HEAT TRANSFER

Heat is a form of ENERGY which can be characterised as an INTERACTION between HOT and COLD bodies, or as the Vibrational Kinetic Energy of molecules.

Heat is transferred by :-

CONDUCTION

CONVECTION

RADIATION

OR, MORE NORMALLY, BY A COMBINATION OF 2 OR MORE OF THE ABOVE!

WHY STUDY HEAT TRANSFER?

In many engineering situations we are interested in either ENHANCING heat transfer (eg in heat-exchangers) , or in INHIBITING heat transfer (eg in loft insulation)

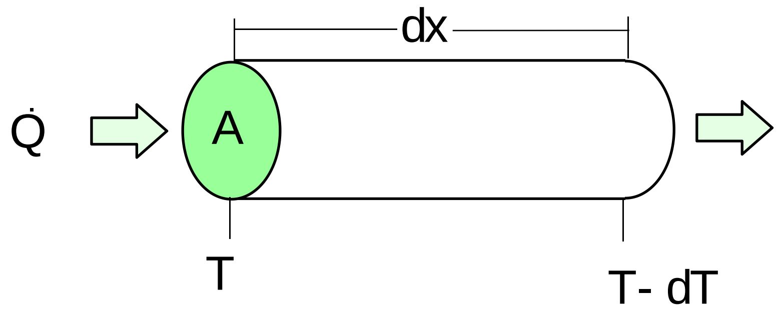
We shall look briefly at each of these and then concentrate on conduction and convection.

CONDUCTION

1-D STEADY STATE HEAT CONDUCTION

Fourier's Law:

The heat transfer rate in a solid is proportional to the temperature gradient and the cross-sectional area normal to the direction of heat flow.



$$\text{Temperature gradient} = \frac{(T\text{-}dT)\text{-}T}{dx} = \frac{-\,dT}{dx}$$

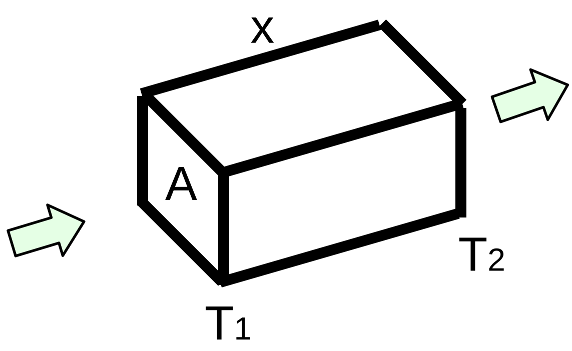
$$\dot{Q} \ \mu \ - \ A \frac{dT}{dx}$$

$$\text{or} \quad \boxed{\dot{Q} = - \ l \ A \frac{¶T}{¶x}}$$

where l = thermal conductivity (units W/mK)

For steady 1-D heat flow the above equation becomes:-

$$\dot{Q} = \frac{-\,l \ A \ (T_1 - T_2)}{x}$$



If we re-arrange the above equation, as shown below , it can be seen to be directly analogous to Ohm's Law:-

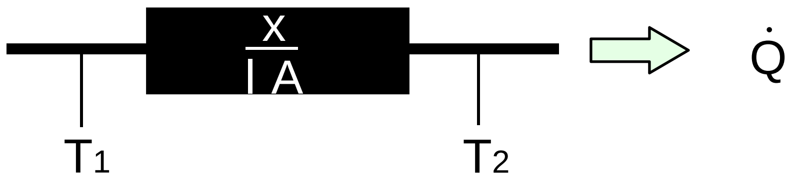
$$(T_1 - T_2) = \dot{Q} \left(\frac{x}{l\,A} \right)$$

$$E = I\,R$$

ie, the term $\left(\frac{x}{l\,A}\right)$ can be thought of as a thermal resistance.

$(T_1 - T_2)$ is the thermal potential, and $\dot{Q}$ is the thermal 'current'.

The symbol [q] is often used for thermal resistance



If we re-arrange the above equation we obtain:-

$$\frac{¶T}{¶x} = \frac{\dot{Q}}{l\,A} = \frac{\dot{Q}''}{l}$$

For steady state conditions the RHS is constant, therefore differentiating gives:-

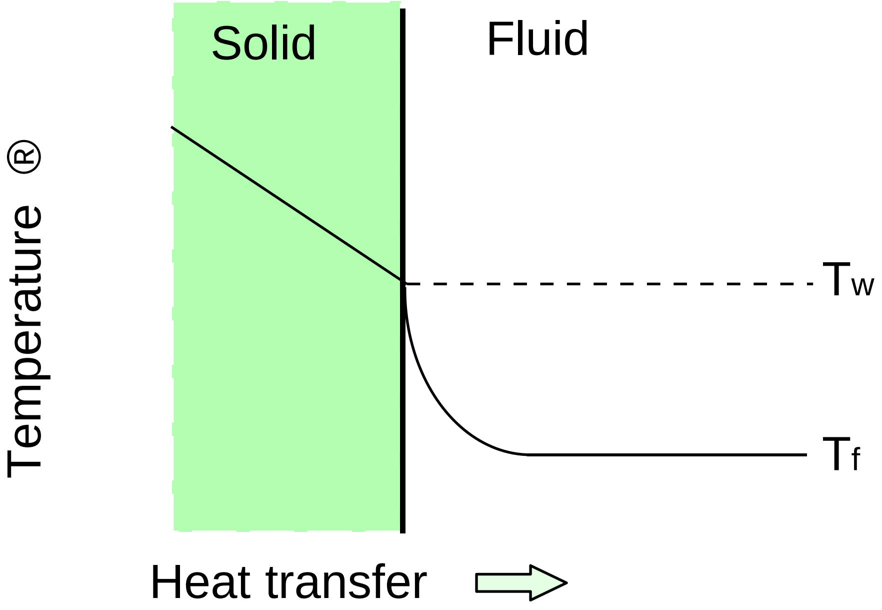
$$\frac{¶^2T}{¶x^2} = 0$$

ie the condition of steady state 1-D conduction constitutes a solution to the above second order differential equation.

Show that for radial heat transfer through a cylindrical section:-

$$\dot{Q}' = \frac{2pl\,(T_i - T_o)}{\ln\left(\frac{r_o}{r_i}\right)}$$

Heat transfer by convection occurs within a fluid or more typically at the interface between a solid boundary and a fluid.



The fluid is normally moving relative to the wall because the temperature changes cause density changes which cause the fluid to rise (bouyancy) or sink (negative bouyancy). Heat transfer to a fluid which moves because of the heat transfer is called NATURAL CONVECTION.

Alternatively the fluid flow can be caused by a fan or a pump. Heat transfer to a fluid under these circumstances is termed FORCED CONVECTION.

BOTH ARE IMPORTANT

Because the temperature changes very rapidly close to the surface (in the boundary layer) x and l cannot be used.

Newtons law of cooling

The heat transfer rate is proportional to the surface area and the temperature difference between the surface and the fluid.

$$\text{ie} \qquad \dot{Q} \ \mu \ A\,(T_w - T_f)$$

The heat transfer rate, area, and temperature difference are correlated by the SURFACE (or FILM) HEAT TRANSFER COEFFICIENT ; [h] .

$$\text{ie} \qquad \boxed{\dot{Q} = h\,A\,(T_w - T_f)}$$

This is the fundamental equation for heat transfer by convection. However, it is 'deceptively' simple.

Although 'h' may be easily measured (in many cases) it is not easy to predict.

It depends on :-
- the type of flow (laminar or turbulent)
- viscosity of the fluid
- thermal conductivity of the fluid
- velocity over the surface
- geometry of the surface
- specific heat capacity of the fluid
- expansion coefficient of the fluid }
- density of the fluid }

Natural convection especially

For some common situations, equations have been derived experimentally (empirical) with the help of 'dimensional analysis'.

Dimensional analysis reveals certain non-dimensional parametric groups which arise from the physical circumstances.

For convective heat transfer the main ones are:-

Reynolds number: [Re] = $\dfrac{r\ v\ d}{m}$

Prandtl number [Pr] = $\dfrac{m\,Cp}{l}$

Nusselt number [Nu] = $\dfrac{h\ d}{l}$

Grashoff number [Gr] = $\dfrac{b\ g\ r^2\ l^3\ AT}{m^2}$

Note that the Nusselt No. contains 'h' , hence typically:-

Nu = ¦ (Re, Pr, Gr)

Some typical ranges for 'h' are:-

0.1 to 7 W/m^2K for natural convection in air

5 to 50 W/m^2K for forced convection in air

50 to 1000 W/m^2K for forced convection in water

500 - 5000 W/m^2K for Boiling or Condensing

Some typical examples of the experimentally determined relationships between the various ND groups are given below:-

For heat transfer to/from a fluid in a pipe with steady turbulent flow:-

$$Nu = 0.023\ Re^{0.8}\ Pr^{0.4}$$

For natural convection from a horizontal round bar:-

$$Nu = \frac{0.527\ Pr^{0.5}\ Gr^{0.25}}{(Pr + 0.952)^{0.25}}$$

For heat transfer from a flat plate:-

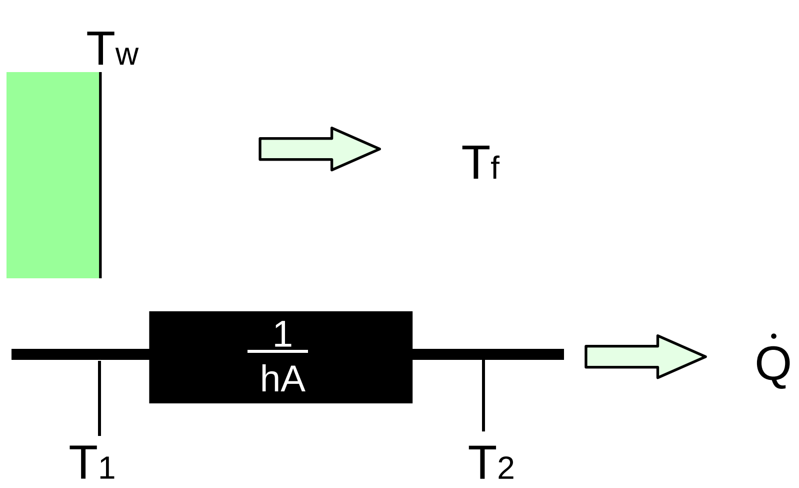
$$Nu = 0.332\ Pr^{0.333}\ Re^{0.5}$$

Again it is possible to formulate an electrical analogy for convective heat transfer.

$$E = I\ R$$

$$\dot{Q} = h\ A\ DT$$

$$DT = \dot{Q}\left(\frac{1}{h\ A}\right)$$



RADIATION

Thermal radiation is the radiation of electro-magnetic (EM) waves within a particular range of the EM spectrum.

The EM spectrum: wavelength

1m 1dm 1cm 1mm 100mm 10mm 1mm 0.1mm

UHF SHF EHF | Infra-red | Visible | UV

X-rays ®

g-rays

It is characterised by:-

(a) its wavelength [l] or frequency [f]

where f l = c ; c is the speed of light » 3 ´ 10^8 m/s

(b) its intensity, which is related to the power with which it is

As with visible light, when IR radiation strikes a body it may be either:-

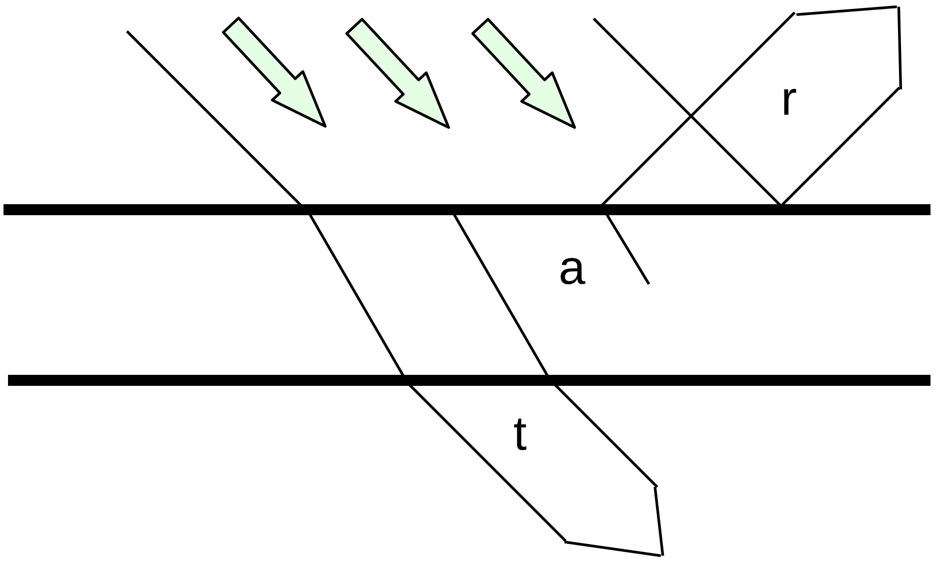
absorbed : determined by the body's absorbtivity (a)

reflected: " " " " reflectivity (r); or

Since these are all the possibilities:-

a + r + t = 1

We can illustrate this thus:-



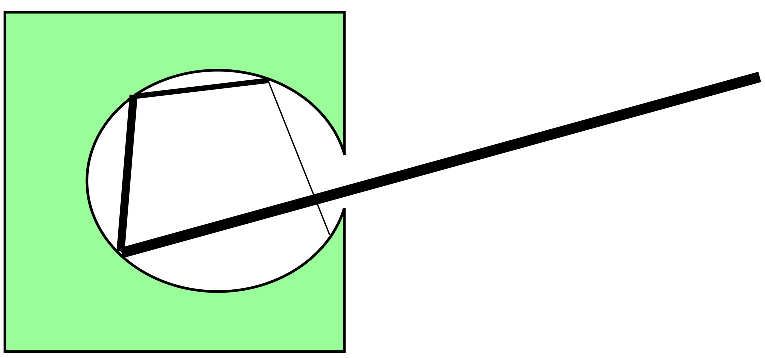
For many (most!) engineering materials $t = 0$, \ $a + r = 1$

Black Bodies

A body which absorbs all visible light falling on it appears black to the eye, similarly a body which absorbs all the thermal (IR) radiation falling on it is thermally black. ie a

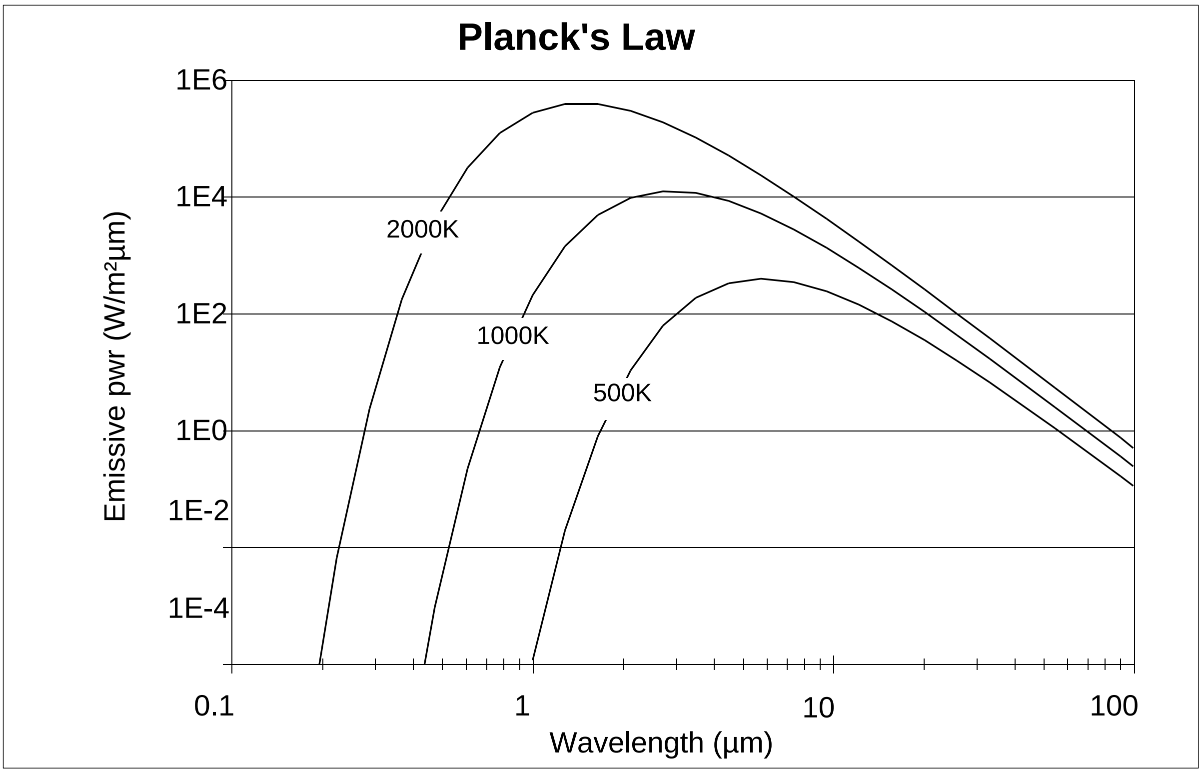
NB It may not be black to look at! eg for snow; $a = 0.985$

No perfect black body exists, but a very close approximation is an aperture to a cavity.



A black body is also the best emitter of IR radiation.

RADIATION FROM A BLACK BODY



$$l_{max} = \frac{2900}{T}$$ (l in mm, T in K) Wien's law

RADIATION FROM A NON-BLACK BODY

At any particular wave-length the emissivity [e] is given by the ratio of the emissive power of the non-black to that of the black body.

$$\frac{AB}{AC}$$

The emissivity of a body radiating at a particular temperature equals its absorbtivity when receiving radiation from a source at the <u>same</u> temperature. If this was <u>not</u> true then the body would be heating up or cooling down. ie not in equilibrium.

This is an embodiment of KIRCHOFF's law.

GREY BODIES

Although emissivity is strictly wave-length sensitive , for many bodies the effect can be 'averaged out' without too much loss of accuracy. Here the emissivity is assumed the same at ALL wave-lengths.
A body dealt with in this way is known as a **grey body**.

The STEPHAN-BOLTZMAN law

'The emissive power of a black body is proportional to the fourth power of its absolute temperature'.

$$\dot{E}''_{black} = \sigma T^4$$

σ is the Stephan-Boltzman constant = 5.67×10^{-8} W/m²K⁴

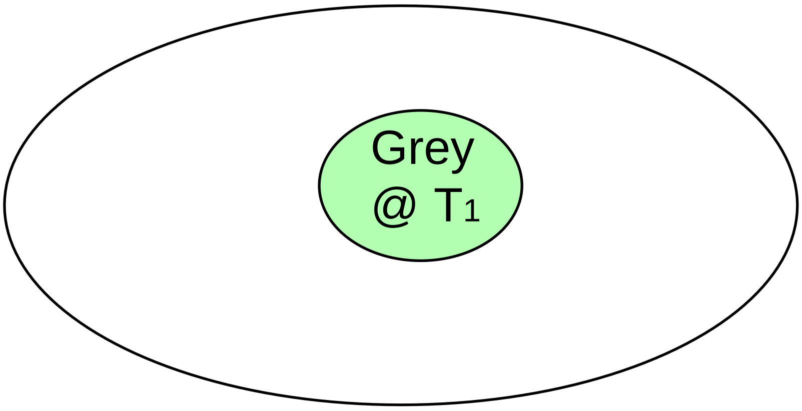
It follows that for a GREY BODY:-

$$\dot{E}''_{grey} = \varepsilon \sigma T^4$$

Some typical emissivities:-
white paint @ 20°C, 0.95; at 540°C, 0.70
polished steel @ 20°C, 0.07; at 540°C, 0.14

RADIANT HEAT EXCHANGE

Many situations can be reasonably represented by a GREY body in BLACK surroundings:



Black surroundings @ T_2

Assume $T_1 > T_2$

Energy emitted from the body/unit area = $e_1 \, s \, T_1^4$

The grey body also absorbs heat from the surroundings.

Energy absorbed/unit area = $a \, s \, T_2^4$

but (from Kirchoff) $a = e$

NETT heat transfer = Energy emitted - Energy absorbed

$$= e \, s \, T_1^4 - e \, s \, T_2^4$$

or

$$\boxed{\dot{Q}'' = e \, s \, (T_1^4 - T_2^4)}$$

NB we have assumed GREY BODY and e constant with temperature.

For a grey body with <u>effective</u> radiating surface area A_R :-

$$\dot{Q}_R = e\ s\ A_R(T_1^4 - T_2^4)$$

$$= e\ s\ A_R(T_1^2 + T_2^2)(T_1^2 - T_2^2)$$

$$= e\ s\ A_R(T_1^2 + T_2^2)(T_1 + T_2)(T_1 - T_2)$$

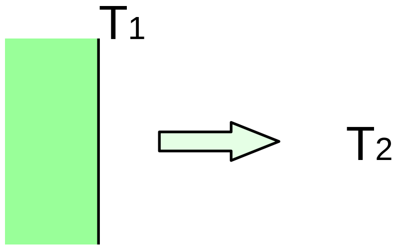
We can define a RADIANT HEAT TRANSFER COEFFICIENT [h_R] as:-

$$h_R = e\ s\ (T_1^2 + T_2^2)(T_1 + T_2)$$

then $$\dot{Q}_R = h_R A_R (T_1 - T_2)$$

ie an equation exactly similar to that for convective heat transfer!
NB because T is in K, h_R is not too affected by temperature changes. If it is we can always use iterative methods.

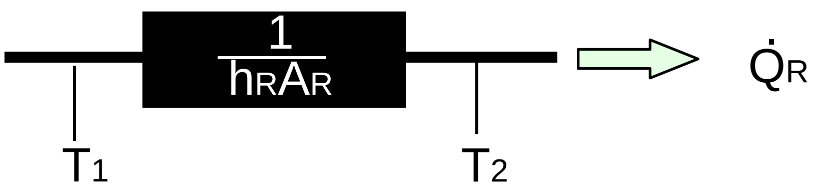
The electrical analogy is the same as for convective heat transfer:



$E = I R$

$\dot{Q}_R = h_R A_R DT$

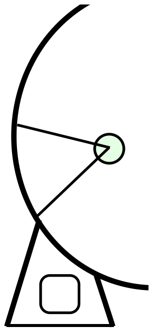
$$DT = \frac{1}{h_R A_R} \dot{Q}_R$$

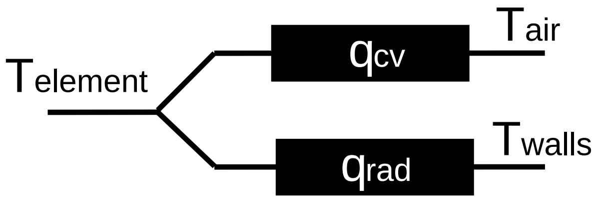


COMBINED MODES OF HEAT TRANSFER

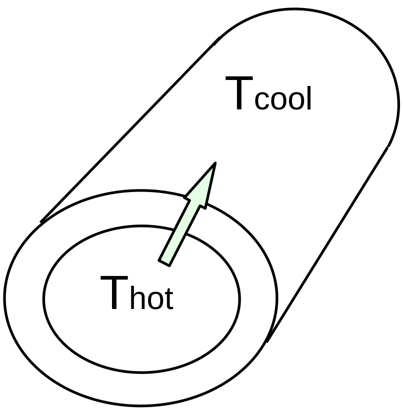
As mentioned at the start, heat transfer rarely occurs in only one mode.

eg1 - Heat transfer from an electrically powered 'radiator'.





eg2 - Heat transfer through the tubes of a heat-exchanger.



The thermal 'network' can sometimes be simplified if we can deduce that particular thermal resistances are either very small or very large compared with others.