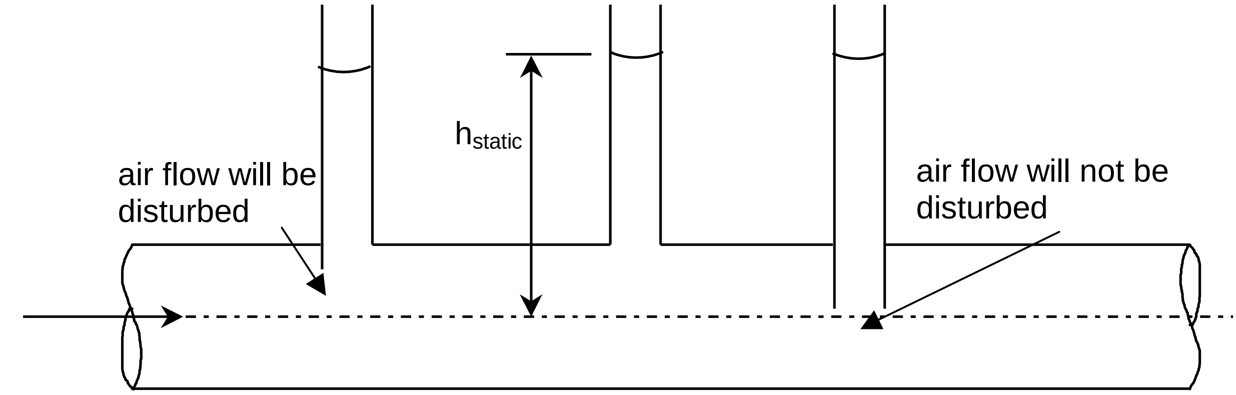
3.7. Applications of Bernoulli's equation

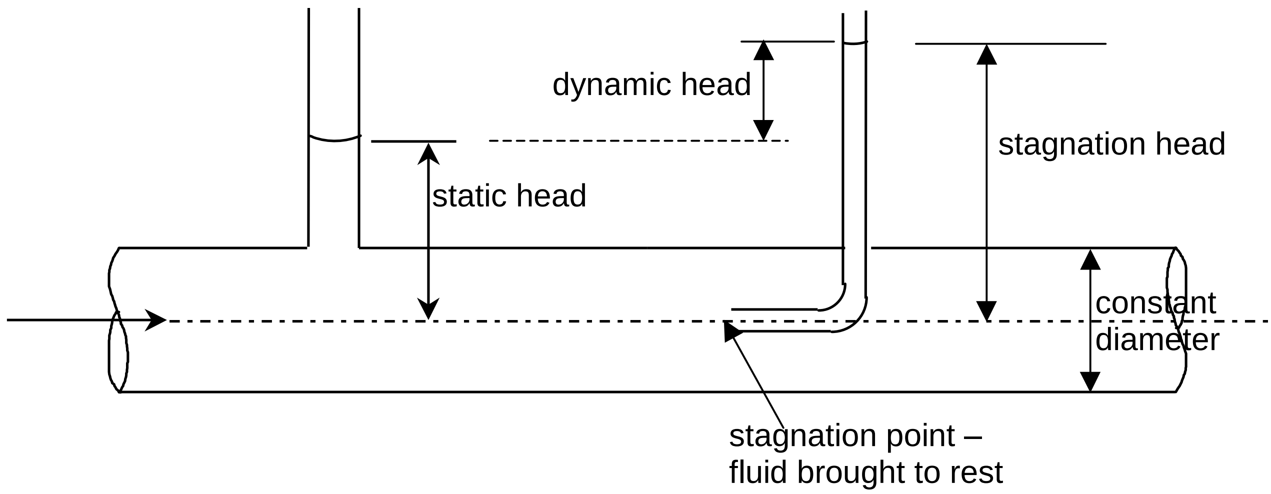
(a) **Recap: Static, dynamic and stagnation pressure**

Static pressure – measured without disturbing the flow



$p = \rho g h_{static}$ where ρ is the density of the flowing fluid

Stagnation pressure – pressure measured at the point where the fluid is brought to rest



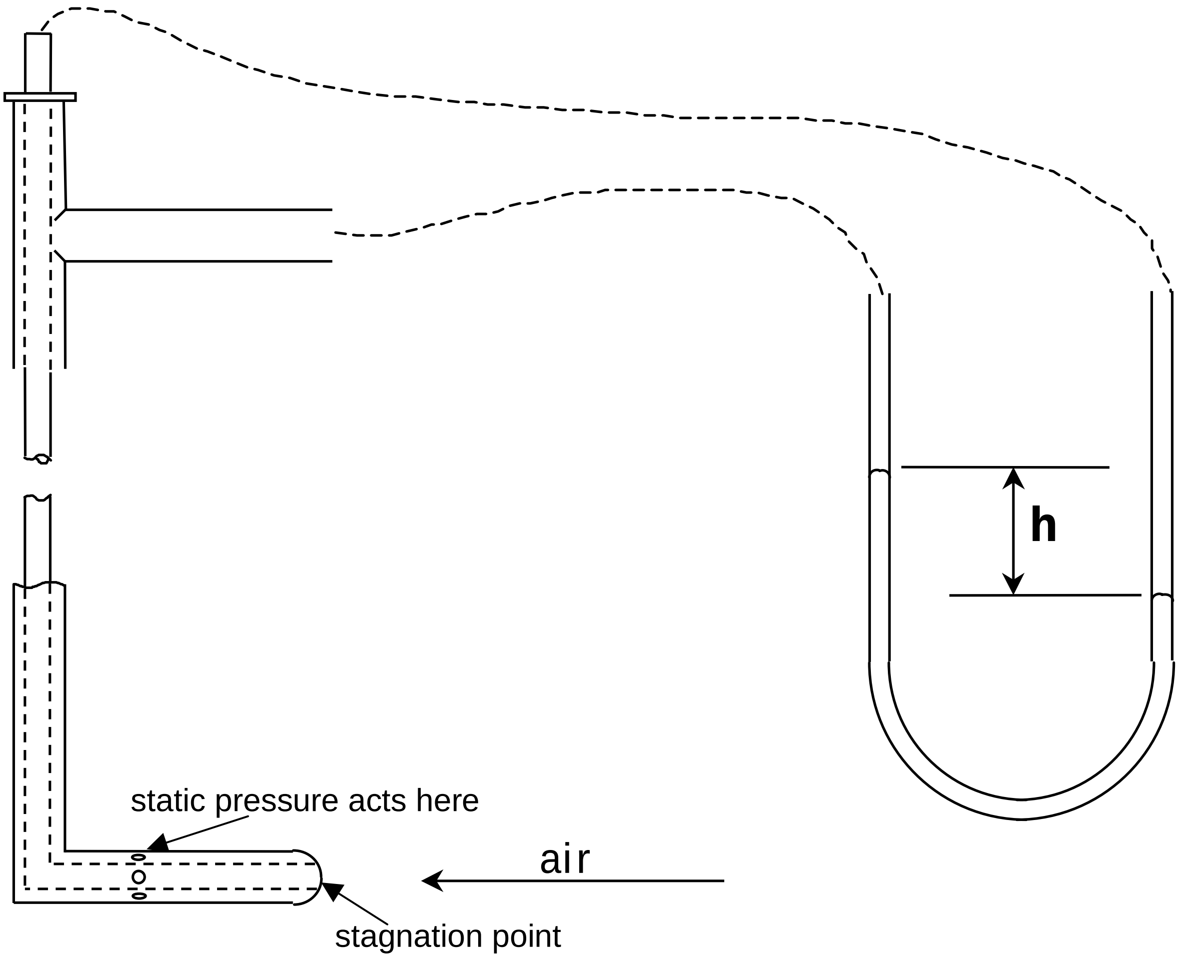
ignoring potential head

$$\frac{p}{\rho g} + \frac{v^2}{2g} = \frac{p_0}{\rho g}$$

static head — dynamic head — total head

(b) **Pitot-static tube**

A device for measuring flow velocity. The stagnation and static pressures are tapped at the same point (or nearly) and the difference between them is measured. The instrument may be calibrated to read flow velocity directly.



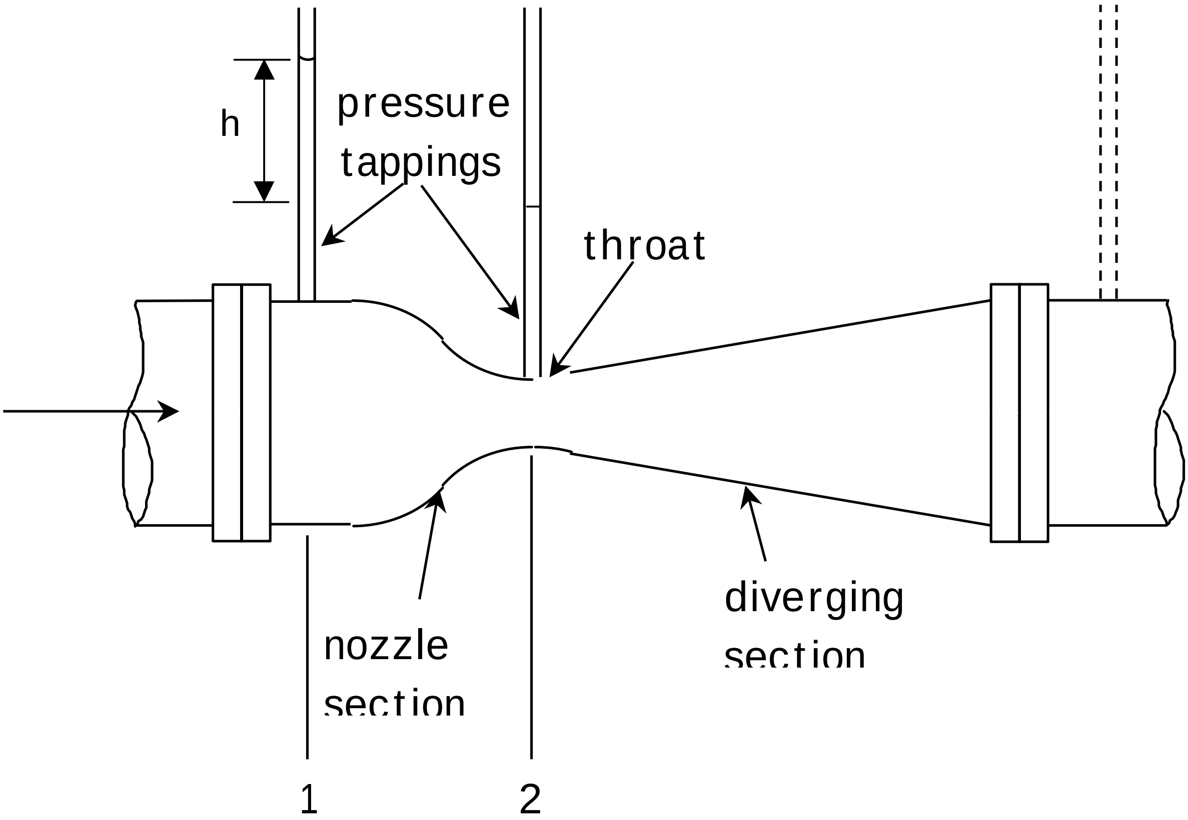
Flow velocity, $v = \sqrt{\dfrac{2(p_{st} - p)}{\rho}} = \sqrt{\dfrac{2\Delta p}{\rho}} = \sqrt{\dfrac{2\rho_m g h}{\rho_f}}$

where ρ_m is the density of the manometer fluid and ρ_f is the density of the fluid under test.

See if you can derive the above relationship.

(c) **Venturi meter**

A device for measuring flow rates. Flow is accelerated through a narrow section (throat) where, according to Bernoulli, the pressure drops. The pressure difference between the main section of flow and flow in the throat gives a measure of the volume flow rate, $\dot{V}$.



By applying the continuity and Bernoulli's equations between sections 1 and 2, show that:

$$\dot{V} = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{\left(A_1^2 - A_2^2\right)}} = k\sqrt{h}$$

where k is a constant for the particular venturi meter.

In fact, because of losses, this calculation gives the *theoretical* flow rate, and the *actual* flow rate is smaller. An empirical discharge coefficient, C_d, is introduced to account for these losses, so that:

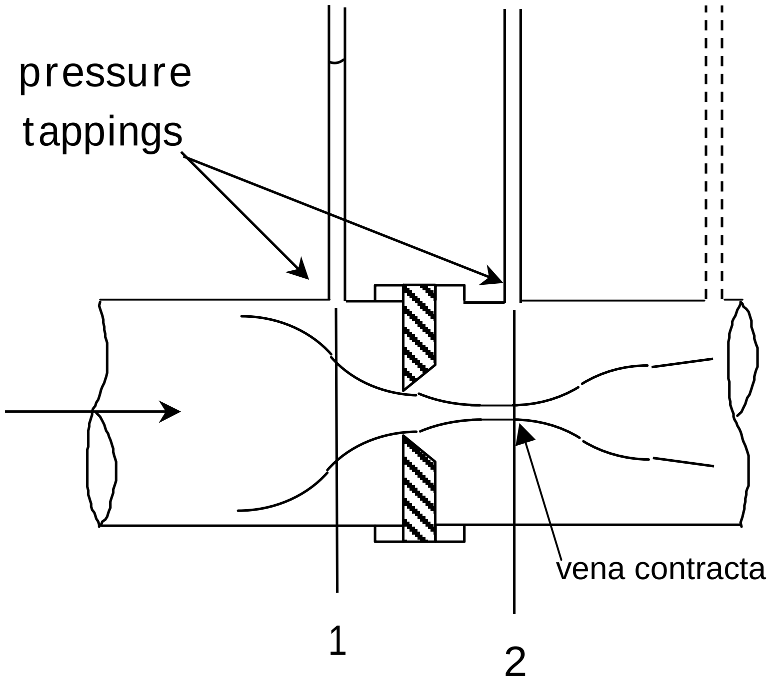
$$\dot{V}_{actual} = C_d k\sqrt{h}$$

If a manometer is used to measure the pressure difference ($p_1 - p_2$), which uses a different fluid, then $\sqrt{h}$ must be replaced by $\sqrt{H\left(\frac{\rho_m}{\rho_f} - 1\right)}$

where H is the manometer reading, ρ_m and ρ_f the densities of the manometer fluid and fluid under test, respectively.

(d) **Orifice meter**

Works in a similar fashion to the Venturi meter



The area of cross section at section 2 (at the *vena contracta*), A_2, is smaller than the area of cross-section of the orifice, A_o, because the stream lines continue to get closer together for a short time after passing through the orifice.

The analysis is as for the Venturi meter with A_2 as the orifice area.

C_d allows for both friction effects and the *vena contracta*, and typically has a value of around 0.65

Compared with the Venturi:

Advantages – simple to implement (incorporate at a pipe flange), cheap, short in length

Disadvantages – high losses, possible obstruction due to debris.

Further reading:

Bacon and Stephens, Fluid Mechanics for Technicians 3/4	Ch 4
Bacon and Stephens, Mechanical Technology	Ch 30
Massey, Mechanics of fluids	3.8
White, Fluid Mechanics	6.10