Engineering Science - MECH 226 - Formula Sheet

Statics

Beams - Load, shear force and bending moment relationships:

$$w = -\frac{dV}{dx} \;;\quad V = \frac{dM}{dx}$$

Simple theory of bending: $\frac{M}{I} = \frac{E}{R} = \frac{-\sigma}{y}$

Bending moment - curvature relationship: $EI\,\frac{d^2v}{dx^2} = M$

Simple theory of torsion for circular a section: $\frac{\tau}{r} = \frac{T}{J} = \frac{G\phi}{l} \;;\; J = \frac{\pi d^4}{32}$

For a slender column with thin ends: $P_{cr} = \frac{\pi^2 EI}{L^2}$

Thin cylinders: $\sigma_\theta = \frac{pd}{2t}$; $\sigma_l = \frac{pd}{4t}$; **Thin sphere:** $\sigma_\theta = \frac{pd}{4t}$

Two dimensional stress and strain - relationships for isotropic elastic materials:

$$\gamma_{xy} = \frac{\tau_{xy}}{G} \;;\; \varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \;;\; \sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \;;\; E = 2G(1+\nu)$$

Two dimensional stress and strain transformations - stresses on plane at ϕ

$$\sigma_\phi = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}(\sigma_x - \sigma_y)\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau_\phi = -\frac{1}{2}(\sigma_x - \sigma_y)\sin 2\phi + \tau_{xy}\cos 2\phi$$

Principal planes: $\tan 2\phi = \dfrac{2\tau_{xy}}{\sigma_x - \sigma_y}$

Strains on plane at θ

$$\varepsilon_\theta = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\frac{1}{2}\gamma_\theta = -\frac{1}{2}(\varepsilon_x - \varepsilon_y)\sin 2\theta + \frac{1}{2}\gamma_{xy}\cos 2\theta$$

Dynamics - acceleration: $F = Ma$; $T = I\alpha$

Moments of inertia:

for a thin disc, polar axis: $I_p = \dfrac{Mr^2}{2}$; about diameter: $I_{dia} = \dfrac{Mr^2}{4}$

for a slender rod about its centre of mass: $I = \dfrac{ml^2}{12}$

Equations of motion - for constant acceleration: $v = u + at$;
$v^2 = u^2 + 2as$; $s = ut + \frac{1}{2}at^2$

Acceleration of a geared system - torque on A to accelerate A and B:

$T_A = (I_A + n^2 I_B)$ where $n = \dfrac{\omega_B}{\omega_A}$

Vibration

Free undamped vibration - general equation: $x'' + \frac{k}{m}x = 0$ the general solution is:

$$x = \frac{x'(0)}{\omega_n}\sin\omega_n t + x(0)\cos\omega_n t \text{ where } \omega_n = \sqrt{\frac{k}{m}}$$

Free vibration with damping - general equation: $x'' + \frac{c}{m}x' + \frac{k}{m}x = 0$

where m = mass, k = spring stiffness and c = viscous damping coefficient.

There are three solutions to this equation depending upon the magnitude of the damping. These are usually written in terms of the damping ratio ζ which is defined as:

$\zeta = \frac{c}{2\omega_n m}$ The three solutions are:

a) Under damped $\zeta < 1$: $x = e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$

where $\omega_d = \omega_n\sqrt{1-(\zeta)^2}$

b) Critically damped $\zeta = 1$: $x = (A + Bt)e^{-\zeta\omega_n t}$

c) Over damped $\zeta > 1$: $x = e^{-\zeta\omega_n t}(Ae^{\omega_n t\sqrt{\zeta^2-1}} + Be^{-\omega_n t\sqrt{\zeta^2-1}})$

Logarithmic decrement: $\delta = Ln\left(\frac{x_n}{x_{n+1}}\right) = \zeta\omega_n T = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

where T is the periodic time: $T = \frac{2\pi}{\omega_d}$