$

G is known as the <u>Shear Modulus</u> of a material

Similarly there is an Ultimate Shear Stress (USS) for materials .i.e. the shear stress just before they break in shear.

Summary

Direct stresses (tensile and compressive) and shear stresses are the only kinds of stress that can occur in a material.

However, they can occur together, and arise whenever forces or moments act on a body. Depending on the stress levels a material will distort or break.

If the distortion disappears when the forces are removed the material is described as distorting (or deforming) elastically.
If some or all of the distortion remains when the forces are removed the material is described as distorting plastically.

If a material breaks when force is applied it is because some or all of the material has reached its UTS or USS. This may occur deliberately (when we are cutting or shaping a material), or it may occur inadvertently when either the forces exceed what we expected, or the material is not as strong as we expected.

The amount of elastic deformation a material undergoes when forces are applied to it depends upon its Elastic Modulus and Shear Modulus