# Thin Cylinders

The problem of finding the stresses in a cylinder subjected to internal pressure is statically indeterminate. It's solution requires the consideration of equilibrium forces, compatibility of displacements, the relationship between stress and strain, and the boundary considerations of the problem. For thin cylinders however, whose thickness may be considered small compared to their diameter, a satisfactory solution can be obtained by making some simplifying approximations.

Consider a thin cylinder of plate thickness t, mean diameter D and length l, subjected to internal pressure p. Now consider that the cylinder is sectioned by the x-plane of symmetry and by the two z-planes (of distance z apart) as shown in figure 1.

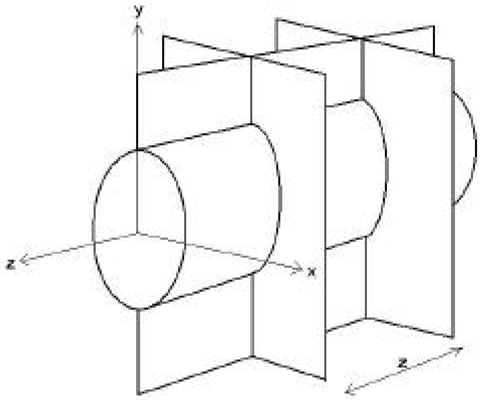


Figure 1.

Consider the equilibrium of forces in the x-direction acting on the sectioned cylinder shown in figure 2. It is assumed that the circumferential stress $\sigma_\theta$ is constant through the thickness of the cylinder.

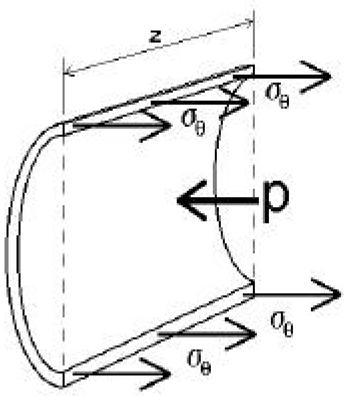


Figure 2.

Force due to internal pressure p acting on area Dz = pDz

Force due to circumferential stress $\sigma_\theta$ acting on area 2tz = $\sigma_\theta$2tz

Equating:

$$\sigma_\theta 2tz = pDz$$

Therefore:

$$\sigma_\theta = \frac{pD}{2t}$$

Now consider the equilibrium of forces in the z-direction acting on the part cylinder shown in figure 3.

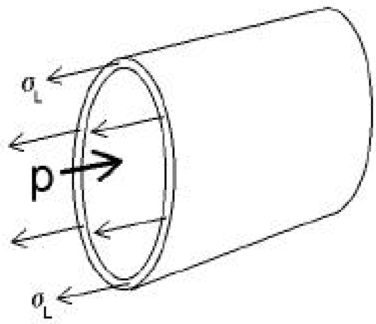


Figure 3.

Force due to internal pressure p acting on area $\pi D^2/4$ = p. $\pi D^2/4$

Force due to longitudinal stress $\sigma_L$ acting on area $\pi Dt$ = $\sigma_L.\pi Dt$

Equating:

$$\sigma_L \pi Dt = p\frac{\pi D^2}{4}$$

Therefore:

$$\sigma_L = \frac{pD}{4t}$$

**Example:**

A cylindrical pressure vessel 2.5m in diameter and 5m long is made from 15mm thick steel plate having a Young's modulus of 207 GN/$m^2$ and a Poisson's ratio of 0.28.

If strain gauges are fixed to the cylinder, aligned circumferentially and longitudinally, what strains would they record when the cylinder is subjected to an internal pressure of 3.2 Mpa?

Circumferential stress $\sigma_\theta$:

$$\sigma_\theta = \frac{pD}{2t}$$

$$\sigma_\theta = \frac{3.2\times10^6\times2.5}{2\times0.015} = 267\times10^6\,N/m^2$$

Longitudinal stress $\sigma_L$:

$$\sigma_L = \frac{pD}{4T}$$

$$\sigma_L = \frac{3.2\times10^6\times2.5}{4\times0.015} = 133\times10^6\,N/m^2$$

Circumferential strain $\varepsilon_\theta$:

$$\varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\sigma_L\right)$$

$$\varepsilon_\theta = \frac{1}{207\times10^9}\left(267 - 0.28\times133\right)\times10^6$$

$$\varepsilon_\theta = 1.108\times10^{-3}$$

(1 108 microstrain)

Longitudinal strain $\varepsilon_L$:

$$\varepsilon_L = \frac{1}{E}\left(\sigma_L - \nu\sigma_\theta\right)$$

$$\varepsilon_L = \frac{1}{207\times10^9}\left(133 - 0.28\times267\right)\times10^6$$

$$\varepsilon_L = 2.83\times10^{-4}$$

(283 microstrain)