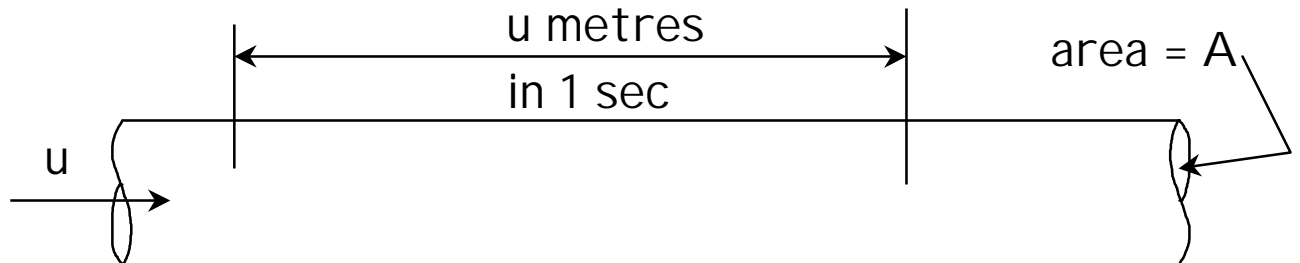


## 2. Continuity

### 2.1. Conservation of mass



Volume flow rate is the volume of fluid flowing through the pipe in 1 s.

$$\text{Volume flow rate } \dot{V} = uA \quad \text{in } \text{m}^3\text{s}^{-1}$$

where  $A$  is the cross-section area and  $u$  is the average velocity of the fluid.

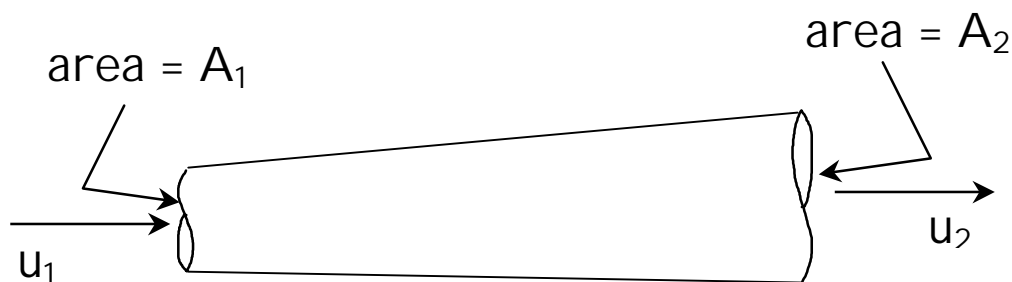
Mass flow rate is the mass of fluid flowing through the pipe in 1 s.

$$\text{Mass flow rate } \dot{m} = \rho\dot{V} = \rho uA \quad \text{in } \text{kg s}^{-1}$$

where  $\rho$  is the density of the fluid

For steady flow in which the flow parameters (velocity, pressure, temperature, density) do not change with time at any particular place in the flow, what flows into the pipe must come out in the same time. That is

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$



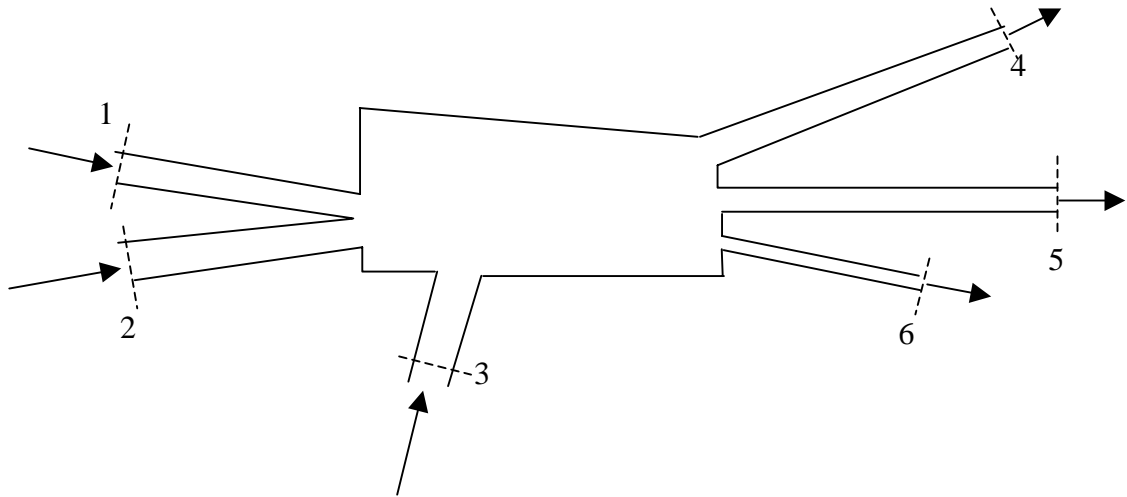
$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$$

*What will this equation become if the density remains constant?*

*Will it always be true that the volume flow rate in is equal to the volume flow rate out?*

For a branched pipe system, the continuity equation is

$$\sum \dot{m}_{\text{in}} = \sum \dot{m}_{\text{out}}$$



$$\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4 + \dot{m}_5 + \dot{m}_6$$

$$\rho_1 \dot{V}_1 + \rho_2 \dot{V}_2 + \rho_3 \dot{V}_3 = \rho_4 \dot{V}_4 + \rho_5 \dot{V}_5 + \rho_6 \dot{V}_6$$

$$\rho_1 A_1 u_1 + \rho_2 A_2 u_2 + \rho_3 A_3 u_3 = \rho_4 A_4 u_4 + \rho_5 A_5 u_5 + \rho_6 A_6 u_6$$

### Example

Water flows through a pipe at  $0.84 \text{ m}^3/\text{min}$ . At point x, the pipe has a diameter of 50 mm; at point y the diameter is 35 mm. Find the velocity at points x and y.

$$\dot{V} = 0.84 \text{ m}^3/\text{min} = \frac{0.84}{60} = 0.014 \text{ m}^3\text{s}^{-1}$$

Since water is an incompressible fluid, the density will be constant, and we can write

$$\dot{V} = u_x A_x = u_y A_y$$

$$\therefore u_x = \frac{\dot{V}}{A_x} = \frac{\dot{V} * 4}{\pi d_x^2} = \frac{0.014 * 4}{\pi * 0.05^2} = 7.13 \text{ m s}^{-1}$$

$$\text{and } u_y = \frac{u_x A_x}{A_y} = \frac{u_x d_x^2}{d_y^2} = \frac{7.13 * 0.05^2}{0.035^2} = 14.55 \text{ m s}^{-1}$$

Further reading:

Bacon and Stephens, Fluid Mechanics for Technicians 3/4 4.1

Bacon and Stephens, Mechanical Technology 30.1

Massey, Mechanics of fluids 3.4

White, Fluid Mechanics 3.3

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