3.6.1 Probabilistic Design

The failure of a single critical component in a modern complex system can have devastating consequences (e.g. failure of disc in a turbine powering an aircraft). To ensure such components are very unlikely to fail, a probabilistic approach to their design is frequently used. A simple approach uses a factor of safety - or design factor - which is in reality a factor of ignorance - to try and make an allowance based on experience for possible variations in materials, loading etc. This is not good enough for safety critical items where some better quantification of reliability is needed.

A suitable methodology uses probabilistic design where the design variables that affect the strength of the component are identified, variation in materials, geometry, surface condition, etc. and their distributions, from which the distribution of the strength of the component is obtained. The distributions of factors affecting the component stresses are also determined, loads, stress concentration. Once these two distributions have been obtained, the component reliability can be calculated.

Frequently it is assumed that all the distributions are 'normally distributed'. The unreliability of the component is then obtained from the overlap of the distributions of the strength and that of the stress.

The mean difference between the distributions of the random variables

$(y = s - \sigma)$, s is strength, σ is stress, is: $\mu_y = \mu_s - \mu_\sigma$

and the difference standard deviation is: $\sigma_y = \sqrt{\sigma_s^2 + \sigma_\sigma^2}$

Expressing the reliability in terms of y then:

$$R = \int_0^\infty \frac{1}{\sigma_{\cdot y}\sqrt{2\pi}} . \exp\left[-\frac{1}{2}\left(\frac{y-\mu_y}{\sigma_{\cdot y}}\right)^2\right] dy$$

transforming this to the standard normal variable by letting $z = (y - \mu_y)/\sigma_{\cdot y}$ then $\sigma_{\cdot y}\, dz = dy$

The lower limit of *z* (the standard normal variable), when *y* = 0, is given by:

$$z = \frac{0 - \mu_{\cdot y}}{\sigma_{\cdot y}} = -\frac{\mu_{\cdot s} - \mu_{\cdot \sigma}}{\sqrt{\sigma_{\cdot s}{}^2 + \sigma_{\cdot \sigma}{}^2}}$$

This is known as the coupling equation.

The upper limit remains as infinity.

The reliability is given by: $R = \frac{1}{\sqrt{2\pi}} \int_{\frac{-(\mu_s-\mu_\sigma)}{\sqrt{\sigma_s^2+\sigma_\sigma^2}}}^{\infty} e^{-z^2/2} dz$

The reliability can now be found from normal tables.

The reliability depends upon the lower limit of the above integral, lowering the lower limit increases the reliability. This comes about by increasing the difference between the mean values of the two distributions or reducing the standard deviations of the distributions.

How the reliability varies for two distributions with varying parameters is shown in the following table:

Example	Mean strength MPa	Mean stress MPa	Standard deviation strength, MPa	Standard deviation stress MPa	FoS: Mean strength/Mean stress	Lower integ. limit	Reliability R
1	500	200	20	20	2.5	10.61	1
2	500	200	50	20	2.5	5.571	1
3	500	200	100	40	2.5	2.785	0.997323
4	500	200	200	80	2.5	1.393	0.91819
5	500	400	100	40	1.25	0.9285	0.82256
6	500	100	100	20	5.0	3.922	0.99995609
7	500	100	200	40	5.0	1.961	0.97506
8	500	250	200	125	2.0	1.060	0.8554
9	500	250	100	50	2.0	2.236	0.98732
10	500	250	50	25	2.0	4.472	0.999996125

It can be seen from the above table that the standard deviations of the strength and stress dominate the reliability, **not** the Factor of Safety.

For metals and alloys in wide use the mean value of the ultimate tensile or of the yield strength will be known within a few percent and typically the standard deviation of the strength will be a few percent of the mean value. However for the fatigue endurance strength the standard deviation will be a rather larger percentage of the mean, possibly several percent.

For many applications there will be a greater lack of knowledge about the loading and consequently the stresses. One of the reasons for the improvements in the reliability of cars over the past 30 years is that manufacturers have spent a lot of time measuring the effects on components of travelling over different surfaces. This work has given vehicle designers a great deal of information about loads which when combined with modern finite element analysis (FEA) enables them to predict mean and standard deviations of stress levels with a good degree of accuracy which facilitates reliable design.

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