Qu.8

For the 'simple' turbine

$$\frac{w}{C_p T_1} = \frac{T_3}{T_1}\eta_t\left(1 - r_p^{\frac{1-\gamma}{\gamma}}\right) - \frac{1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right) \quad \text{——} (i)$$

for $w > 0$

$$\frac{T_3}{T_1}\eta_t\left(1 - r_p^{\frac{1-\gamma}{\gamma}}\right) > \frac{1}{\eta_c}\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right)$$

i.e. $$\eta_t \eta_c > \frac{T_1}{T_3}\left(\frac{r_p^{\frac{\gamma-1}{\gamma}} - 1}{1 - r_p^{\frac{1-\gamma}{\gamma}}}\right)$$

but $$\frac{r_p^{\frac{\gamma-1}{\gamma}} - 1}{1 - r_p^{\frac{1-\gamma}{\gamma}}} = \frac{r_p^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}} = \frac{r_p^{\frac{\gamma-1}{\gamma}}\left(\cancel{r_p^{\frac{\gamma-1}{\gamma}} - 1}\right)}{\cancel{r_p^{\frac{\gamma-1}{\gamma}} - 1}}$$

$$= r_p^{\frac{\gamma-1}{\gamma}}$$

∴ Condition is that $\underline{\underline{\eta_t \eta_c > \frac{T_1}{T_3} r_p^{\frac{\gamma-1}{\gamma}}}}$

If we differentiate (i) wrt r_p we obtain

$$\frac{\partial\left(\frac{w}{C_p T_1}\right)}{\partial r_p} = -\frac{1-\gamma}{\gamma} r_p^{\left(\frac{1-\gamma}{\gamma}-1\right)} \frac{T_3}{T_1}\eta_t - \frac{\gamma-1}{\gamma} r_p^{\left(\frac{\gamma-1}{\gamma}-1\right)} \frac{1}{\eta_c}$$

the RHS $= \phi$ at a maxima or minima.

$$\cancel{\frac{1-\gamma}{\gamma}} r_p^{\left(\frac{1}{\gamma}-2\right)} \frac{T_3}{T_1}\eta_t - \cancel{\frac{1-\gamma}{\gamma}} r_p^{\left(-\frac{1}{\gamma}\right)} \frac{1}{\eta_c} = \phi$$

or $$r_p^{\left(\frac{2}{\gamma}-2\right)} \frac{T_3}{T_1}\eta_t - \frac{1}{\eta_c} = \phi$$

or $$r_p^{2\left(\frac{1-\gamma}{\gamma}\right)} = \frac{T_1}{T_3}\frac{1}{\eta_c}\frac{1}{\eta_t}$$

or $$\underline{\underline{r_p = \left(\frac{T_3}{T_1}\eta_c \eta_t\right)^{\frac{\gamma}{2(\gamma-1)}}}}$$