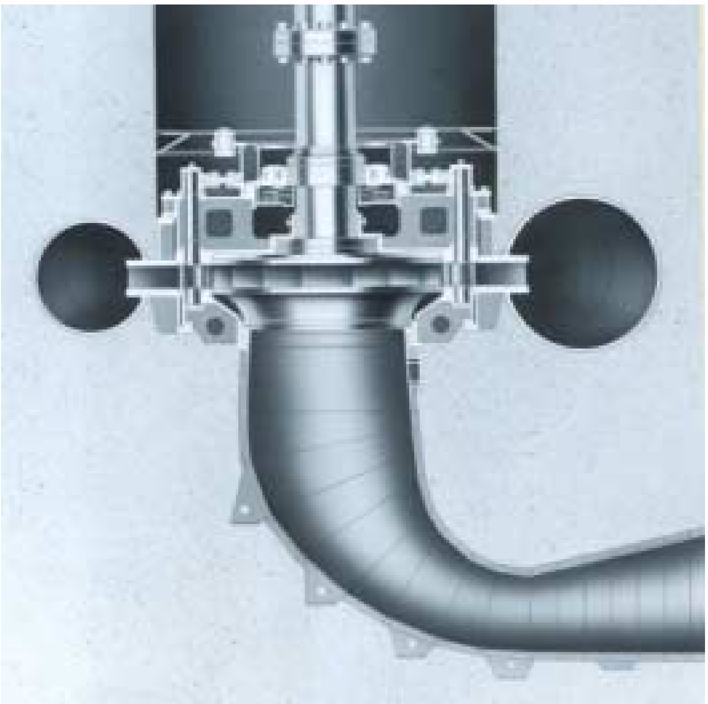
4.4. Reaction turbine

A reaction turbine is a device for converting energy stored in a fluid in the form of pressure to useful work. There are two main types of turbine, an impulse turbine and a reaction turbine, and good descriptions of these devices can be found in Massey, section 14.3. They work by passing a fluid through a runner (also known as an impeller) which has blades which cause a change in the direction of the fluid. Its momentum is therefore changed, and the rate of change of momentum produces a force which acts on the runner. The runner therefore turns, and can drive, for example, an electric generator.

In a reaction turbine, the static pressure decreases as the water passes through it. An example of a reaction turbine is the Francis turbine.

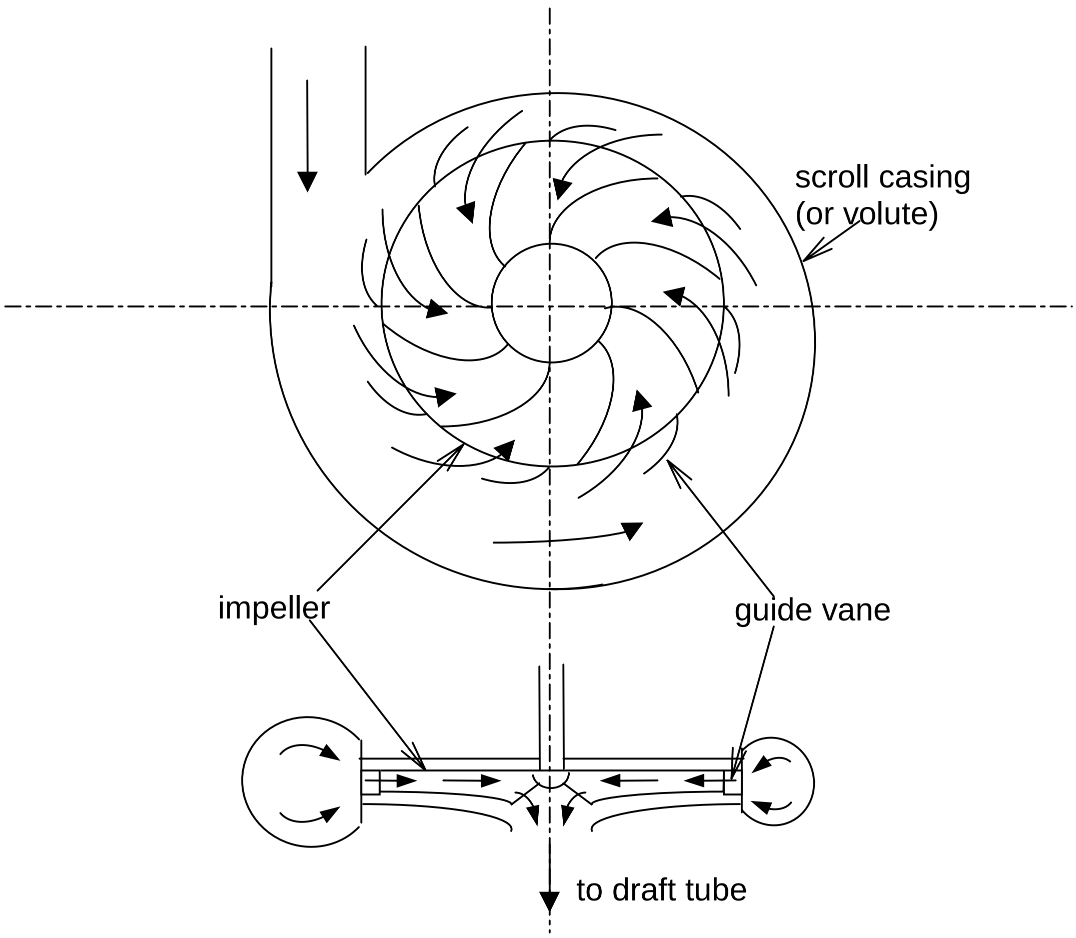


(Reference: http://www.atals.com/newtic/hyd_tbfr.htm)

A good description of this turbine can be found in Massey, sections 14.3.3. and 14.3.4.

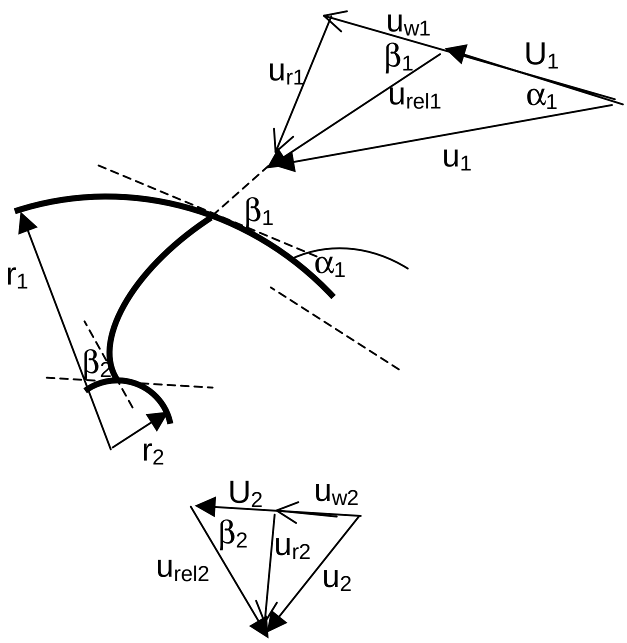
You may like to check out the following web-sites too:
http://www.wkv-ag.com/englisch/produkte/francis/francis01_e.htm
http://www.newmillshydro.freeserve.co.uk/francispics.html
http://www.cink-turbiny.cz/english/vyrobky/francis.html
http://starfire.ne.uiuc.edu/~ne201/1996/schoenau/wheel~1.html

A simplified version of Fig. 14.13 in Massey is given below:



To calculate the torque produced, it is necessary to consider the angles at which the flow enters and leaves the impeller. The simplest way to do this is to draw the vector diagrams for flow velocity at inlet and outlet to the impeller. To do this we need to consider in more detail the geometry of the impeller blades and the guide vanes.

A part of the impeller, showing the inlet and outlet is reproduced below:



Suffixes 1 and 2 refer to inlet and outlet respectively

U is the blade velocity (tangential to the impeller motion)

u is the absolute water velocity

u_w is the tangential component of u

u_r is the radial component of u

u_{rel} is the velocity of the water relative to the blade

Let ω be the angular speed of rotation of the impeller in radians per second.
Let $\dot{V}$ be the volume flow rate of water through the impeller in $m^3\ s^{-1}$.
Let B be the height of the impeller blades in m.

At inlet, the flow area (neglecting the width of the blades) is

$A_1 = 2\pi r_1 B_1$

Therefore the radial speed at inlet is $$u_{r1} = \frac{\dot{V}}{2\pi r_1 B_1}$$

And at the outlet, $$u_{r2} = \frac{\dot{V}}{2\pi r_2 B_2}$$

From the velocity diagram at inlet, the tangential velocity of the water is

$$u_{w1} = \frac{u_{r1}}{\tan\alpha}$$ where α is the guide vane angle.

The tangential speed of the impeller at the inlet is $U_1 = \omega r_1$.

For a smooth flow onto the blade, the angle β_1 should be such that

$$\tan\beta_1 = \frac{u_{r1}}{u_{w1} - U_1}$$

At the outlet, $\tan\beta_2 = \frac{u_{r2}}{U_2 - u_{w2}}$ where $U_2 = \omega r_2$

Therefore, $u_{w2} = U_2 - \frac{u_{r2}}{\tan\beta_2}$

To find the power developed by the turbine, we need to consider the rate of change of momentum of the water as it flows through the impeller. Just as the resultant force is equal to the rate of change of linear momentum of a fluid, the resultant torque is equal to the rate of change of angular momentum, where the angular momentum is equal to the moment of the linear momentum.

Thus the angular momentum at inlet is the momentum tangential to the inlet times the radius at inlet. The rate at which angular momentum enters the impeller is therefore, $\dot{m}u_{w1}r_1$ and the rate at which it leaves the impeller is $\dot{m}u_{w2}r_2$.

Therefore the torque developed, T_f which is the torque exerted *on the fluid*, is

$$T_f = \dot{m}(u_{w2}r_2 - u_{w1}r_1)$$

Therefore the torque exerted by the fluid on the impeller, T, is $-T_f$, so

$$T = \dot{m}(u_{w1}r_1 - u_{w2}r_2)$$

The power developed, $P = T\omega$. Therefore,

$$P = \dot{m}\omega(u_{w1}r_1 - u_{w2}r_2) = \dot{m}(u_{w1}U_1 - u_{w2}U_2)$$

For the optimum design, the tangential velocity at exit, u_{w2}, is reduced to as small a value as possible, i.e. 0. So the optimum blade angle at outlet is β_2

where $\tan\beta_2 = \frac{u_{r2}}{U_2}$

And the maximum torque developed is $T = \dot{m} u_{w1} r_1$

Note that a **pump** is a turbine operated in reverse. This means that work is done to turn the impeller, which then imparts pressure to the fluid. The principles of operation are identical.

Example

A Francis turbine has the following dimensions:

Impeller flow depth, B = 200 mm ($B_1 = B_2$)

Impeller outer radius, r_1 = 1200 mm
Impeller inner radius, r_2 = 600 mm
Guide vane angle, α_1 = 15°
The volume flow rate is 18 $m^3 s^{-1}$; the impeller angular speed, ω = 30 rad s^{-1}.

Calculate: (a) the optimum blade angles, β_1 and β_2.
(b) the torque on the impeller with these blade angles
(c) the power developed.

$$u_{r1} = \frac{\dot{V}}{2\pi r_1 B_1} = \frac{18}{2\pi * 1.2 * 0.2} = 11.937$$

$$u_{r2} = \frac{\dot{V}}{2\pi r_2 B_2} = \frac{18}{2\pi * 0.6 * 0.2} = 23.873$$

$$u_{w1} = \frac{u_{r1}}{\tan\alpha} = \frac{11.937}{\tan 15°} = 44.549$$

$$U_1 = \omega r_1 = 30 * 1.2 = 36$$

(a) The optimum blade angle at inlet is

$$\tan\beta_1 = \frac{u_{r1}}{u_{w1} - U_1} = \frac{11.937}{44.549 - 30} = 1.396$$

Therefore, β_1 = 54.4°

For the optimum design, u_{w2} is zero and $\tan\beta_2 = \frac{u_{r2}}{U_2} = \frac{u_{r2}}{\omega r_2}$

Therefore, $\tan\beta_2 = \frac{23.873}{30 * 0.6} = 1.326$ β_2 = 52.98° = 53°

(b) the torque on the impeller with these blade angles,

$T = \dot{m} u_{w1} r_1 = \rho \dot{V} u_{w1} r_1 = 1000 * 18 * 44.549 * 1.2 = 962258$ Nm = 962.3 kNm

(c) The power developed P = Tw = 962.258*30 = 28867.7 kW = 28.9 MW

Return to module introduction:
http://www.tech.plym.ac.uk/sme/mech225/mechsci0.htm