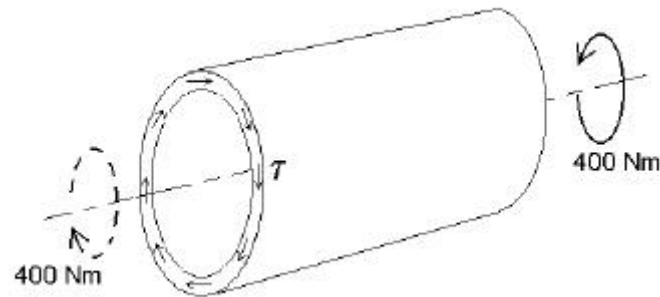


### Example 2

Consider the torsion of a thin cylinder 50 mm diameter by 1 mm thick under the action of a twisting moment of 400 Nm. Show that a shear stress of 102 MPa is set up and show this on a biaxial element.

Hence demonstrate that this 'pure shear' is equivalent to a system of tensile and compressive stresses of magnitude equal to  $t$  and orientated at  $45^\circ$  to the cylinder axis.

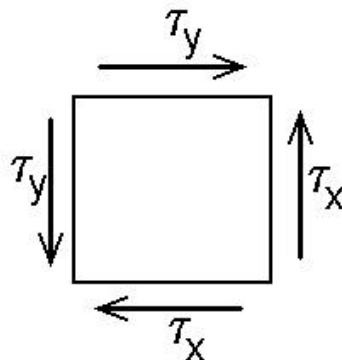


For rotational equilibrium:

$$\begin{aligned} 400 &= \text{stress} \times \text{area} \times \text{radius} \\ &= \tau_x \times 2\pi r t \times r \end{aligned}$$

Hence:

$$\tau_x = \frac{400}{2\pi \times 0.025^2 \times 0.001} = 102 \times 10^6 \text{ N/m}^2$$



Also  $\tau_y = \tau_x$  (complementary shear stress)

Hence,  $\tau_y = \tau_x = \tau = 102 \text{ MPa}$

Then, from equations (1) and (2):-

$$\sigma_{\phi} = \tau \sin 2\phi$$

$$\tau_{\phi} = \tau \cos 2\phi$$

When  $\phi = 45^{\circ}$  and  $135^{\circ}$ ,

$$\sigma_{45} = \tau \text{ and } \sigma_{135} = -\tau$$

$$\tau_{45} = 0 \text{ and } \tau_{135} = 0$$

