Design – Springs

1. Introduction

Springs are widely used in engineering to exert a force. In many critical applications, vehicle suspension springs and engine valve springs, they are subjected to rapid changes in length and their mass must be kept as low as possible to minimise undesirable dynamic effects. This inevitably means that such springs are working at high stress levels which has implications concerning the choice of materials and manufacturing processes. This section will concentrate on coil springs working in compression, dealing with design, stress analysis, choice of materials and manufacturing processes.

2. Forces Acting

The loads in the wire can be deduced from the FBD shown at right and are equal to a force \( F \) parallel to the spring axis (acting transverse to the wire axis) and a torque \( T = \frac{FD}{2} \), about the axis of the wire. Both of these generate shear stress in the wire.

\[
\tau = \pm \frac{FDd}{4J} + \frac{F}{A},
\]

where \( J \) is the polar moment of area of the wire cross section. The first term is due to torsion and the second term is due to the transverse shear force and this expression ignores the curvature of the wire in the spring.

As \( J = \frac{\pi d^4}{32} \) and \( A = \frac{\pi d^2}{4} \) the equation for the shear stress can be written as:

\[
\tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}
\]
The spring index, $C$ is defined as $D/d$ and for many springs is in the range 6 –12. The two terms in the previous expression for shear stress can be combined by introducing a shear stress correction factor $K_s = \frac{2C+1}{2C}$ which gives $\tau = K_s \frac{8FD}{\pi d^3}$

3. Curvature Effect

As the wire is coiled, the rotation of the wire on the inner side of the coil occurs over a shorter distance than rotation at the outer side of the coil. This means the shear stress in the wire at the inner side of the coil must be greater than that at the outside of the coil. This also means that the centre of rotation of the wire must be displaced away from the wire axis towards the centre of the coil, although the actual displacement is quite small.

Two factors have been proposed to include the effects of both the curvature and the transverse shear.

The ‘Wahl’ factor

$$K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

The Bergstrasser factor

$$K_B = \frac{4C+2}{4C-3}$$

The results from using these two methods usually differ by less than 1%.

The curvature correction factor can be determined by cancelling the effect of transverse shear, done below for the Bergstrasser factor:

$$K_c = \frac{K_B}{K_s} = \frac{2C(4C+2)}{(4C-3)(2C+1)}$$

When fatigue is likely (or the spring material must be considered to be brittle) $K_c$ is used as a stress concentration factor. Normally in fatigue calculations the stress concentration factor would be corrected to $K_f$ because of notch sensitivity, but for high strength steels the notch sensitivity is close to 1, so the full value of $K_c$ (or $K_B$ or $K_w$ in some procedures) is used.

4. Equations Used in Design

The relationship between load and deflection is given by:

$$F = \frac{Gd^4\delta}{8D^3N_a}$$

where:

$G$ is the shear modulus, for spring steel a value of 80 GPa can be used.
$
\delta$ is the deflection and

$N_a$ is the number of active turns, in determining this account must be taken of the ways that the end of the coil are finished and appropriate corrections made.

The linear spring rate is then:

$$R = \frac{F}{\delta} = \frac{Gd^4}{8D^3N_a}$$

The basic stress is given by: $S = \frac{8DF}{\pi d^3}$ and the corrected stress: $S_c = \frac{8DFKw}{\pi d^3}$
Design methods used to make use of nomographs, however spreadsheets are now used. The Society of Automotive Engineers (SAE) publish a ‘Spring Design Manual’ that contains information about design and design methodology, reliability and materials. Normally a design will start with some constraints about space available, governing D, required spring rate, limits of motion, availability of wire diameter, material, maximum allowable stress when the spring is ‘solid’. Some iterations will probably be needed to reach the best solution. Fatigue testing is commonly carried out on new designs of springs destined for critical applications.

5. Materials and Manufacture

‘Music wire’, AISI 1085 steel is used in diameters up to 3 mm for the highest quality springs.

For diameters up to 12 mm AISI 1065 may be used in the hardened and tempered condition or cold drawn.

For larger wire diameter, or for highly stressed applications, low alloy steels containing chrome - vanadium, chrome – silicon and silicon – manganese, hot rolled, hardened and tempered (to 50 to 53 Rc hardness, equivalent to about 1600 to 1700 MPa UTS) are used.

For most spring materials increasing the wire diameter reduces the UTS, and if the UTS is plotted against the wire diameter on semi – log graph paper, the line is often nearly straight for many of the metals used for springs.

Research suggests that the yield shear stress of most metals lies in the range of 0.35 to 0.55 times the UTS, in the absence of specific information, a value of 0.5 times the UTS can be used for hardened and tempered carbon and low alloy steels.

The fatigue strength of springs can be increased by cold setting and particularly by shot peening, which can increase the fatigue strength by as much as 50%. Both processes are routinely carried out on highly loaded automotive suspension springs.

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