

Fin Performance

It is possible using methods we have already used to analyse the heat transfer characteristics (convection/conduction) of fins.

For long constant cross-section fins (where heat transfer through the tip is ignored (insulated), and assuming a constant heat transfer coefficient along its length, the temperature difference between the fin and the fluid (q) is given by :-

$$\frac{q}{q_0} = \frac{\cosh m(L - x)}{\cosh mL}$$

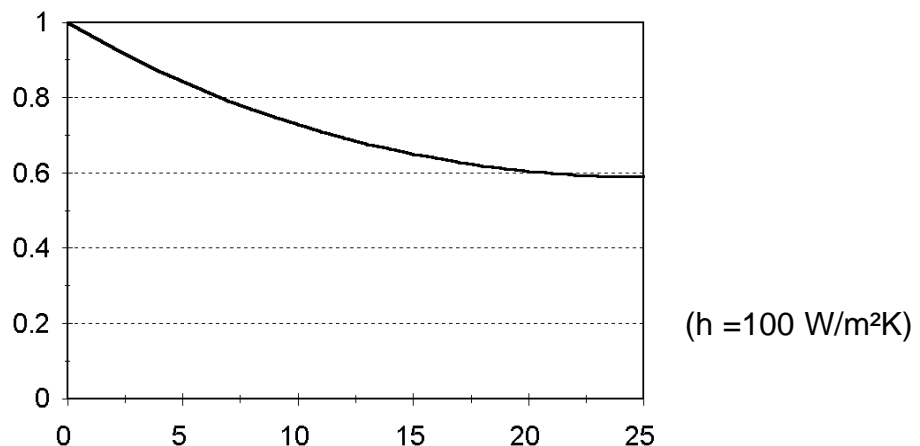
where q_0 is the temperature difference at the root of the fin (the wall end);

and $m = \sqrt{hp/l A}$

p = perimeter of the fin cross-section;

A = fin cross-sectional area;

L = fin length;



The heat transfer rate from the fin may be obtained by determining the conduction at the root.

$$\dot{Q}_0 = ml Aq_0 \tanh mL$$

For a comparatively short constant cross-section fin we cannot ignore the heat transfer at its tip, and when this is included we obtain:-

$$\frac{q}{q_0} = \frac{\cosh m(L - x) + \frac{h}{lm} \sinh m(L - x)}{\cosh mL + \frac{h}{lm} \sinh mL}$$

and

$$\dot{Q}_0 = ml Aq_0 \frac{\tanh mL + \frac{h}{lm}}{1 + \frac{h}{lm} \tanh mL}$$

Note: if $h/lm = 1$, $\dot{Q}_0 = h A q_0$
ie the same as with **no** fin.

if $h/lm < 1$ a fin is worthwhile

if $h/lm > 1$ it would reduce heat transfer!
(ie act as small insulators)

Fin Efficiency / Effectiveness

If the whole fin were at the wall (or root) temperature, the increase in heat transfer rate would be in direct proportion to the increase in surface area.

Owing to the temperature gradient along a fin this is not achievable.

We therefore define a **fin efficiency** as:-

$$h_{fin} = \frac{\text{Actual fin heat transfer rate}}{\text{Heat transfer rate if the whole fin were at the root temperature}}$$

For long constant cross-section fins this is:

$$h_{fin} = \frac{m l A q_0 \tanh mL}{h p L q_0} = \frac{\tanh mL}{mL}$$

For short constant cross-section fins this is:

$$h_{fin} = \frac{\tanh mL + h/l m}{(1 + h/l m \tanh mL)(h/l m + mL)}$$

Finned Surfaces

If A = **total** surface area (including fin area),
and A_{fin} = fin surface area, let $b = A_{fin} / A$

The total heat transfer from a finned surface is then given by:-

$$\begin{aligned} \dot{Q} &= [h_{fin} A_{fin} + (A - A_{fin})] h q_0 \\ &= \frac{[h_{fin} A_{fin} + (A - A_{fin})] A h q_0}{A} \\ &= [h_{fin} b + (1 - b)] A h q_0 \\ &= h' A q_0 \end{aligned}$$

$h' = 1 - b (1 - h_{fin})$ is the area weighted fin efficiency

h' can be incorporated into the calculation for the overall heat transfer coefficient (U) for heat exchangers.

$$\frac{1}{UA} = \frac{1}{h' \phi_1 A_1} + \frac{1}{h_2 A_2}$$

In this case the fins are on the ' h_1 ' side.

U can be based on either A_1 or A_2 , ie $A = A_1$ or $A = A_2$ but it is normal to base it on the unfinned side's area.

Remember $A_{1,2}$ is the **total** surface area

Varying cross-section fins

eg Annular or tapered fins

Although analysis of these fins is more complex than for constant cross-section fins, it can be done (!), and graphs are available which give fin efficiency as a function of their geometry and dimensions.

These can be used to estimate the heat transfer enhancement achieved by using these types of fins.