<u>The Strain Gauge Rosette</u>

In the practical context, the most important tool in experimental stress analysis is the electrical resistance strain gauge. This device measures accurately the surface strain in the direction in which it is applied.

In many cases the directions in which the strains are to be measured are known and no difficulty exists. However in other cases there may be no clearly defined direction and the investigator may wish to determine the greatest strain at the point.

To allow this to be accomplished strain-gauge rosettes have been developed. These multi-element gauges are arranged in a given geometry as shown in Figure 4.

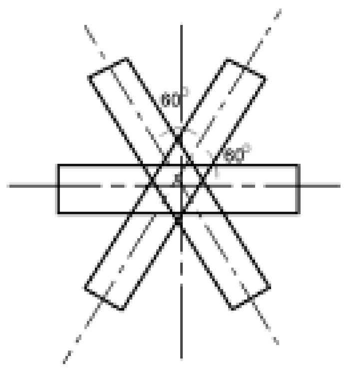

Figure 4.

From any three linear strain measurements it is possible to determine the complete strain system as follows:

Consider the case shown in Figure 5 where strains are measured in directions OA, OB and OC.

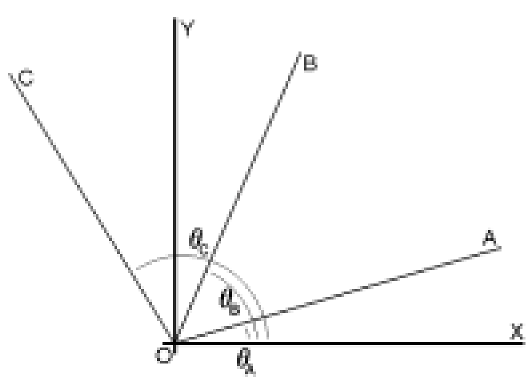

Figure 5.

The procedure is as follows:

Applying equation (1) to directions A, B and C:

$$\varepsilon_a = \frac{1}{2}\left(\varepsilon_x + \varepsilon_y\right) + \frac{1}{2}\left(\varepsilon_x - \varepsilon_y\right)\cos 2\theta_a + \frac{1}{2}\gamma_{xy}\sin 2\theta_a$$

$$\varepsilon_b = \frac{1}{2}\left(\varepsilon_x + \varepsilon_y\right) + \frac{1}{2}\left(\varepsilon_x - \varepsilon_y\right)\cos 2\theta_b + \frac{1}{2}\gamma_{xy}\sin 2\theta_b$$

$$\varepsilon_c = \frac{1}{2}\left(\varepsilon_x + \varepsilon_y\right) + \frac{1}{2}\left(\varepsilon_x - \varepsilon_y\right)\cos 2\theta_c + \frac{1}{2}\gamma_{xy}\sin 2\theta_c$$

Then, since ε_a, ε_b, ε_c, θ_a, θ_b and θ_c are known, we have three simultaneous equations in ε_x, ε_y and γ_{xy} which we solve.

We then find θ_1 and θ_2 from:

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

And substitute back into equation (1) for ε_1 and ε_2.

Example 1

The following results are obtained from a 60^0 strain gauge rosette:

Strain in direction of strain gauge A = 750 microstrain;
Strain in direction of SG B, 60^0 to A = 350 microstrain;
Strain in direction of SG C, 120^0 to A = 100 microstrain.

Determine the principal strains and their directions.

From equation (1):

$$\varepsilon_a = \frac{1}{2}\left(\varepsilon_x + \varepsilon_y\right) + \frac{1}{2}\left(\varepsilon_x - \varepsilon_y\right)\cos 2\theta_a + \frac{1}{2}\gamma_{xy}\sin 2\theta_a$$

Hence:

$$\varepsilon_a = 750 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta_a + 0$$

Therefore:

$$\varepsilon_x = 750\mu\varepsilon \qquad \text{(i)}$$

And:

$$\varepsilon_b = 350 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 120 + \frac{1}{2}\gamma_{xy}\sin 120$$

$$350 = 0.25\varepsilon_x + 0.75\varepsilon_y + 0.433\gamma_{xy} \qquad \text{(ii)}$$

$$\varepsilon_c = 100 = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 240 + \frac{1}{2}\gamma_{xy}\sin 240$$

$$100 = 0.25\varepsilon_x + 0.75\varepsilon_y - 0.433\gamma_{xy} \qquad \text{(iii)}$$

Then (ii) – (iii) gives:

$$250 = 0.866\gamma_{xy}$$

Hence:

$$\gamma_{xy} = 288.7\mu\varepsilon$$

And ε_x and γ_{xy} in (ii) gives:

$$\varepsilon_y = \frac{350 - 0.25 \times 750 - 125}{0.75} = 50\mu\varepsilon$$

Then if $\varepsilon_x = 750$, $\varepsilon_y = 50$ and $\gamma_{xy} = 289$:

$$\tan 2\theta = \frac{289}{750-50}$$

Therefore:

$2\theta = 22.4^o$ and $180^o + 22.4^o$

$\theta = 11.2^o$ and 101.2^o

And from equation (1):

$$\varepsilon_{11.2} = \frac{1}{2}(750+50) + \frac{1}{2}(750-50)\cos 22.4 + \frac{1}{2} 289 \sin 22.4$$

$$\varepsilon_{11.2} = 779\mu\varepsilon$$

(ε_1)

$$\varepsilon_{101.2} = \frac{1}{2}(750+50) + \frac{1}{2}(750-50)\cos 202.4 + \frac{1}{2} 289 \sin 202.4$$

$$\varepsilon_{101.2} = 21\mu\varepsilon$$

(ε_2)