3.8 Statistics and the Effects of Tolerances

Where a physical phenomena is a function of several independent normally distributed variables (BS 5760 suggests this is the case in 80 % or more cases) then the effects of variation in the variables on the phenomena can be calculated. If *y* is a function of several variables *x*:

$$y = f(x_1, x_2, x_3, ...x_n)$$

the extreme variability (essentially the 'worst case' if all the tolerances 'add up') of *y* is given approximately by:

$$\delta y \approx \sum_{i=1}^{n} \left| \frac{\partial f}{\partial x_i} \right| \delta x_i$$

The standard deviation is given approximately by the relationship:

$$\sigma_y \approx \sqrt{\sum_{i=1}^{n} \left(\frac{\partial f}{\partial x_i} \right)^2 \sigma_{xi}^2}$$

Example: The 'spring rate', or stiffness, of a helical compression spring is given by the equation:

$$R = \frac{F}{\delta} = \frac{Gd^4}{8D^3 N_a}$$

where *G* = shear modulus
d = wire diameter
D = coil diameter
N_a = number of active coils

For the mean and standard deviation values shown below, calculate the average value, the extreme variability value and the standard deviation of the spring rate. Assume that the variability (total of + to - tolerance) of each variable is six times their standard deviation.

$$\mu_G = 80GPa \qquad \sigma_G = 2GPa$$
$$\mu_d = 12mm \qquad \sigma_d = 0.1mm$$
$$\mu_D = 110mm \qquad \sigma_D = 1.0mm$$
$$\mu_{N_a} = 8.5 \qquad \sigma_{N_a} = 0.1$$

Solution: Substituting the average values to give an average value of the stiffness:

$$R_{average} = \frac{80x10^9 x(0.012)^4}{8x(0.110)^3 x8.5}$$ = 21.073572 N/mm

Calculating the variabilities as 6 times the standard deviations:

$\delta_G = 12GPa$ $\delta_d = 0.6mm$ $\delta_D = 6mm$ $\delta_{N_a} = 0.6$

Calculating the partial derivatives:

$\frac{\partial R}{\partial G} = \frac{d^4}{8D^3 N_a}$ = 263.4e-9

$\frac{\partial R}{\partial d} = \frac{4Gd^3}{8D^3 N_a}$ = 7.025e6

$\frac{\partial R}{\partial D} = -\frac{3Gd^4}{8D^4 N_a}$ = - 602.1e3

$\frac{\partial R}{\partial N_a} = -\frac{Gd^4}{8D^3 N_a^2}$ = - 2.479e3

Extreme variability: $\delta R = \left|\frac{\partial R}{\partial G}\right|\delta G + \left|\frac{\partial R}{\partial d}\right|\delta d + \left|\frac{\partial R}{\partial D}\right|\delta D + \left|\frac{\partial R}{\partial N_a}\right|\delta N_a$

= 263.4e-9 x 12e9 + 7.025e6 x 0.0006 + 602.1e3 x 0.006 + 2.479e3 x 0.6

= 12.48e3 N/m or 12.48 N/mm

Hence R = 21.07 + or - 6.24 N/mm.

The standard deviation of the spring stiffness is given by:

$$\sigma_R = \sqrt{\left(\frac{\partial R}{\partial G}\right)^2 \sigma_G^2 + \left(\frac{\partial R}{\partial d}\right)^2 \sigma_d^2 + \left(\frac{\partial R}{\partial D}\right)^2 \sigma_D^2 + \left(\frac{\partial R}{\partial N_a}\right)^2 \sigma_{N_a}^2}$$

$$\sigma_R = \sqrt{(263.4e-9)^2 (2e9)^2 + (7.025e6)^2 (0.0001)^2 + (-602.1e3)^2 (0.001)^2 + (-2.479e3)^2 (0.1)^2}$$

σ_R = 1.093e3 N/m or 1.093 N/mm

Return to introduction to reliability:
http://www.tech.plym.ac.uk/sme/tsoc302/reliable1.htm

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