MECH 225 Engineering Science 2

4. Momentum

4.1. The force-momentum principle

The force-momentum principle allows us to determine the **forces** exerted by flowing fluids.

Momentum is mass x velocity. Like velocity and force momentum is a **vector**, so its direction must be specified.

Newton's second law states that the resultant force on a body is equal to its rate of change of momentum. We can apply this law to a control volume through which fluid flows. This gives us the **force-momentum principle** which can be stated as follows:

For a given control volume in the steady state, the sum of the forces acting on the contents equals the change in the momentum flow rates.

Mathematically: $\sum \mathbf{F} = \sum \dot{\mathbf{M}}_{out} - \sum \dot{\mathbf{M}}_{in}$

where **ΣF** is the resultant force acting on the fluid in the control volume, $\dot{\mathbf{M}}$ is the momentum flow rate (equal to $\dot{m}\mathbf{u}$, the mass flow rate times the average flow velocity).

The procedure for applying this principle is as follows:

Step 1 Specify the control volume of fluid under consideration

Step 2 Specify co-ordinate axes

Step 3 Show all the forces acting on the fluid in the control volume. The forces may include:

(a) Body forces – the weight of the fluid inside the control volume

(b) Pressure forces – forces due to the pressure on the control volume surface = pA

(c) Structural forces – forces exerted on the fluid by surrounding or structures

Step 4 Show on the diagram the direction of flow into and out of the control volume

Step 5 Evaluate flow velocities and pressures at specified flow stations, using the principles of continuity and energy, if necessary, and calculate the magnitudes of all known forces.

Step 6 Apply the force-momentum principle to evaluate unknown forces.

Remember that force and momentum are vectors so the force-momentum principle must be applied separately for all 3 co-ordinate axes:

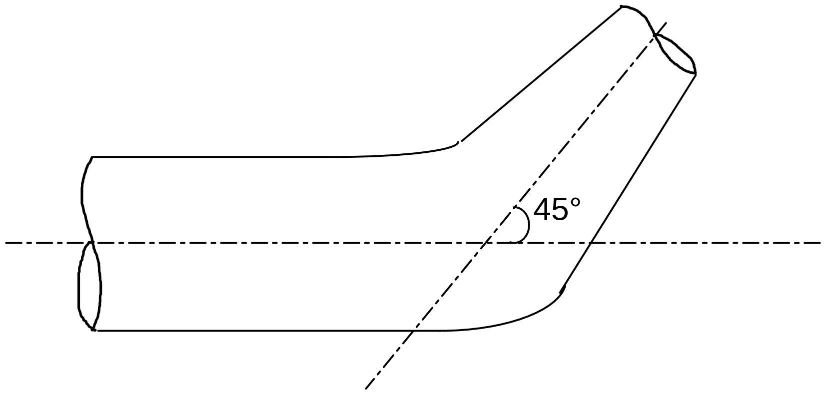
$$\sum F_x = \sum \dot{M}_{outx} - \sum \dot{M}_{inx}$$

$$\sum F_y = \sum \dot{M}_{outy} - \sum \dot{M}_{iny}$$

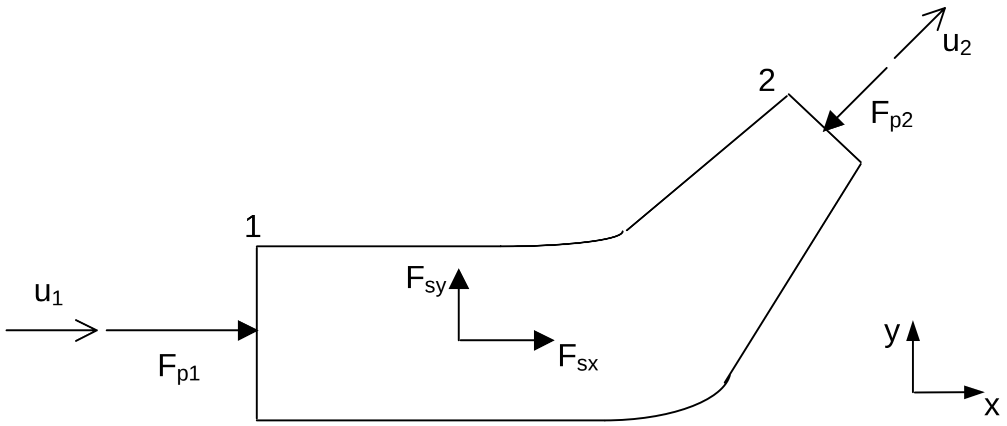
$$\sum F_z = \sum \dot{M}_{outz} - \sum \dot{M}_{inz}$$

Example

A 45° reducing pipe bend (in the horizontal plane) has an inlet diameter of 400 mm and an outlet diameter of 150 mm. The mass flow rate of water is 200 kgs^{-1}, and the inlet pressure is 120 kPa (gauge). Neglecting friction, calculate the horizontal force exerted by the fluid on the pipe. (Take density of water to be 1000 kgm^{-3})



Steps 1 and 2 Define the control volume. This contains the fluid in the pipe between the inlet,1, and the outlet, 2, shown below with the co-ordinate axes.



Steps 3 and 4 The forces acting on the fluid in the control volume are:

(a) Body forces = 0 since the pipe is in the horizontal plane

(b) Pressure forces are $F_{p1} = p_1A_1$ at 1 and $F_{p2} = p_2A_2$ at 2. Note that pressure forces always act at right angles to the cross-sectional area and *in* to the control volume.

(c) Structural force, F_s. Since we don't know either the magnitude or the direction this is drawn as its two components, F_{sx} and F_{sy} in the x and y directions, respectively.

These forces are shown on the CV, as well as the directions of flow at 1 and 2.

Step 4 From continuity: $\dot{m} = \rho_1 u_1 A_1 = \rho_2 u_2 A_2 = 2 \text{ kg s}^{-1}$

$\rho_1 = \rho_2 = 1000 \text{ kg m}^{-3}$

$$\therefore \quad u_1 = \frac{\dot{m}}{\rho A_1} = \frac{\dot{m} * 4}{\rho \pi d_1^2} = \frac{200 * 4}{1000 * \pi * 0.4^2} = 1.592 \text{ ms}^{-1}$$

and $u_2 = \frac{\dot{m}}{\rho A_2} = \frac{\dot{m} * 4}{\rho \pi d_2^2} = \frac{200 * 4}{1000 * \pi * 0.15^2} = 11.318 \text{ ms}^{-1}$

From Bernoulli's equation (neglecting friction):

$$p_1 + \frac{1}{2}\rho u_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho u_2^2 + \rho g z_2$$

$z_1 = z_2$ (horizontal plane)

$$p_2 = p_1 + \frac{1}{2}\rho\left(u_1^2 - u_2^2\right) = 120 * 1000 + \frac{1}{2} * 1000 * \left(1.592^2 - 11.318^2\right) = 57.219 \text{ kPa}$$

$\therefore$ $F_{p1} = p_1 A_1 = 120 * 1000 * \frac{\pi * 0.4^2}{4} = 15.08 \text{ kN}$

and $F_{p2} = p_2 A_2 = 57.219 * 1000 * \frac{\pi * 0.15^2}{4} = 1.011 \text{ kN}$

Step 5

In the x-direction $\sum F_x = \sum \dot{M}_{outx} - \sum \dot{M}_{inx}$

$$F_{p1x} + F_{p2x} + F_{sx} = \dot{m} u_{2x} - \dot{m} u_{1x}$$

$$15.08 * 1000 + (-1.011 * 1000 * \cos 45°) + F_{sx} = 200 * (11.318 \cos 45° - 1.592)$$

$$\therefore F_{sx} = 1282.2 - 15080 + 778.5 = -13019.3$$

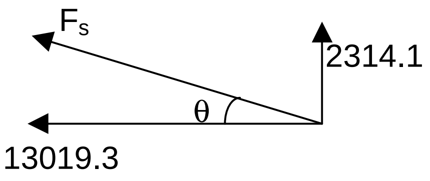
In the y-direction $\sum F_y = \sum \dot{M}_{outy} - \sum \dot{M}_{iny}$

$$F_{p1y} + F_{p2y} + F_{sy} = \dot{m} u_{2y} - \dot{m} u_{1y}$$

$$(-1.011 * 1000 * \sin 45°) + F_{sy} = 200 * (11.318 \sin 45°)$$

$$\therefore F_{sy} = 1599.2 + 714.9 = 2314.1$$

Therefore, the force exerted by the pipe *on the fluid* is:



$$F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{13019.3^2 + 2314.1^2}$$
$$= 13223.3\text{N}$$

$$\tan\theta = \frac{F_{sy}}{F_{sx}} = \frac{2314.1}{13019.3} = 0.178 \quad \theta = 10.1°$$

Therefore the force exerted by the fluid on the pipe is $-\mathbf{F_s}$ = 13.223 kN at 10.1°

Further reading:

Bacon and Stephens, Fluid Mechanics for Technicians 3/4 Ch 5

Bacon and Stephens, Mechanical Technology Ch 32

Massey, Mechanics of fluids 4.1-4.2

White, Fluid Mechanics 3.4

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