1.
TIME DEPENDENT MECHANICAL BEHAVIOUR

Introduction: In the elastic analysis of metals it is assumed that elastic strain is a function of stress only. This is not strictly true since there is time dependence to the elasticity. In metals the effect is very small and generally neglected, but in polymers the effect is much more significant. The general name for this time dependence is **anelasticity**.

Anelasticity: **Figure 1** shows that if a strain ε_1 is applied to an anelastic material at time $t = t_0$ then the strain will slowly increase to ε_2, the completely relaxed strain, at time $t = t_1$. The anelastic strain is $\varepsilon_2 - \varepsilon_1$. If the material is now suddenly unloaded the strain decreases immediately by ε_1, followed by a slower decrease to zero strain known as the **elastic aftereffect**. If the loading is applied so rapidly that there is no time for thermal equilibrium with the environment, i.e. constant entropy, adiabatic loading, there will be a temperature decrease. For uniaxial tensile strain, this is given by:

$$\left(\frac{\delta T}{\delta \varepsilon}\right)_S = -\frac{V_m \alpha E T}{c_v} \qquad ...(1)$$

where: V_m = the molar volume of the material.

α = coefficient of linear expansion.

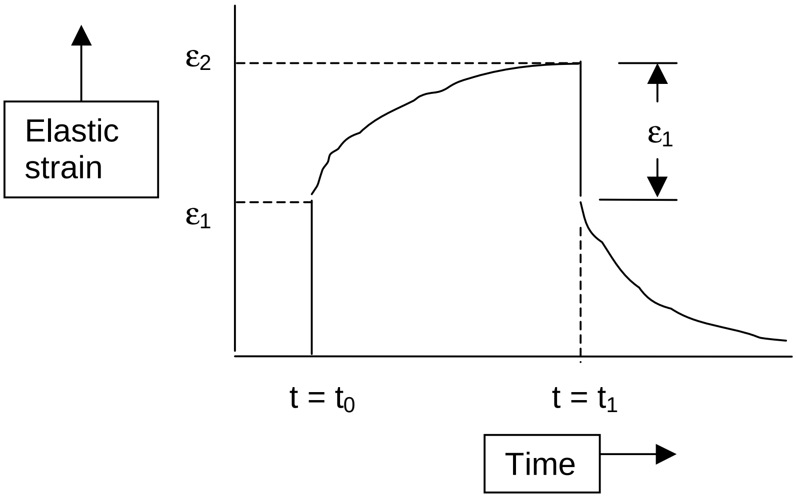
E = isothermal Young's modulus.

T = absolute temperature.

c_v = specific heat at constant volume.

Compression would raise the temperature, and this effect is called the **thermoelastic effect**, though the temperature changes are very small.

Figure 1: Anelastic behaviour:



If a specimen is stressed elastically at a slow rate as shown in **Figure 2**, then thermal equilibrium is maintained and an **isothermal strain,** $\boldsymbol{\varepsilon_I}$ is produced. If rapid, adiabatic, constant entropy loading occurs, there is no time for thermal equilibrium and a small temperature drop occurs with an **adiabatic strain** $\boldsymbol{\varepsilon_A}$ being produced. The specimen then warms up and expands along path BC, and if rapid unloading then occurs, the specimen warms slightly as path CD is followed adiabatically. Finally the specimen loses heat to the environment and cools and contracts along path DA. This results in a closed **mechanical hysteresis loop.** In practice if stressing is continuous, a more lenticular shaped loop is produced as shown in **Figure 3.**

Figure 2: Adiabatic and Isothermal stress strain curves:

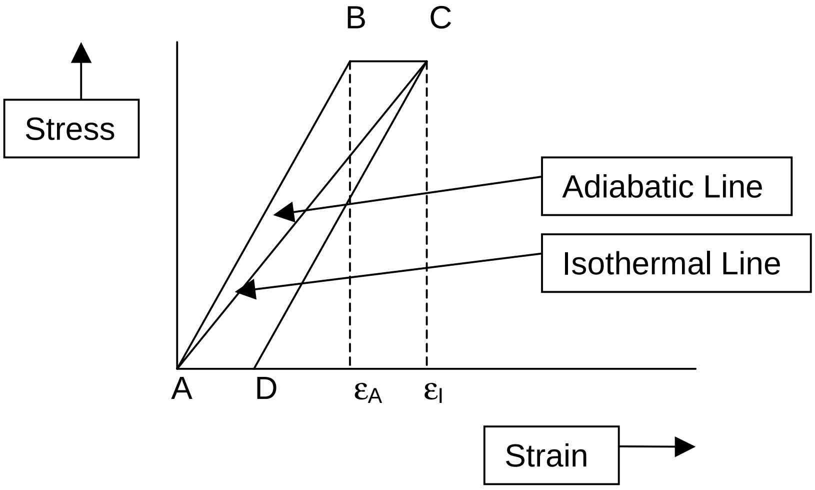
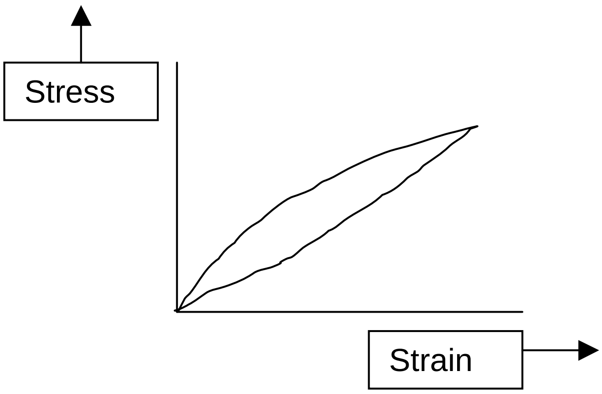


Figure 3: Hysteresis Loop from continuous cyclic loading.



The area of a hysteresis loop is small, especially for metals, but with continuous rapid vibration, the energy loss per cycle is given by the area within the loop. This is because the area under a stress strain curve represents elastic strain energy per unit volume, i.e. $U = \frac{1}{2}\sigma\varepsilon$ ...(2). From an engineering point of view, the energy loss leads to heating, damping of vibrations and also contributes to friction, particularly in materials like rubber.

Mechanical Hysteresis, Internal Friction and Damping: The area of the hysteresis loop depends on frequency of cycling. For very low frequency, isothermal cycling the area is low, and area is also low for very fast adiabatic cycling. At some intermediate frequency the area will be a maximum. Vibrational energy in crystalline solids may be dissipated by several mechanisms, other than the thermoelastic effect. These include:

(i) Stress induced ordering of solute atoms.

(ii) Grain boundary sliding.
(iii) Dislocation movements.
(iv) Bond rotation in polymers.
(v) Moisture absorption in polymers.

The generic name for these energy dissipating mechanisms is **Internal Friction,** and precision measurements of this parameter can give information on structure changes such as precipitation, diffusion etc. If a bar of material is set into vibration with an initial amplitude A_0, after a time 't' the amplitude will have decayed to A_t, where $A_t = A_0 e^{-\beta t}$ ...(3), where β is the attenuation coefficient. The ratio of successive amplitudes enables the logarithmic decrement δ to be found from: $\delta = \ln \frac{A_n}{A_{n+1}}$ ...(4)

The strain lags behind the stress by a phase angle α, which is related to the logarithmic decrement by: $\delta = \pi\alpha$ ...(5)

True anelastic behaviour is independent of amplitude of vibration, but other energy dissipating mechanisms may not be. For **Forced Vibrations**, the specimen is driven at constant amplitude, and a measure of the internal friction is given by: $\delta = \frac{\Delta W}{2W}$ ...(5), where ΔW is the energy loss per cycle, or hysteresis loop area, and $2W$ is the total energy per cycle. For a fixed amount of energy per cycle, the maximum amplitude of vibration occurs at the **resonant frequency**. Internal friction studies are carried out at low stress and strain levels (0.05 to 0.5 MNm^{-2}), while energy dissipation at higher stress and strain levels is generally referred to as **damping**. In engineering terms, a high **Damping Capacity** would help to reduce machinery noise, suppress vibrations in machinery, reduce the amplitude of resonant vibrations and reduce likelihood of fatigue. Table 1 shows the damping capacity of some engineering materials at different stress levels. It can be seen that cast iron makes a better energy absorbing material than other alloys due to the inability of graphite flakes in the structure to transmit elastic waves.

Table 1: Damping Capacity of Materials:

Material	Specific Damping Capacity $\Delta W / W$		
	31 MNm^{-2}	46 MNm^{-2}	77 MNm^{-2}
Carbon steel (0.1%)	2.28	2.78	4.16
Ni-Cr steel, quenched/tempered	0.38	0.49	0.70
12% stainless steel	8.0	8.0	8.0
18-8 stainless steel	0.76	1.16	3.8
Cast iron	28.0	40.0	
Yellow brass	0.50	0.86	