

10. WEIGHT, LIFE CYCLE & DISPOSAL

In designing for strength, it is generally thought that it is always better to err on the side of safety, and therefore to select a size larger than actually needed.

However, in many ways this is worse (or at least as bad) as choosing too small a size that might be at risk of deformation or fracture. The extra weight in the design is carried around for the whole of its life without being really noticed, and the extra material used represents an extra cost and, of course, it cannot be used elsewhere.

We can see this more clearly if we can find the strength to weight ratio of two different elements.

Strength to weight ratio of a drive shaft.

The torsion which can be safely transmitted by a drive shaft can be found from the torsion formula.

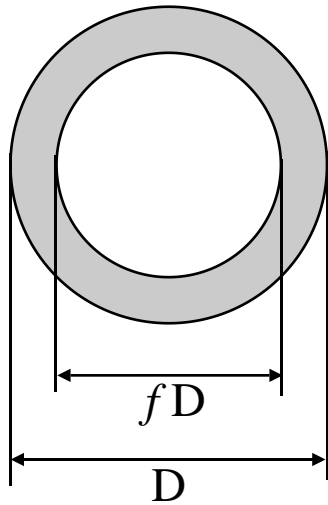
$$\frac{T}{J} = \frac{G\theta}{L} = \frac{t}{r}$$

For a given maximum shear stress t_{\max}

$$T_{\max} = t_{\max} \frac{J}{r}$$

i.e. the strength is proportional to: $\frac{J}{r}$

Let us assume we have a hollow shaft where the inside diameter is a fraction f of the outside diameter.



$$\frac{J}{r} = \frac{\frac{\rho [D^4 - (fD)^4]}{32}}{\frac{D}{2}}$$

$$= \frac{\rho [1 - f^4] D^3}{16}$$

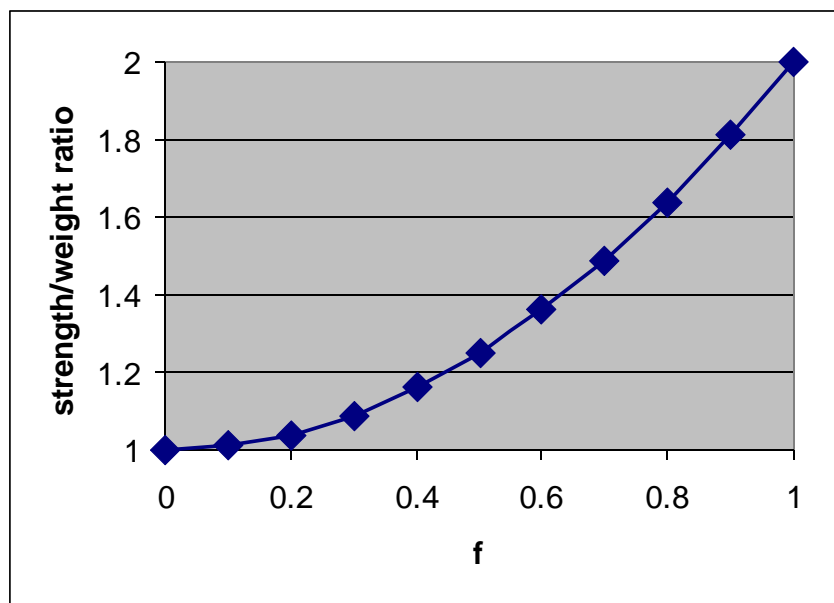
The weight of the shaft = $\frac{\rho [D^2 - (fD)^2]}{4}$ x density x length

i.e for a given material and length the weight is proportional to:

$$\frac{\rho [1 - f^2] D^2}{4}$$

The strength to weight ratio is therefore proportional to:

$$\frac{\frac{\rho [1 - f^4] D^3}{16}}{\frac{\rho [1 - f^2] D^2}{4}} = \frac{[1 - f^4] D}{[1 - f^2] 4} = [1 + f^2] \frac{D}{4}$$

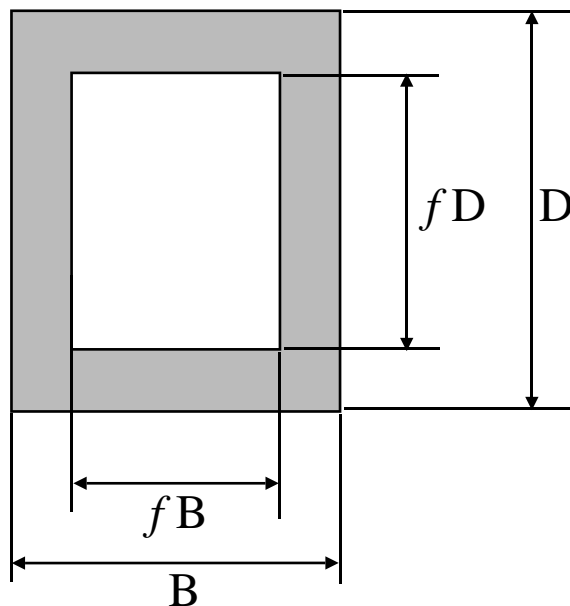


$f=0$ corresponds to a solid shaft, and f close to a value of 1 is a thin-walled tube.

It follows that a thin-walled tube has a strength to weight ratio nearly twice that of a solid shaft of the same outside diameter.

Strength to weight ratio of a RHS beam

By using the bending equation, it is possible to carry out a similar analysis for a beam as we have just done for a drive shaft:



$$M_{\max} = \frac{s_{\max} \mathbf{I}}{y_{\max}}$$

$$\frac{\mathbf{I}}{y_{\max}} = \frac{[BD^3 - fB(fD)^3]}{12 \cdot \frac{D}{2}}$$

$$= \frac{[1 - f^4] BD^2}{6}$$

The weight of the RHS = $[BD - (fB fD)] \times \text{density} \times \text{length}$

$$= BD[1 - f^2]$$

The strength to weight ratio of a RHS beam is therefore proportional to:

$$[1 + f^2] \frac{D}{6}$$

Permissible Stress Levels

The stress which we permit in a given material depends upon the material properties and also on how well we can predict the actual loads the material has to sustain.

Both of these have elements of uncertainty about them.

Material properties are variable because the manufacturing process and the material itself can vary.

The maximum load may not be precisely predictable.

These uncertainties are allowed for by what is known as the **factor of safety**.

$$\text{Factor of Safety} = \frac{\text{UTS}}{\text{Working Stress}}$$

$$\text{or, more usefully} \quad \text{Working Stress} = \frac{\text{UTS}}{\text{Factor of Safety}}$$

The difference between the UTS and the working stress is the stress margin allowing for uncertainties.

The **margin of safety** is given by:

$$\frac{\text{UTS} - \text{Working Stress}}{\text{Working Stress}} = \text{Factor of Safety} - 1$$

The more confident we are in respect of material properties and loads the lower the factor of safety can be, and hence the lower the weight of material needed to make any particular load bearing member.

Aircraft need to be built as light as possible, therefore strenuous efforts are made to ensure consistently high quality material, loads are calculated as accurately as possible, and specific limits are placed on these loads in the form of what is known as a design envelope within which the aircraft can be flown safely. A typical margin of safety for aircraft design is 2.

General engineering factors of safety will typically be 3 to 5 , and for safety critical components (like the cables supporting a lift cage) margins of safety of 9 or 10 may be used.

The general rule to minimise weight, material waste and cost is to use the lowest applicable factor of safety and ensure that the design is appropriate for the situation.

Life Cycle & Disposal

In the past very little thought was given to what happened to a structure or machine when it came to the end of its useful life either through age and deterioration or through becoming obsolete. Often the cost of reclamation of materials was not worth the bother, and the materials were disposed of by being dumped, buried or sunk.

There is greater consciousness today that if these matters are thought about at the design stage, disposal, recycling, and re-use can be economic, avoiding the need to use primary materials.

Some general guidelines which aid re-cycling of materials are:

1. Modular design: The modules can be disassembled and handled according to their characteristics.
2. Avoiding, as far as possible, mixing different materials in a way that makes them difficult to separate out.
3. Using metals as opposed to plastics which cannot be re-used.
4. Using materials which can be re-cycled in bulk, e.g. glass.
5. Finding other uses for spent materials, e.g. uses for the rubber of car tyres, old concrete.
6. Avoiding the use of high cost materials or if necessary allowing for their reclamation. e.g. gold, platinum, palladium.

The main thing is that some consideration is given to disposal, recycling and reclamation at the design stage as opposed to none.