Heat Exchanger Performance

The LMTD method for HX design is difficult to use if we want to predict the performance of a HX. For example, at off-design conditions, or if we wish to evaluate how well one of a range of ‘standard’ HX’s might perform in a given situation.

In predicting performance we would know:-

$$\dot{m}_h \quad \dot{m}_c \quad T_{h1} \quad T_{c1} \quad U \;\; \& \;\; A$$

However, we would not know:-

$$T_{h2} \text{ or } T_{c2}$$

hence we cannot find:

$$\Delta T_{\log} \text{ or } \dot{Q}$$

We could guess a value for T_{h2} or T_{c2}, find $\dot{Q}$ from a heat balance, and then find $\dot{Q}$ from $UA\Delta T$. We would need to progressively alter our guess until the two were equal. This iterative method can readily be done by computer, but a direct method can also be used.

This direct method is known as the NTU / Effectiveness method.

The NTU/Effectiveness method of HX performance

In order to use the NTU / E method we need three new definitions:

1. The Thermal Capacity Ratio [***C***]

The thermal capacity of a fluid stream is the quantity of heat it can transport per unit change in temperature:

i.e. its mass flow x specific heat capacity $\dot{m}c$

The thermal capacity ratio is defined as:-

$$C = \frac{(\dot{m}c)_{\min}}{(\dot{m}c)_{\max}}$$

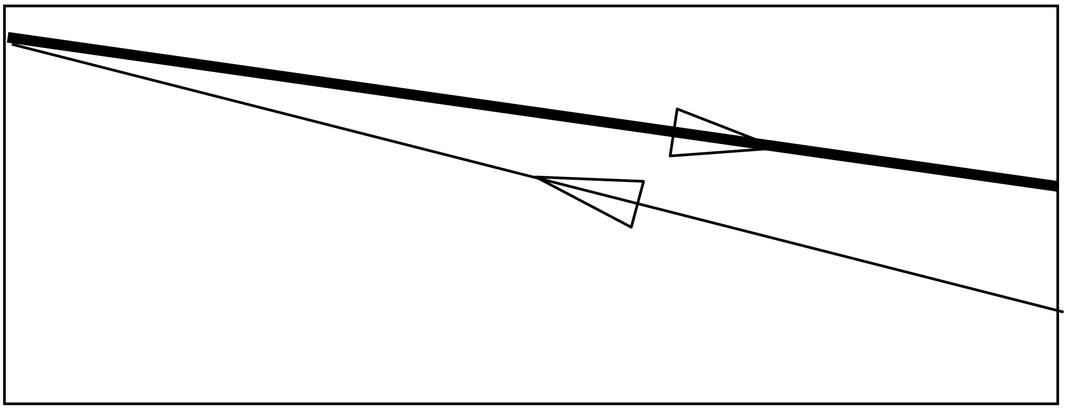
NB : ***C*** will always be <1

2. Thermal Effectiveness [***E***]

The Thermal Effectiveness is defined as:

$$E = \frac{\textit{Actual Heat Transfer Rate}}{\textit{Theoretical maximum Heat Transfer rate}}$$

The maximum theoretical heat transfer rate occurs in counter-flow with infinite heat transfer surface area. It cannot occur in parallel flow because the exit temperature must be between the two inlet temperatures.



In the infinite surface area HX above, the thermal capacity of the cold fluid is less than that of the hot fluid. It is kept in thermal contact sufficiently for it to emerge at the hot fluid inlet temperature.

The maximum theoretical heat transfer is given by:

$$\dot{Q}_{\max} = (\dot{m}c)_{\min}(T_{h1} - T_{c1})$$

The actual heat transfer rate is given (as above) from:

$$\dot{Q}_{actual} = (\dot{m}c)_h(T_{h1} - T_{h2}) = (\dot{m}c)_c(T_{c2} - T_{c1})$$

It follows that if the hot fluid has the lower thermal capacity:

$$E = \frac{(\dot{m}c)_h(T_{h1} - T_{h2})}{(\dot{m}c)_h(T_{h1} - T_{c1})} = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$$

and if the cold fluid has the lower thermal capacity:

$$E = \frac{(\dot{m}c)_c(T_{c2} - T_{c1})}{(\dot{m}c)_c(T_{h1} - T_{c1})} = \frac{T_{c2} - T_{c1}}{T_{h1} - T_{c1}}$$

Generally: $$E = \frac{\Delta T \textit{ of minimum thermal capacity stream}}{\textit{Hot - Cold inlet temperatures}}$$

3. <u>The Number of Transport Units</u> [***NTU***]

The heat transfer rate through the walls of a HX is given by:

$$\dot{Q} = UA\Delta T_{\log}$$

Therefore the heat transfer rate/ unit temperature difference is:

$$UA$$

This may compared with the thermal capacity of the minimum thermal capacity stream to obtain a ratio known as the ***NTU***:

$$NTU = \frac{UA}{(\dot{m}c)_{\min}}$$

Note: It is dimensionless.. hence 'number' !

Using these definitions a HX can be analysed in exactly the same way as for the LMTD situation to obtain (for a parallel HX) as before on p.8.....

$$-\ln\left[\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right] = \left(\frac{1}{(\dot{m}c)_c} + \frac{1}{(\dot{m}c)_h}\right)UA$$

$$\text{Assuming } (\dot{m}c)_h < (\dot{m}c)_c \quad C = \frac{(\dot{m}c)_h}{(\dot{m}c)_c} \text{ and } E = \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$$

$$-\ln\left[\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}}\right] = \frac{UA}{(\dot{m}c)_c} + NTU$$

$$= \frac{UA}{(\dot{m}c)_h}\frac{(\dot{m}c)_h}{(\dot{m}c)_c} + NTU$$

$$= NTU \times C + NTU$$

$$= NTU\,(1 + C)$$

$$\text{or} \quad \frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = e^{-NTU\,(1+C)}$$

We can rearrange the LHS solely in terms of E and C:

$$\frac{T_{h2} - T_{c2}}{T_{h1} - T_{c1}} = \frac{T_{h2} - T_{h1}}{T_{h1} - T_{c1}} + \frac{T_{h1} - T_{c2}}{T_{h1} - T_{c1}}$$

$$= -E + \frac{T_{h1} - T_{c1}}{T_{h1} - T_{c1}} + \frac{T_{c1} - T_{c2}}{T_{h1} - T_{c1}}$$

$$= -E + 1 + \frac{T_{c1} - T_{c2}}{T_{h1} - T_{c1}}$$

but $(\dot{m}c)_c (T_{c2} - T_{c1}) = (\dot{m}c)_h (T_{h1} - T_{h2})$

$$= -E + 1 + \frac{(\dot{m}c)_h}{(\dot{m}c)_c} \frac{T_{h1} - T_{h2}}{T_{h1} - T_{c1}}$$

$$= -E + 1 - CE$$

Finally for the parallel HX:

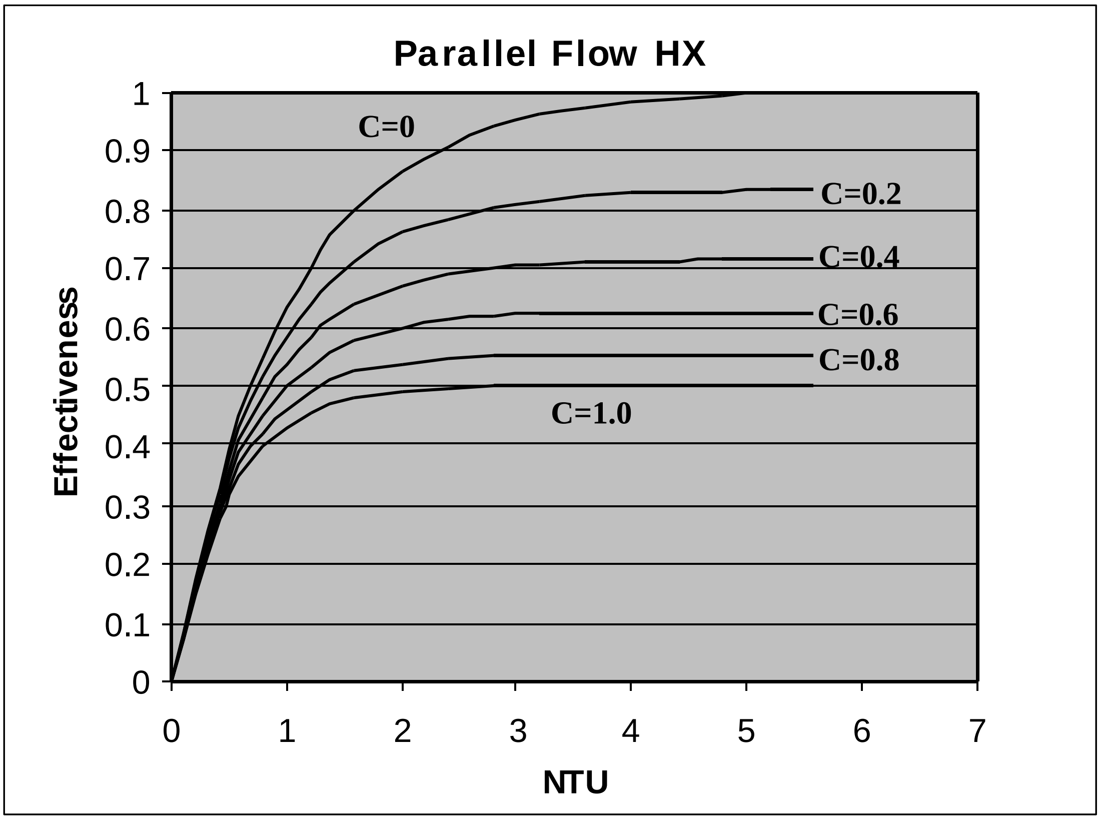
$$E = \frac{1 - e^{-NTU(1+C)}}{1 + C}$$

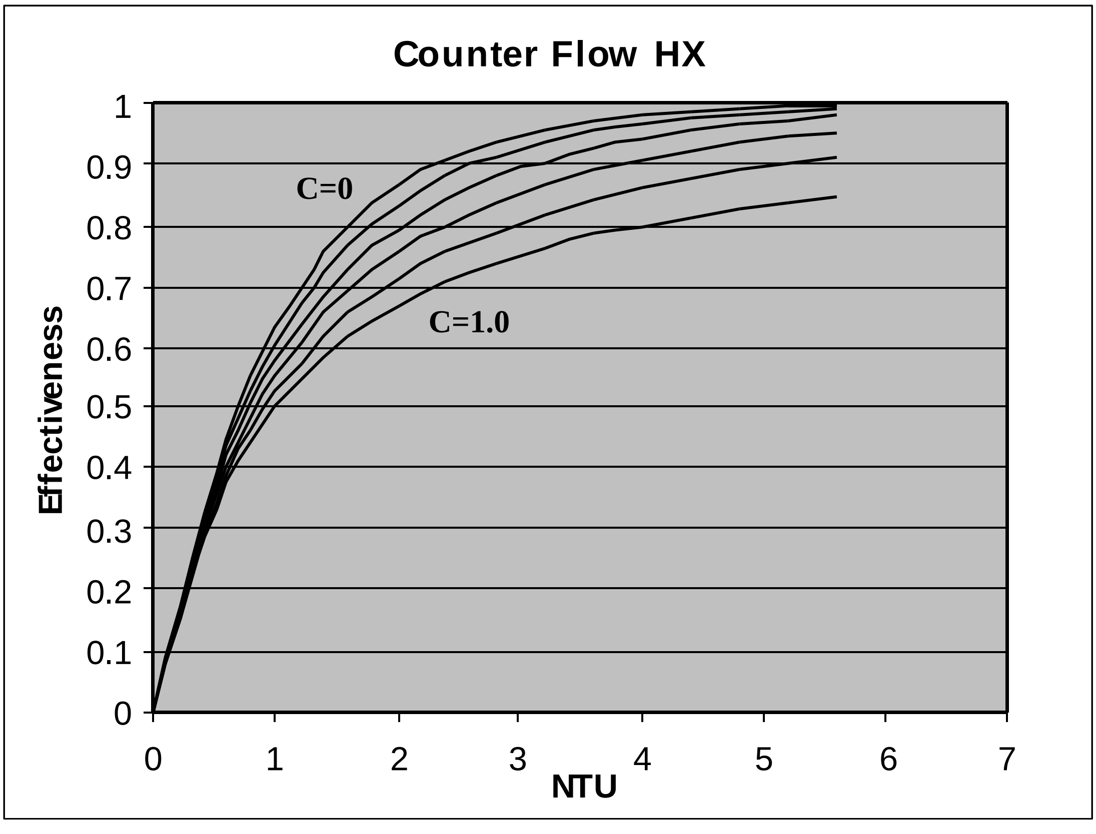
Had we analysed a counter-flow HX a different result would have been obtained:

$$E = \frac{1 - e^{-NTU(1-C)}}{1 - Ce^{-NTU(1-C)}}$$

We would normally know ***C & NTU***, and hence can find ***E***. Hence we can find the exit temperature of the lower thermal capacity stream.

Graphs are often more convenient to use than formulae.


Parallel Flow HX
C=0
C=0.2
C=0.4
C=0.6
C=0.8
C=1.0
Effectiveness
NTU
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7


Counter Flow HX
C=0
C=1.0
Effectiveness
NTU
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
1
2
3
4
5
6
7