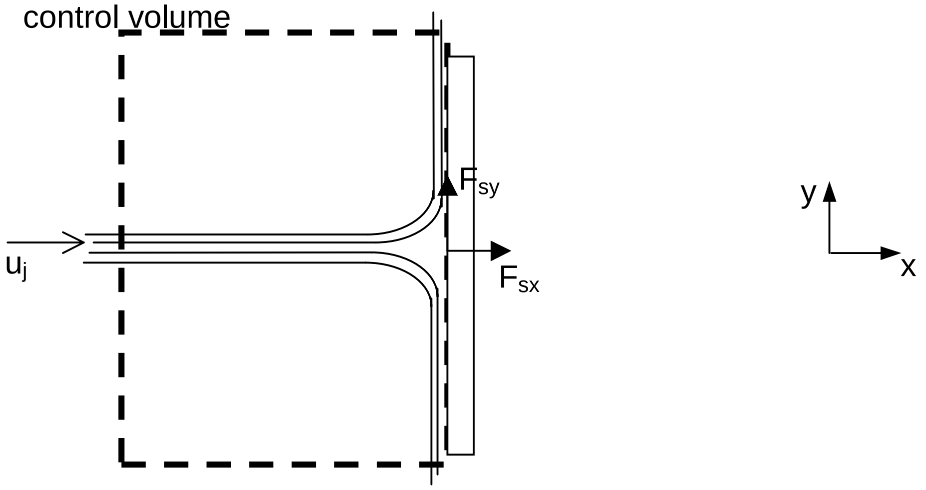
4.3. Fluid deflection

Whenever the direction of flow of a fluid is altered by, for example, it striking a fixed plate, then the plate exerts a force on the fluid. By Newton's third law, the fluid exerts an equal and opposite force on the plate.

Example 1. Suppose that a horizontal jet of fluid strikes a vertical circular plate in such a way that it is deflected evenly all around the plate:



Consider a control volume that encloses the fluid between where it strikes the plate and leaves it as illustrated above.

The pressure is atmospheric pressure all around the control volume, where the fluid enters it and where it leaves it. So there are no pressure forces. Neglecting the weight of the fluid, there are no body forces.
There is a structural force Fs exerted by the plate on the fluid, shown as its two components, F_{sx} and F_{sy}, both drawn in the positive x and y directions.

What is the velocity of the water as it leaves the plate? Apply Bernoulli, neglecting changes in height.

You should find that the velocity of the jet exiting the control volume is exactly the same as that entering it, except that its direction has changed.

The fluid leaving the control volume has no component of velocity in the x-direction, since the fluid will flow over the vertical plate. Therefore, in the x-direction,

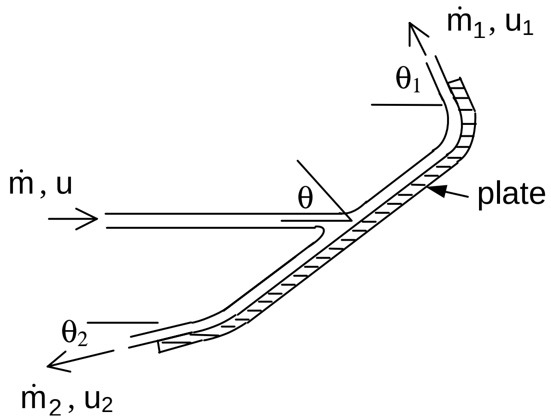
$$\sum F_x = \sum \dot{M}_{outx} - \sum \dot{M}_{inx}$$

$\therefore F_{sx} = 0 - \dot{m}u_j$ F_{sx} therefore acts to the left.

In the vertical direction, all the upward components of fluid velocity leaving the control volume will balance all the downward components. There is no y-component to the velocity entering the control volume. Furthermore, a force

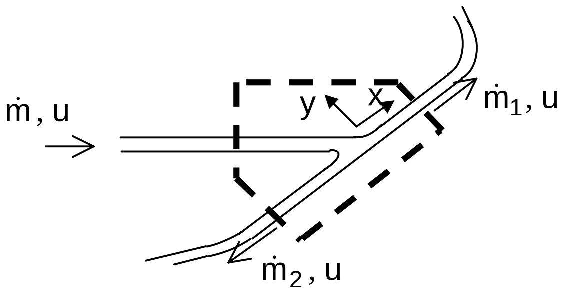
parallel to the plate would be a shear force acting on the fluid; but this is not possible in an ideal "frictionless" fluid, so F_{sy} must be 0.
Thus, the force on the plate is a horizontal force of $-F_{sx}$. It will act to the right and have a magnitude of $\dot{m}u_j$.

Example 2



A jet strikes a plate as shown in the diagram above. $\theta = 30°$, $\theta_1 = 40°$ and $\theta_2 = 25°$. The mass flow rate in the jet is 1.5 kgs^{-1} and its velocity is 25 ms^{-1}. Find the force exerted on the plate.

First, we need to find the mass flow rates $\dot{m}_1$ and $\dot{m}_2$. Consider a control volume which encloses the part of the fluid with axes parallel and perpendicular to the plate surface, as shown below:



As before, applying Bernoulli shows that $u_1 = u_2 = u$.

As above, there is no structural force parallel to the plate. So in the x-direction:

$$\sum F_x = \sum \dot{M}_{outx} - \sum \dot{M}_{inx}$$

$$\therefore 0 = \dot{m}_1 u - \dot{m}_2 u - \dot{m} u \sin\theta$$

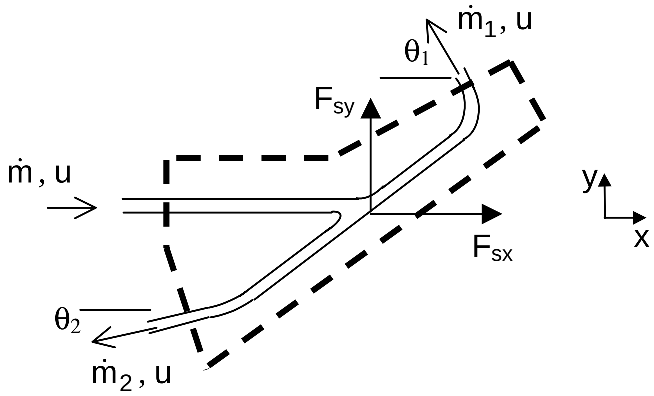
$$\therefore \dot{m}_1 - \dot{m}_2 = \dot{m}\sin\theta \qquad \therefore \dot{m}_1 - \dot{m}_2 = 1.5\sin 30° = 0.75 \text{ kgs}^{-1} \qquad (1)$$

From continuity, $\dot{m}_1 + \dot{m}_2 = \dot{m}$ $\qquad \therefore \dot{m}_1 + \dot{m}_2 = 1.5 \text{ kgs}^{-1}$ $\qquad$ (2)

From equations (1) and (2) we find that $\dot{m}_1 = 1.125 \text{ kgs}^{-1}$ and $\dot{m}_2 = 0.375 \text{ kgs}^{-1}$

To find the force exerted on the plate, first find the force exerted on the fluid in the control volume the surface of which cuts the jets after they leave the plate.

Try drawing a suitable control volume yourself.



Apply the force-momentum principle.

In the x-direction: $\sum F_x = \sum \dot{M}_{outx} - \sum \dot{M}_{inx}$

$$F_{sx} = -\dot{m}_1 u \cos\theta_1 - \dot{m}_2 u \cos\theta_2 - \dot{m}u = -1.125 * 25 * \cos 40° - 0.375 * 25 * \cos 25° - 1.5 * 25$$

$\therefore$ F_{sx} = -67.54 N

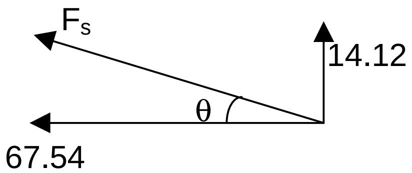
Note that $\sum \dot{M}_{outx} = -\dot{m}_1 u \cos\theta_1 - \dot{m}_2 u \cos\theta_2$ where the minus signs arise because the components of flow velocity are in the negative x-direction, according to my specified co-ordinate axes.

In the y-direction: $\sum F_y = \sum \dot{M}_{outy} - \sum \dot{M}_{iny}$

$$F_{sy} = \dot{m}_1 u \sin\theta_1 - \dot{m}_2 u \sin\theta_2 - 0 = 1.125 * 25 * \sin 40° - 0.375 * 25 * \sin 25°$$

$\therefore$ F_{sy} = 14.12 N

Therefore, the force exerted by the plate *on the fluid* is:



$$F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{67.54^2 + 14.12^2}$$
$$= 69 \text{N}$$

$$\tan\theta = \frac{F_{sy}}{F_{sx}} = \frac{14.12}{67.54} = 0.209 \qquad \theta = 11.8°$$

Therefore the force exerted on the plate is $-\mathbf{F_s}$ = 69 N at 11.8° ↘

Further reading:

Bacon and Stephens, Fluid Mechanics for Technicians 3/4	5.2-5.4
Bacon and Stephens, Mechanical Technology	32.2-32.4
Massey, Mechanics of fluids	4.3.1
White, Fluid Mechanics	3.4

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