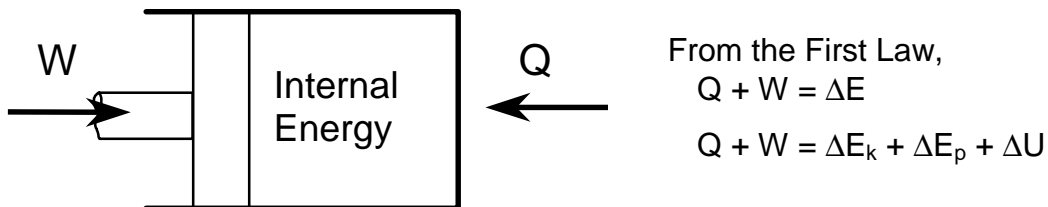


3.3. Non-Flow Energy Equation (NFEE)

You may have noticed that the term “system” keeps cropping up. It is necessary, therefore, that before we start any analysis we define the system that we are looking at. To do this we construct an imaginary boundary around what we are interested in – for example, the cricket ball (struck by Nasser Hussein) or the water in the kettle). When dealing with a non-flow situation, then the system will be of fixed mass - no matter crosses the boundary - so it is useful to define a **control mass**. If we are considering a flow situation, then a **control volume** through which the fluid flows is more useful. I hope these ideas will become clearer when we consider some examples.

A typical closed system is a gas enclosed in a cylinder by means of a piston. The gas inside the cylinder is the control mass.



From the First Law,

$$Q + W = \Delta E$$

$$Q + W = \Delta E_k + \Delta E_p + \Delta U$$

But for a non-flowing gas, its velocity will be 0 to start with, and when it has settled down after the process, its velocity will again be 0, so the change in kinetic energy, $\Delta E_k = 0$. Similarly, there is no significant change in potential energy, so $\Delta E_p = 0$. There will, however, be a change in the internal energy, ΔU . So, the non-flow energy equation (NFEE) becomes, simply,

$$Q + W = \Delta U$$

or
$$Q + W = U_2 - U_1$$

where U_2 is the internal energy in state 2, after the process and U_1 is the internal energy in state 1, before the process.

Example 1

During a complete cycle, a system is subjected to the following heat transfers: 800 kJ from the surroundings and 500 kJ to the surroundings. At two points in the cycle, work is transferred to the surroundings of 96 kJ and 20 kJ. At a third point there is a further work transfer. Determine its magnitude and sense.

Example 2

In an air compressor, the compression takes place at constant internal energy and 100 kJ of energy are rejected to the cooling water per kg of air. Determine the specific work transfer during the compression stroke.

Solution 1

Here, the system is defined for us – no details are given as to its nature. For a *complete cycle*, we know that:

$$\Sigma Q + \Sigma W = 0$$

$$\Sigma Q = + 800 - 500 = 300 \text{ kJ}$$

$$\Sigma W = -96 - 20 + W = -116 + W$$

where W is the value of the unknown work transfer in kJ.

$$\therefore 300 - 116 + W = 0$$

$$\therefore W = -300 + 116 = -184 \text{ kJ}$$

The negative sign tells us that it is a work transfer *to* the surroundings.

Solution 2

The system is the air in the compressor. We do not know its mass, so let us suppose that it is m kg.

The internal energy remains constant, so ΔU , the change in internal energy is 0.

The non-flow energy equation becomes

$$Q + W = 0$$

The energy transferred by heating is *from* the air in the compressor to the cooling water, and is therefore negative. It is -100 kJ per kg, so for m kg:

$$Q = - 100m \text{ kJ}$$

Applying the NFEE, $-100m + W = 0$

$$\therefore W = 100m \text{ kJ}$$

You are asked to find the *specific* work transfer. This means the work transfer for every kilogram, and this is given the symbol, w . Therefore,

$$w = \frac{W}{m} = 100 \text{ kJ}$$

What does the positive sign for the work transfer tell you? Is this what you would expect? Can you explain why it is positive?

3.4 Applications of the NFE

The non-flow energy equation can be applied to any non-flow **thermodynamic process**. We can classify processes depending on the conditions under which they take place, and the type of fluid. Typical processes that are performed on gases are at constant volume, constant pressure, constant temperature (isothermal) and with no energy transfer by heating (adiabatic).

Calculating energy transfers

(a) Energy transfer by heating, Q

We saw in 1.6 that we can calculate the energy transferred by heating using the equation:

$$Q = mc(T_2 - T_1)$$

(b) Energy transfer by working, W

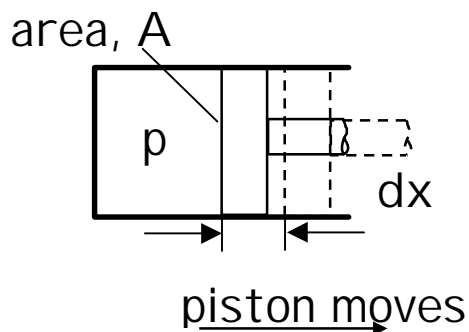
How can we calculate the energy transferred by working?

$W = \text{Force} \times \text{distance moved}$

and $\text{Force} = \text{pressure} \times \text{area}$

Therefore, $W = \text{pressure} \times \text{area} \times \text{distance moved}$.

Consider a gas enclosed by a piston:



The force on the piston = pA

The distance moved = dx

Therefore the work done by the gas is $dW = pA dx = p dV$

The total work done, when the piston moves so that the volume increases from V_1 to V_2 is:

$$W = -\int_1^2 p dV$$

This is the area under the p - V diagram between states 1 and 2.

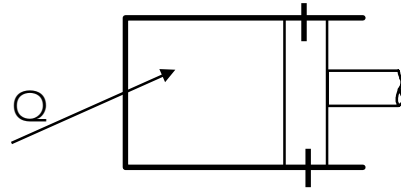
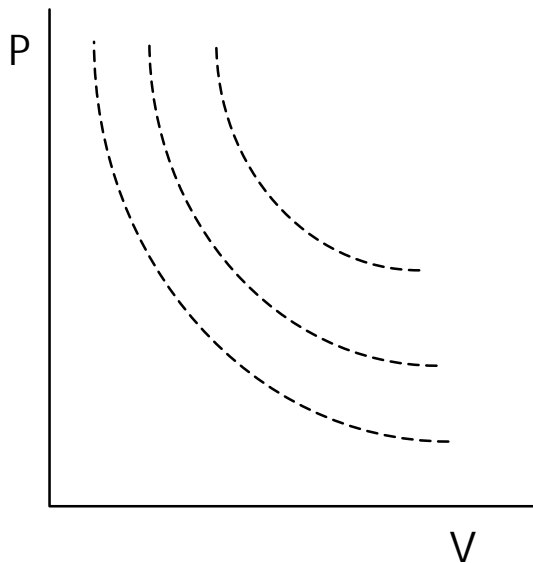
The negative sign is there, because as the gas expands it does work on the surroundings. We need to integrate $p dV$ because as the gas expands the pressure may not remain constant, in general, so we need to add up all the small amounts of work done at each of the intervening pressures when the piston moves by a small distance, dx .

For a perfect gas, we know the relationships between pressure, volume and temperature. If we know the conditions of the process, then we can calculate the work done and the heat transferred. But these quantities, **Q and W** , **depend on the process**, not only on the beginning and end states.

A process can be shown on a p-V diagram. On the diagrams below, isotherms are marked as dashed lines. Isotherms are lines of constant temperature. For a perfect gas at constant temperature, $pV = \text{constant}$. So these dashed lines represent constant temperature processes.

Put in the lines on the p-V diagrams, to represent the relevant process in each of the following and see if you can write down, or calculate, W and Q for each.

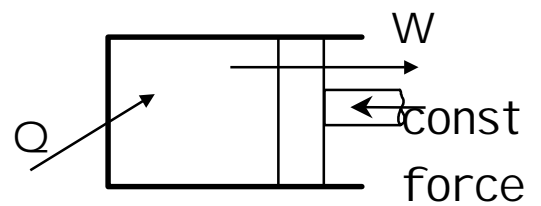
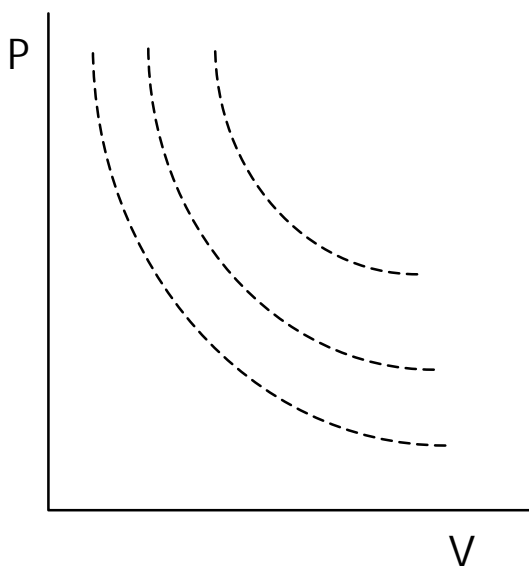
(a) Constant Volume



$W =$

$Q =$

(b) Constant Pressure

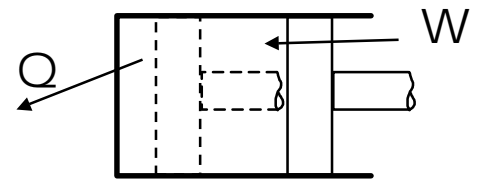
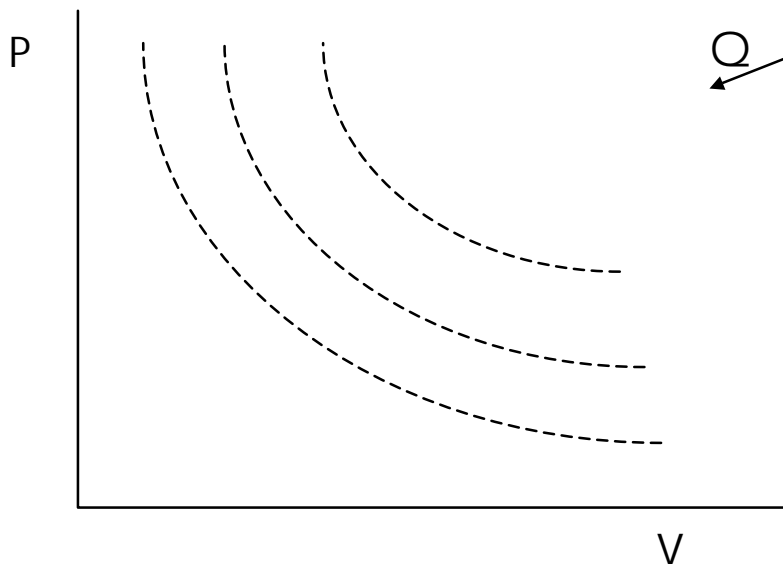


piston moves

$W =$

$Q =$

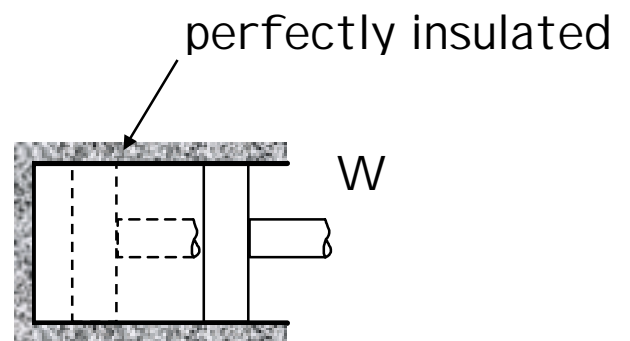
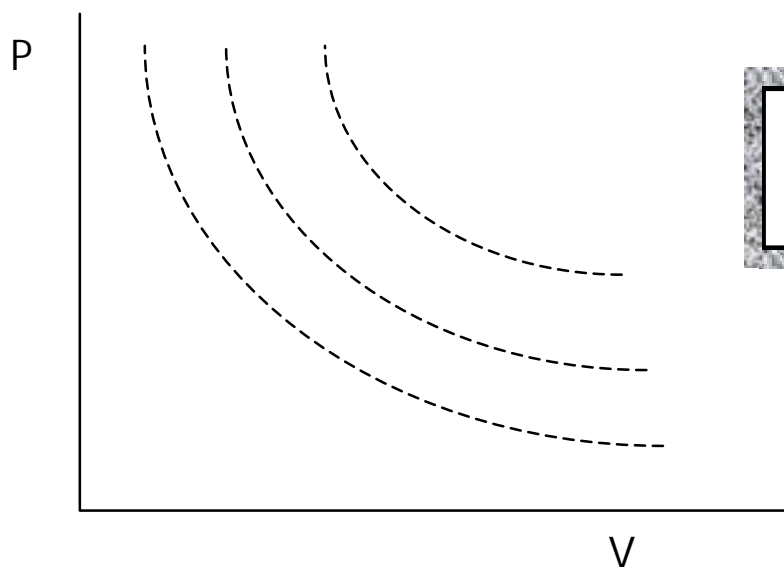
(c) Isothermal (at constant temperature)



$W =$

$Q =$

(d) Adiabatic (no heat transfer)

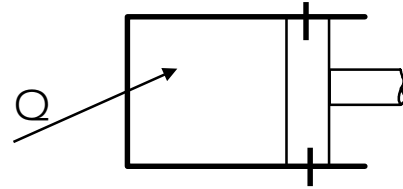
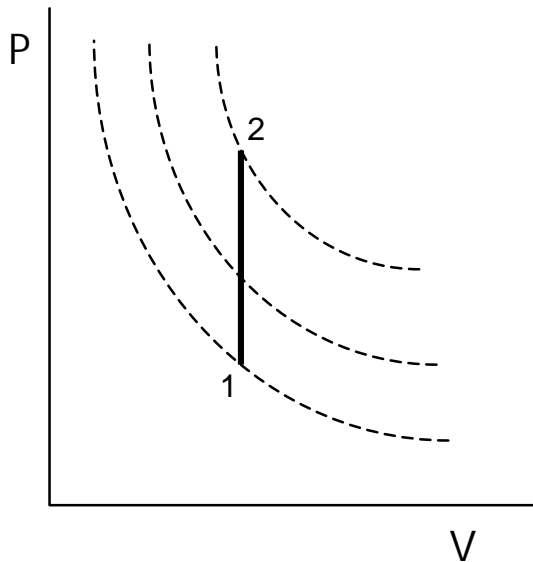


$Q =$

$W =$

You should have the following:

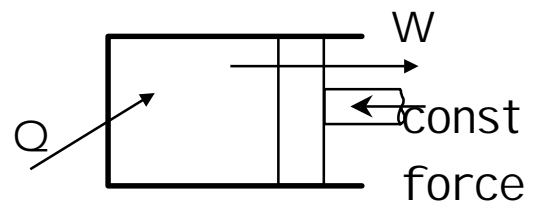
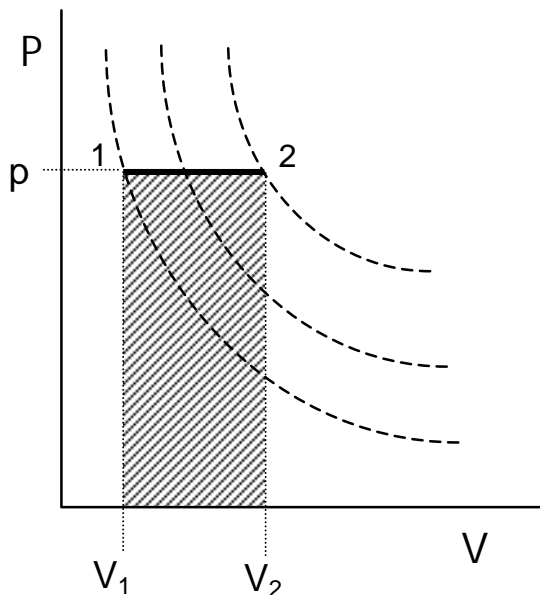
(a) Constant Volume



$$W = 0$$

$$Q = \Delta U \\ = mc_v(T_2 - T_1)$$

(b) Constant Pressure



piston moves

$$W = -p(V_2 - V_1) \\ \text{(shaded area under the line)}$$

For a perfect gas,

$$W = -mR(T_2 - T_1)$$

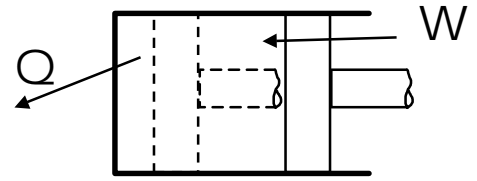
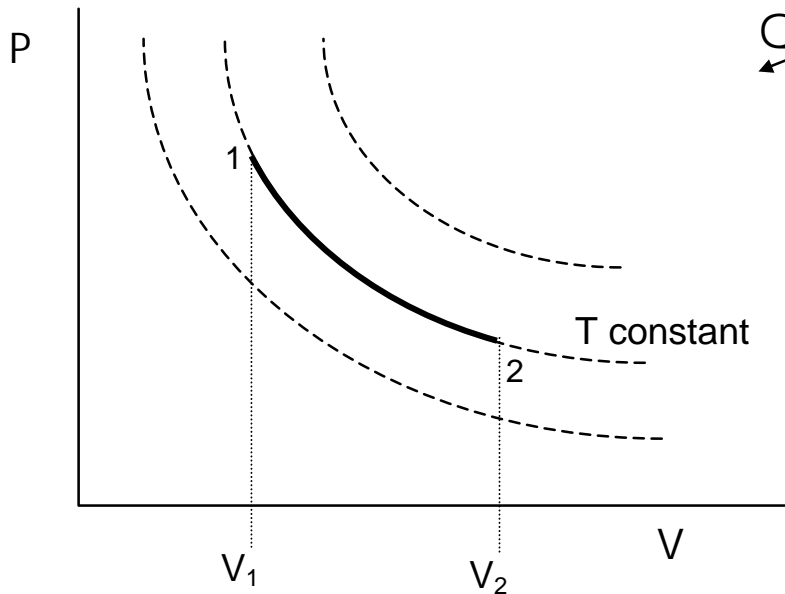
$$Q = \Delta U - W$$

$$= mc_v(T_2 - T_1) + mR(T_2 - T_1)$$

$$= m(c_v + R)(T_2 - T_1)$$

$$= mc_p(T_2 - T_1)$$

(c) Isothermal (at constant temperature)

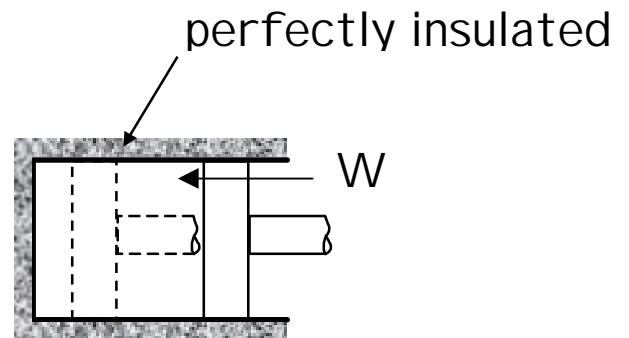
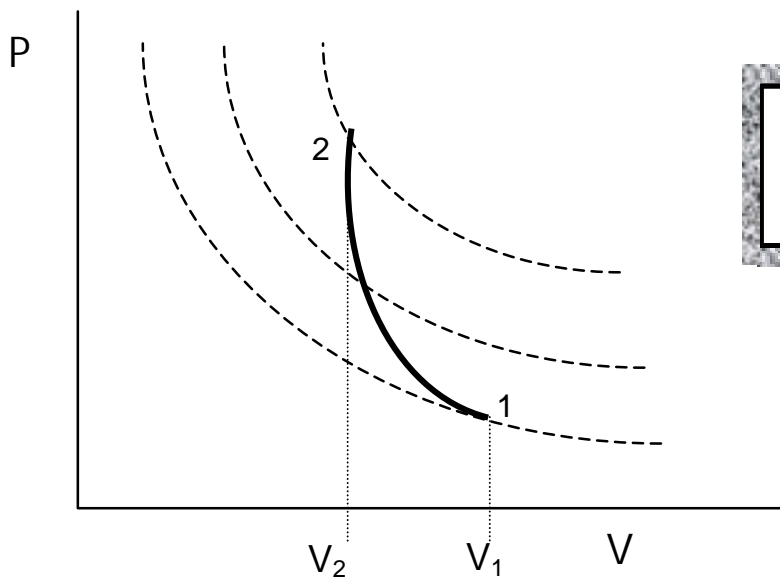


$$\begin{aligned}
 W &= -\int_1^2 p dV \\
 &= -\int_1^2 \frac{mRT}{V} dV \\
 &= -mRT \ln\left(\frac{V_2}{V_1}\right)
 \end{aligned}$$

$$\Delta U = 0$$

$$Q = -W$$

(d) Adiabatic (no heat transfer)



$$Q = 0$$

$$\begin{aligned}
 W &= \frac{(p_2 V_2 - p_1 V_1)}{\gamma - 1} \\
 &= \frac{mR(T_2 - T_1)}{\gamma - 1} \\
 &= mc_v(T_2 - T_1) \\
 &= \Delta U
 \end{aligned}$$

where $\gamma = \frac{c_p}{c_v} \approx 1.4$ for cool air

Some questions for you to consider:

- (1) Why is the work done in a constant volume process equal to 0?
- (2) Why is the change in internal energy in an isothermal process equal to 0?
- (3) Derive the expression for the work done in a polytropic process, that is a process which can be modelled by the equation $pV^n = \text{constant}$, where n can take any value, and is normally found by experiment.

Example (polytropic process)

Air at 1.4 bar with a specific volume of $1.2 \text{ m}^3\text{kg}^{-1}$ is compressed to 20 bar according to $Pv^{1.3} = \text{constant}$. Find:

- the new specific volume
- the specific work done
- the initial and final temperatures
- the specific heat transfer

Repeat for an isothermal process.

For air, take $R = 287 \text{ Jkg}^{-1}\text{K}^{-1}$ and $c_p = 1005 \text{ Jkg}^{-1}\text{K}^{-1}$.

Solution:

$$p_1 = 1.4 \text{ bar} \quad v_1 = 1.2 \text{ m}^3\text{kg}^{-1}$$

$$p_2 = 20 \text{ bar} \quad v_2 = ?$$

$$pv^{1.3} = \text{constant} \quad \therefore p_1v_1^{1.3} = p_2v_2^{1.3}$$

$$\therefore v_2^{1.3} = \frac{p_1v_1^{1.3}}{p_2} = \frac{1.4 * 1.2^{1.3}}{20} = \frac{1.4 * 1.267}{20} = 0.089$$

$$\therefore v_2 = 0.0887^{\frac{1}{1.3}} = 0.0887^{0.769} = 0.155 \text{ m}^3\text{kg}^{-1}$$

$$w = \frac{p_2v_2 - p_1v_1}{n-1} \quad (\text{see summary below})$$

NOTE: lower case letters are used to denote **specific** quantities, i.e. values for 1 kg mass.

$$\therefore w = \frac{20 * 10^5 * 0.155 - 1.4 * 10^5 * 1.2}{1.3 - 1} = \frac{3.1 - 1.68}{0.3} * 10^5 = 4.733 * 10^5 \text{ J}$$

$$\therefore w = 473.3 \text{ kJ kg}^{-1}$$

The initial and final temperatures we can find using the equation of state for a perfect gas (see 1.5 gas laws)

$$\therefore T_1 = \frac{p_1 v_1}{R} = \frac{1.4 * 10^5 * 1.2}{287} = 585.4 \text{ K} = 312^\circ\text{C}$$

$$\text{and } T_2 = \frac{p_2 v_2}{R} = \frac{20 * 10^5 * 0.155}{287} = 1080 \text{ K} = 807^\circ\text{C}$$

To find the specific heat transfer, apply the NFEE:

$$q = \Delta u - w$$

$$\Delta u = c_v(T_2 - T_1) = (c_p - R)(T_2 - T_1) = (1005 - 287)(1080 - 585) = 718 * 495 \text{ J}$$

$$\therefore \Delta u = 355.41 \text{ kJ kg}^{-1}$$

$$\therefore q = 355.4 - 473.3 = -117.9 \text{ kJ kg}^{-1}$$

For an isothermal process, we have $T_2 = T_1 = 585.4 \text{ K}$

$$\text{and } pv = \text{constant}, \therefore p_1 v_1 = p_2 v_2 \quad v_2 = \frac{p_1 v_1}{p_2} = \frac{1.4 * 1.2}{20} = 0.084 \text{ m}^3 \text{kg}^{-1}$$

$$w = -RT \ln\left(\frac{v_2}{v_1}\right) = -287 * 585.4 \ln\left(\frac{0.084}{1.2}\right) = -287 * 585.4 * (-2.659) = 446.8$$

$$\therefore w = 447 \text{ kJ kg}^{-1}$$

For an isothermal process $\Delta u = 0$, so the NFEE gives $q = -w$

$$\therefore q = -447 \text{ kJ kg}^{-1}$$

The thermodynamic relationships for perfect gases are summarised below:

Summary of thermodynamic relationships for ideal gases

For any ideal gas:

$pV = mRT$ (Equation of State)	p = pressure (Nm^{-2} or Pa) V = volume (m^3) T = temperature (K) m = mass (kg) R = gas constant ($\text{Jkg}^{-1}\text{K}^{-1}$) = $287 \text{ Jkg}^{-1}\text{K}^{-1}$ for air
$\gamma = \frac{c_p}{c_v}$	γ = the adiabatic index = 1.4 for air
$c_v = \frac{R}{\gamma - 1}$	c_v = specific heat at constant volume ($\text{Jkg}^{-1}\text{K}^{-1}$)
$c_p - c_v = R$	c_p = specific heat at constant pressure ($\text{Jkg}^{-1}\text{K}^{-1}$)

For any process:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\Delta U = mc_v \Delta T$$

ΔU is the change in internal energy (J)
 ΔT is the change in temperature

$$Q + W = \Delta U$$

(First Law of Thermodynamics)

W is the work done (J)

Q is the energy transfer by heating (J)

For an adiabatic process:

$$p_1 V_1^\gamma = p_2 V_2^\gamma \quad \text{also} \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

$$W = \frac{p_2 V_2 - p_1 V_1}{\gamma - 1} = mc_v (T_2 - T_1)$$

$$Q = 0$$

For a constant pressure process:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$W = p(V_2 - V_1) = mR(T_2 - T_1)$$

$$Q = mc_p(T_2 - T_1)$$

For a constant temperature (isothermal) process:

$$p_1 V_1 = p_2 V_2$$

$$W = mRT \ln\left(\frac{V_1}{V_2}\right) = p_1 V_1 \ln\left(\frac{V_1}{V_2}\right) \quad Q = -W$$

For a constant volume process:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

$$W = 0$$

$$Q = mc_v(T_2 - T_1)$$

For a polytropic process:

$$p_1 V_1^n = p_2 V_2^n$$

n is the polytropic index

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

$$W = \frac{p_2 V_2 - p_1 V_1}{n-1} \quad Q = mc_v(T_2 - T_1) - W$$

Further reading:

Bacon and Stephens, Mechanical Technology

23.5-23.8

Rogers, G and Mayhew, Y,

Engineering Thermodynamics Work and Heat Transfer

Ch 3

The Open University, T236 Introduction to thermofluid mechanics

Block 4