3.9 Reliability of a Design

In this section we will look at the problem of designing a component - a tension element in the form of a circular rod where failure occurs by tensile fracture - to have a specific reliability, taking into account the variability (assumed to be normally distributed) of the relevant material property (the UTS) and component geometry (the rod radius).
We need to determine the required rod radius.

Load: P_{mean} = 15000 N, load standard deviation: σ_p = 500 N

UTS: S_{mean} = 600 MPa, UTS standard deviation: σ_S = 10 MPa

Required reliability: 0.99990

Procedure:

The tensile stress, Ts, is given by $Ts = P/A$ where A is the area of the circular cross section: $A = \pi . r^2$, where *r* is the radius of the rod.

The mean area is given by: $A_{mean} = \pi . r_{mean}{}^2$

The standard deviation of the area is given by $\sigma_A = 2\pi . r_{mean} \sigma_r$ where σ_r is the standard deviation of the rod radius.

Assuming the tolerance on the radius of the circular cross section is a fraction α of the mean radius, then $3\sigma_r = \alpha .. r_{mean}$ or $\sigma_r = (\alpha / 3) r_{mean}$. Now

$$Ts_{mean} = \frac{P_{mean}}{A_{mean}} = \frac{P_{mean}}{\pi . r_{mean}{}^2}$$

and the standard deviation of the stress, resulting from a combination of the standard deviation of the independent variables, load and area:

$$\sigma_{\cdot Ts}{}^2 = \sigma_p{}^2 \left(\frac{1}{A_{mean}} \right)^2 + \sigma_A{}^2 \left(\frac{P_{mean}}{A_{mean}{}^2} \right)^2$$

Substituting values:

$$\sigma_{\cdot Ts}{}^2 = \frac{\pi .^2 . r_{mean}{}^4 \sigma ._p{}^2 + 4\pi .^2 . r_{mean}{}^4 \left(\frac{\alpha}{3} \right)^2 P_{mean}{}^2}{\pi^4 r_{mean}{}^8} = \frac{\sigma_p{}^2 + (4/9)\alpha .^2 P_{mean}{}^2}{\pi .^2 r_{mean}{}^4}$$

Substituting this in the 'coupling equation' gives:

$$z = -\frac{S_{mean} - \dfrac{P_{mean}}{\pi . r_{mean}{}^{2}}}{\sqrt{(\sigma_S)^2 + \left(\dfrac{\sigma ._p{}^2 + (4/9)\alpha^2 P_{mean}{}^2}{\pi .^2 . r_{mean}{}^4}\right)}}$$

A specified reliability of R = 0.99990 means that z must be z = - 3.72. If the fraction α is assumed to be 0.01 then the above equation becomes:

$$-3.72 = \frac{600e6 - \dfrac{15000}{\pi . r_{mean}{}^{2}}}{\sqrt{(10e6)^2 + \dfrac{(500)^2 + (4/9)(0.001)^2(15000)^2}{\pi . r_{mean}{}^4}}}$$

This simplifies to: 3.539e12 r^4 - 56.55e6 r^2 + 221.5 = 0

The two positive roots are 0.003011 m and 0.002617 m. Using these radii to find the stresses gives values of 526 and 697 MPa respectively. Using the smaller radius would result in an unreliability of 0.99990

By calculating the reliability for a given value of rod radius for different values of α it can be shown that reliability decreases as the tolerance increases, as would be expected.

Doing a similar calculation to see the effect of varying the standard deviation of the material strength shows that the reliability decreases as the σ_S increases.

Return to introduction to reliability:
http://www.tech.plym.ac.uk/sme/tsoc302/reliable1.htm

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