1.6 Specific Heat

(a) For liquids and solids:

Specific heat capacity is

the energy transfer by heating required to raise the temperature of 1 kg of the substance by 1 K.

If, for a mass m, heat transfer dQ raises the temperature by dT then,

$$c = \frac{1}{m}\frac{\delta Q}{\delta T} \rightarrow \frac{1}{m}\frac{dQ}{dT}$$

What are the units of specific heat capacity?

If c is constant, then for a temperature rise from T_1 to T_2, of a mass m, the energy required is

$$Q = mc(T_2-T_1)$$

For mixing problems, energy is conserved: heat transferred to cooler bodies or liquids heating up equals heat transferred from hotter bodies cooling down.

Example

A copper container of mass 400 g contains 2 kg of water, both at 15°C. A brass cube of mass 500 g at 90°C is placed in the water. If no heat is lost to the atmosphere (i.e., the copper container is well-insulated) what is the final temperature?

The specific heat capacities are:
c_{copper} = 394 J kg^{-1} K^{-1}
c_{water} = 4180 J kg^{-1} K^{-1}
c_{brass} = 385 J kg^{-1} K^{-1}

Let t_f°C be the final temperature. Since we are dealing with temperature differences it will be alright to work in Celsius. We do not need to change all the temperatures to K.

Energy lost by the brass = energy gained by water and copper

$Q_b = Q_w + Q_c$

$Q_b = m_b c_b(t_b-t_f) = 0.5*385*(90-t_f) = 17325 - 192.5t_f$
$Q_w = m_w c_w(t_f-t_w) = 2*4180*(t_f-15) = 8360t_f - 125400$
$Q_c = m_c c_c(t_f-t_c) = 0.4*394*(t_f-15) = 157.6t_f - 2364$

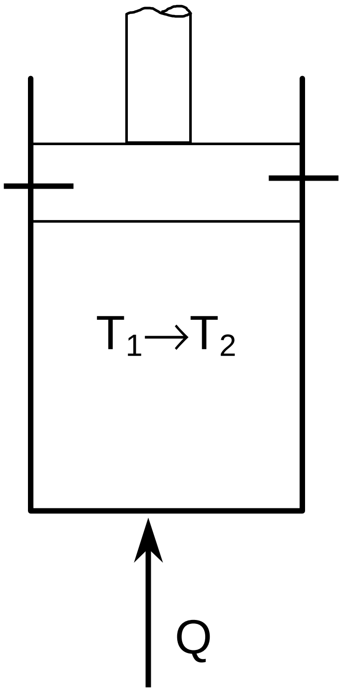
$\therefore 17325 - 192.5t_f = 8360t_f - 125400 + 157.6t_f - 2364$
$17325 + 125400 + 2364 = 8360t_f + 157.6t_f + 192.5t_f$
$145089 = 8710.1t_f$

$$\therefore t_f = \frac{145089}{8710.1} = 16.66$$

(b) For gases

Gases have two specific heat capacities depending on whether the heating takes place at constant volume or at constant pressure:

constant volume



$$c_v = \frac{1}{m}\frac{dQ}{dT}$$

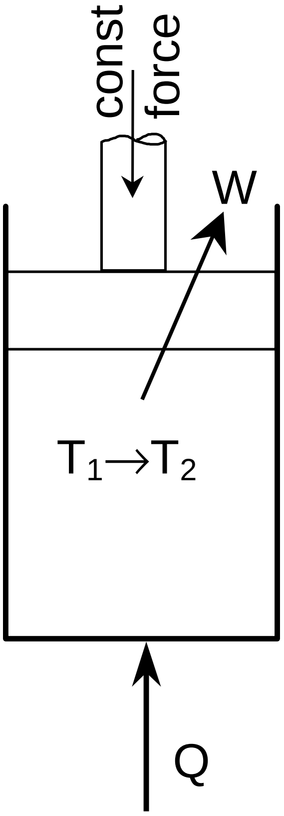
at constant volume

It can be shown that

$$c_v = \frac{du}{dT}$$

u is the specific internal energy

constant pressure



$$c_p = \frac{1}{m}\frac{dQ}{dT}$$

at constant pressure

$$c_p = \frac{dh}{dT}$$

h is the specific internal enthalpy

Which is greater, the specific heat at constant volume, c_v, or the specific heat at constant pressure, c_p?

The specific heat at constant pressure is the greater. This is because when a gas is heated at constant pressure it must expand, in order to keep the pressure constant. When it expands it does work, so the energy supplied by heating must be sufficient not only to raise the temperature, but also to do some work.

For air, $c_v = 718 \text{ J kg}^{-1} \text{ K}^{-1}$ and $c_p = 1005 \text{ J kg}^{-1} \text{ K}^{-1}$

There are two useful relationships between the specific heats of ideal gases:

$$c_p - c_v = R$$

and

$$\frac{c_p}{c_v} = \gamma \qquad \text{where } \gamma \text{ is the adiabatic index, and is 1.4 for air}$$

Example
A gas whose pressure, volume and temperature are 250 kPa, 0.15 m^3 and 150°C respectively, undergoes cooling at constant pressure. The final temperature of the gas is 27°C. (a) What is the mass of the gas? (b) What is its final volume? (c) What is the energy transferred by heating during this process? Assume the gas behaves as a perfect gas with $R = 0.29 \text{ kJ kg}^{-1} \text{ K}^{-1}$ and $c_p = 1.005 \text{ kJ kg}^{-1} \text{ K}^{-1}$

(a) $p_1 = 250$ kPa $\qquad V_1 = 0.15 \text{ m}^3 \qquad T_1 = 150 + 273.15 = 423.15\text{K}$

For a perfect gas, pV = mRT $\therefore m = \frac{pV}{RT} = \frac{250 * 10^3 * 0.15}{0.29 * 10^3 * 423.15} = 0.306 \text{ kg}$

(b) $p_1 = 250$ kPa $\qquad V_1 = 0.15 \text{ m}^3 \qquad T_1 = 27 + 273.15 = 300.15\text{K}$

Since the process is at a constant pressure, $p_2 = p_1 = 250$ kPa

Also, $T_2 = 27 + 273.15 = 300.15$ K

For a perfect gas: $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

For a constant pressure process, this becomes: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\therefore \qquad V_2 = \frac{V_1T_2}{T_1} = \frac{0.15 * 300.15}{423.15} = 0.106 \text{ m}^3$$

(c) $Q = mc_p(T_2 - T_1) = 0.306*1.005*(300.15 - 423.15) = -37.826$ kJ

The minus sign indicates that energy is transferred *from* the gas (it is cooled).

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