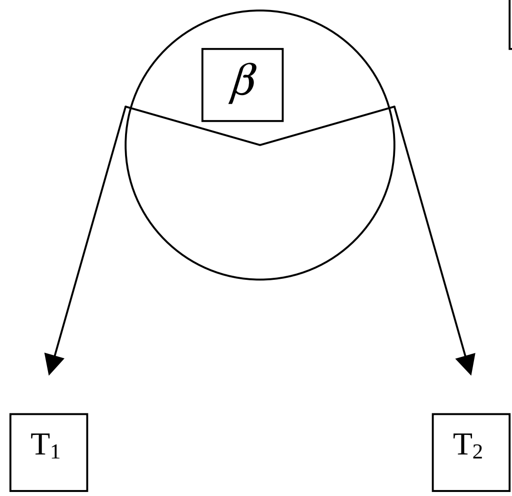
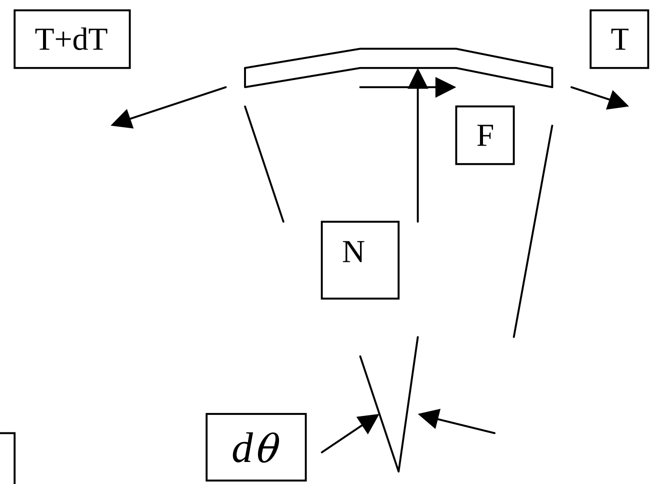
Design Notes - Belt Drives - Simple Analysis





Consider the equilibrium of a small element of belt subtending a small angle $d\vartheta$
Assume that the coefficient of friction between pulley and the belt is μ and that the belt tension on the tight side of the pulley is T_1 and slack side of the pulley T_2.
Assume the angle of wrap of the belt around the pulley is β and ignore centrifugal effects.
Resolving forces on the small element of belt in the tangential direction:

$$(T+dT)\cos\frac{d\theta}{2} - F - T\cos\frac{d\theta}{2} = 0$$ as $d\theta$ is small, $\cos d\theta \approx 1$

so $dT = F$, when the belt is on the point of slipping: $F = \mu N$ so $dT = \mu N$

Resolving the forces on the small element of belt in a radial direction:

$$(T+dT)\sin\frac{d\theta}{2} + T\sin\frac{\theta}{2} - N = 0$$ for small angles $\sin\theta \approx \theta$

so $Td\theta = N$

Eliminate N, which is not wanted at this stage, gives: $\frac{dT}{T} = \mu d\theta$

This can be integrated: $\int_{T_2}^{T_1} \frac{dT}{T} = \mu \int_0^{\beta} d\theta$ giving: $\ln\frac{T_1}{T_2} = \mu\beta$

or: $$\frac{T_1}{T_2} = e^{\mu\beta}$$