SECTION 3

RELIABILITY

3.1 INTRODUCTION

When manufacturers claim that their products are *very reliable* they essentially mean that the products can function as required for a long period of time, when used as specified. In order to assess and improve the reliability of an item we need to be able to measure it. Thus a more formal definition is required.

When an item stops functioning satisfactorily it is said to have *failed*. The *time-to-failure* or *life-time* of an item is intimately linked to its *reliability* and this is a characteristic that will vary from item to item even if they are supposedly identical. For example, say we have 100 identical brand new light bulbs that we plug into a test circuit, turn on simultaneously and observe how long they last for. We would not, of course, expect them all to blow out at the same time. Their *times to failure* will differ. Furthermore, there is some random element to their failure times so their lifetime is a random variable whose behaviour can be modelled by a probability distribution. This is the basis of **reliability theory**.

Reliability analysis enables us to answer such questions as:

- what is the probability that a unit will fail before a given time?
- what percentage of items will last longer than a certain time?
- what is the expected lifetime of a component?

First we must define some fundamental concepts.

3.2 FUNDAMENTAL CONCEPTS ASSOCIATED WITH RELIABILITY

3.2.1 Some Important Functions

In the analysis of reliability data, it is the *lifetime* or *time to failure* of a unit that is of interest. This characteristic, lifetime, is a random variable. As a result, it will have a probability density function (pdf) associated with it.

There are two approaches to determining the pdf associated with a set of time-to-failure data. The first is to plot the lifetimes and try to fit one of the standard pdf's (exponential, normal, Weibull, lognormal) described in Section 2. If the data do not appear to follow a standard pdf, a so-called non-parametric approach can be taken, whereby the pdf is determined directly from the data or the form of f(x) is deduced from the shape of the histogram.

In each case, once the pdf is found, the cumulative distribution function or cdf, F(t), can be found. Recall that

$$F(t) = P(T \leq t)$$

where F(t) denotes the probability that the unit fails before some time t. This is related to the pdf, f(t), by

$$\boxed{F(t) = \int_0^t f(t)dt}$$

or

$$\boxed{f(t) = \frac{dF(t)}{dt}}$$

Once the pdf and cdf are known, there are three other characteristics that can be determined from them, which are commonly used when dealing with reliability data: (i) the reliability function, (ii) hazard function and (iii) 'mean time between failures'.

(i) <u>Reliability Function (or Survival Function)</u>

The cdf gives the probability that a unit will fail before a certain time t. Of greater interest in reliability work is the probability that a unit survives to time t. This is known as the reliability function, R(t) or the survivor function. Then

$$R(t) = P(T > t)$$

$$= 1 - P(T < t)$$

i.e $\boxed{R(t) = 1 - F(t)}$

The reliability function is always a decreasing function of time.

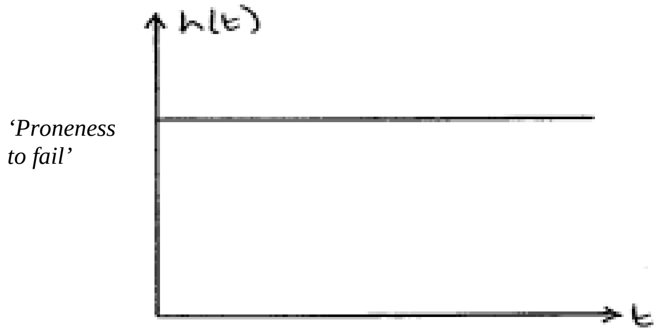
(ii) Hazard Function

Another quantity that we are interested in is the rate of failure of units; this is estimated by the so-called hazard function, h(t), where h(t) is defined by

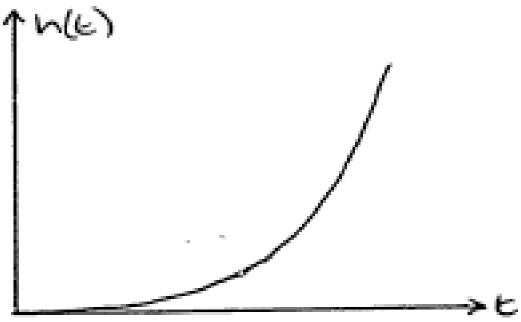
$$h(t) = \frac{f(t)}{R(t)}$$

h(t) is the ***hazard*** or ***instantaneous failure rate function***. It indicates the 'proneness to failure' or 'risk' of a unit after time t has elapsed. However, it is not a probability (eg. it can take values greater than 1). (If we want to work in probabilities we must use the survivor function, whose relationship to h(t) will be derived below.)

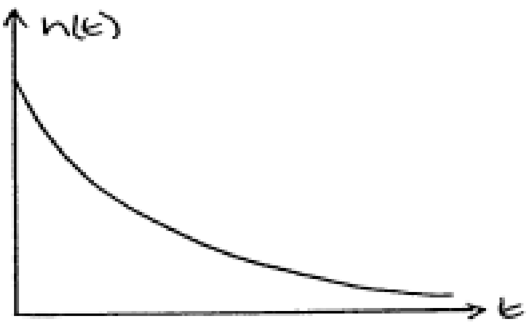
From the graph of h(t) we obtain a useful picture of the behaviour of a unit. For example



constant failure rate ⇒ Probability of breakdown is independent of age of unit. This is, unit is equally likely to fail at any moment during its lifetime, regardless of how old it is. (eg. windscreen)

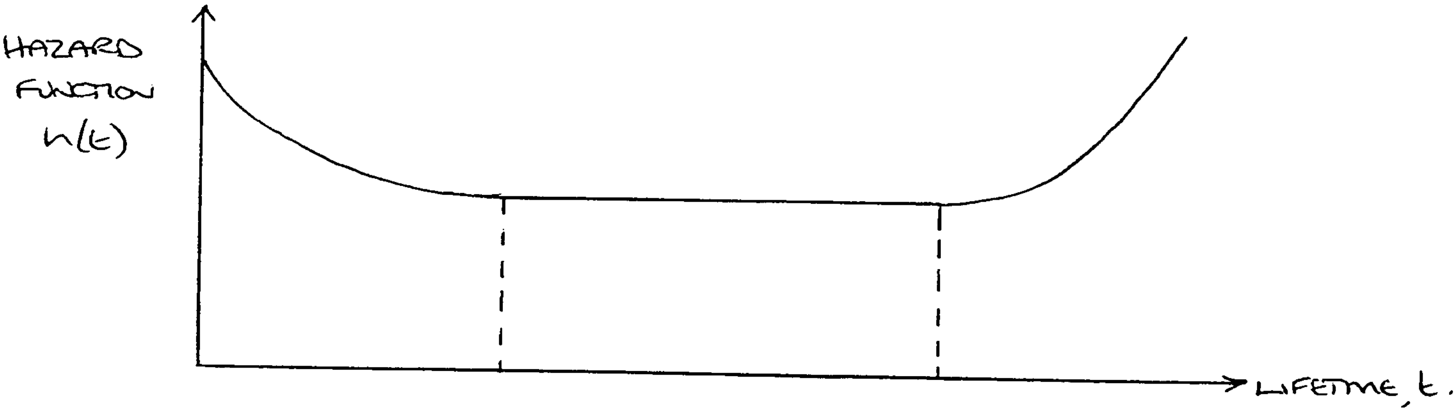


increasing failure rate ⇒ unit becomes more likely to fail as it gets older, so it is ageing. (eg. any mechanical item)



decreasing failure rate ⇒ unit gets less likely to fail as it gets older.
(eg. new vehicle, as it passes through 'running in' period)

In practice, the hazard function of a unit or system is often assumed to change in the following manner:



<u>Decreasing failure rate</u>;
often due to so-called "infant mortality" or "burn in"
(Weibull)

<u>Constant Failure rate</u> for majority of lifetime
(exponential)

<u>Increasing failure rate</u>;
often due to items "wearing out" or "ageing"
(Weibull)

- the so-called **<u>BATHTUB</u>** curve. For such a curve, the different periods of the item's life are modelled by different standard distributions as shown in brackets. This will be discussed in more detail later.

It should be noted that, in reality, an item’s hazard function (instantaneous failure rate) is unlikely to remain <u>constant</u> throughout the majority of its lifetime. Most items age gradually throughout their entire life rather than suddenly starting to deteriorate towards the end, as implied by the Bathtub curve. This will be discussed in more detail in Section 3.4.4.

The ***cumulative hazard function*** is defined from the hazard function as

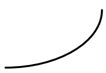
$$H(t) = \int_0^t h(t)dt$$

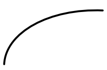
and it can be shown that

$$R(t) = e^{-H(t)}$$

or

$$H(t) = -\ln R(t)$$


An upwardly-curving cumulative hazard function (chf) then indicates an increasing failure rate as the component ages,


whilst a downwardly-curving chf indicates decreasing failure rate.

Note that f(t), F(t), R(t), h(t) and H(t) give mathematically equivalent descriptions of T in the sense that, given any one of these functions, the other four functions may be deduced.

(iii) <u>Mean Time Between Failures</u>

The ***Mean Time Between Failures (MTBF)*** is a concept which is frequently used in reliability work. It is defined to be the 'average' or 'expected' lifetime of an item. Alternative notation is μ or E(T).

Then, from the definition of a mean or expected value,

$$\text{MTBF} = \int_0^\infty t f(t) dt$$

Alternatively

$$\text{MTBF} = \int_0^\infty [1 - F(t)] dt$$

We most commonly express the mean time between failures in terms of the reliability function, namely

$$\text{MTBF} = \int_0^\infty R(t) dt$$

3.2.2 <u>Summary of Reliability Formulae</u>

- $F(t) = \int_0^t f(t)\, dt$
- $R(t) = 1 - F(t)$
- $f(t) = \dfrac{dF(t)}{dt} = -\dfrac{dR(t)}{dt}$
- $h(t) = \dfrac{f(t)}{R(t)}$
- $H(t) = \int_0^t h(t) dt$
- $R(t) = e^{-H(t)}$
- $H(t) = -\ln R(t)$
- $\text{MTBF} = \int_0^\infty t\, f(t) dt = \int_0^\infty R(t)\, dt$

3.3 LIFETIME FOLLOWING AN EXPONENTIAL DISTRIBUTION

We have previously seen (Section 2.7) that the exponential distribution is a useful model of the time (or length) between failures in situations where the failures are happening

- at random

and

- at a known rate, λ.

The p.d.f. is then given by

$$f(t) = \lambda e^{-\lambda t} \qquad \text{for } t>0$$

3.3.1 Associated Functions

(i) Then the *cdf* is

$$\begin{aligned} F(t) &= P(T \le t)) \\ &= \int_0^t f(t)dt \\ &= \int_0^t \lambda e^{-\lambda t}.dt \\ &= \lambda\left[\frac{e^{-\lambda t}}{-\lambda}\right]_0^t \\ &= \frac{\lambda}{-\lambda}\left[e^{-\lambda t} - e^0\right] \\ &= -1\left[e^{-\lambda t} - 1\right] \end{aligned}$$

i.e. $F(t) = 1 - e^{-\lambda t}$.

(i) Survival/Reliability Function

Then since

$$\begin{aligned} R(t) &= 1 - F(t) \\ &= 1 - [1 - e^{-\lambda t}] \end{aligned}$$

i.e.

$$R(t) = e^{-\lambda t}$$

if the item's lifetime follows an exponential distribution.

(iii) Also, the *hazard function* is given by

$$h(t) = \frac{f(t)}{R(t)}$$

So in this case

$$h(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}}$$

That is, in the case of an item whose lifetime is found to follow an exponential distribution:

$$h(t) = \lambda \quad \text{(i.e. a constant)}$$

(iv) The *mean time between failures* (MTBF) is

$$\text{MTBF} = \int_0^\infty R(t)dt$$

$$= \int_0^\infty e^{-\lambda t}dt$$

$$= \left[\frac{e^{-\lambda t}}{-\lambda}\right]_0^\infty$$

$$= \frac{e^{-\infty} - e^{-0}}{-\lambda}$$

$$= \frac{1}{\lambda}.$$

(i) Finally, the *cumulative hazard function* is

$H(t) = \int_0^t h(t)dt = \int_0^t \lambda dt = \lambda t$ (i.e. a straight line)

{Alternatively obtained from $H(t) = -\ln R(t) = -\ln(e^{-\lambda t}) = \lambda t$ }

Examples *(Assuming constant failure rates.)*

(i) A system has a failure rate of 2×10^{-3} failures/hours. What is the mean time between failures?

(ii) A component has a failure rate of 5 failures/10^6 hours. What is the probability that the component will still be working after 10,000 hours of operation?

(iii) Assume that a guided missile has a true MTBF of 2 hours and constant failure rate. What is the probability of the missile failing to complete a two-hour mission?

Solution *(Using time units of 1 hour)*

(i) $\lambda = 0.002\ failures/hour$

Then the $MTBF = \frac{1}{\lambda} = \frac{1}{0.002}$ = **500 hours**

(ii) $\lambda = 5 \times 10^{-6}$ *per hour*

We know that $R(t) = e^{-\lambda t}$

Then $R(10000) = e^{-(5\times10^{-6})(10000)}$

$= e^{-0.05}$

= **0.951**

(iii) $MTBF = 2hours \rightarrow \lambda = 0.5$ *per hour.*

$P(X < 2) = F(2)$ where $F(t) = 1 - e^{-\lambda t}$

Then $F(2) = 1 - e^{-\left(\frac{1}{2}\right)\times 2}$

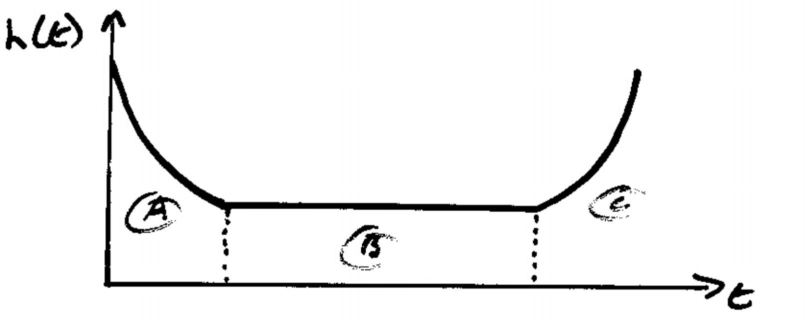
= $1 - e^{-1}$

= 1-0.368

= **0.632**

3.3.2 <u>Constant Hazard Function</u>

Refer back to the so-called "Bathtub curve" for the hazard function of a system or item.



It can be seen that within region (B), ***the hazard function is constant***. That is, within region (B) **the lifetime of the system can be modelled by an exponential distribution**.

To handle other situations where the hazard function is changing with time, such as (A) and (C) above, we need a more general lifetime distribution than the exponential. The following distribution – the **<u>WEIBULL</u>** distribution – is very flexible, and has been found to fit many real sets of lifetime data. It is therefore commonly used in reliability analysis.

3.3.2 <u>Summary</u>

For an item or system whose lifetime follows on <u>exponential</u> distribution

- $f(t) = \lambda e^{-\lambda t}$
- $F(t) = 1 - e^{-\lambda t}$
- $R(t) = e^{-\lambda t}$
- $h(t) = \lambda$
- $H(t) = \lambda t$
- $MTBF = \frac{1}{\lambda}$

3.4 LIFETIME FOLLOWING A WEIBULL DISTRIBUTION

If lifetimes have a Weibull distribution then the p.d.f. is:

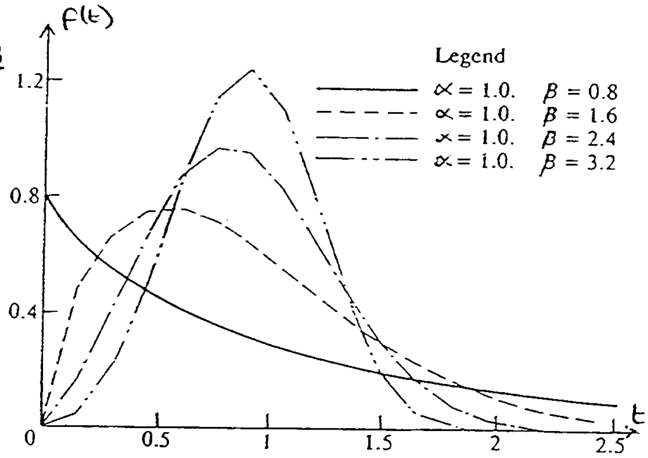
$$f(t) = \beta\alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right]$$

The Weibull pdf contains two parameters, α and β.

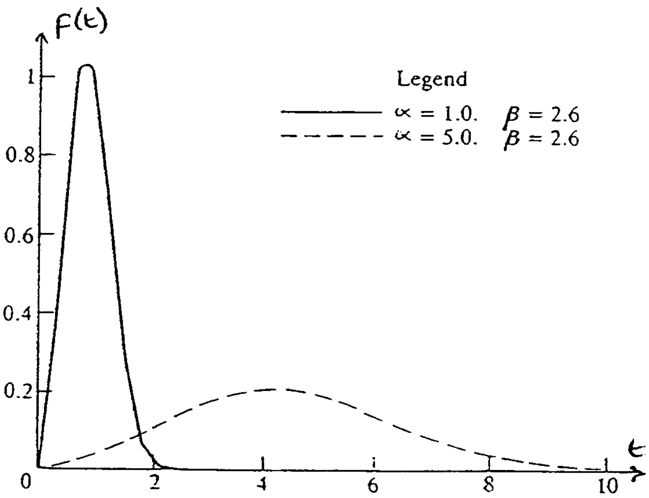
> ***The <u>scale parameter</u>, α, reflects the size of the units in which the random variable, t, is measured.***
>
> ***The <u>shape parameter</u>, β, causes the shape of the distribution to vary. By changing the value of β we can generate a widely varying set of curves to model real lifetime failure distributions.***

The effects of different scale and shape parameters on the Weibull distribution are:

- ***<u>Effect of different shape parameters, β</u>***



- ***<u>Effect of different scale parameters, α</u>***



Special case:

β = 1 results in the pdf collapsing to $f(t) = \alpha' e^{-\alpha' t}$ where $\alpha' = \dfrac{1}{\alpha}$ which is an exponential distribution with rate α' (or mean $1/\alpha'$).

3.4.1 Associated functions

If $$f(t) = \beta\alpha^{-\beta} t^{\beta-1} \exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right].$$

then it can be (fairly easily!) shown that the (i) cumulative distribution function is

$$F(t) = \int_0^t f(t)dt$$

is given by: $$\boxed{F(t) = 1 - e^{-\left(t/\alpha\right)^{\beta}}}$$

Then R(t) = 1 – F(t) gives the (ii) reliability function

$$\boxed{R(t) = e^{-\left(t/\alpha\right)^{\beta}}}$$

and the (iii) hazard function $h(t) = \dfrac{f(t)}{R(t)}$ is

$$h(t) = \frac{\beta\alpha^{-\beta}\left(t^{\beta-1} e^{-\left(t/\alpha\right)^{\beta}}\right)}{e^{-\left(t/\alpha\right)^{\beta}}}$$

i.e. $$\boxed{h(t) = \beta\alpha^{-\beta} t^{\beta-1}}$$

It can also be shown that the (iv) MTBF is

$$\boxed{MTBF = \alpha\Gamma\left(1 + 1/\beta\right)}$$

where Γ denotes the gamma function. Tables of gamma functions are available to assist in calculating the mean time between failures – see table at back of notes.
NB If x > 2, $\Gamma(x) = (x-1)\Gamma(x-1)$

Finally, the (v) cumulative hazard function is

$$H(t) = -\ln R(t) = -\ln\left[e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\right] = \left(\frac{t}{\alpha}\right)^{\beta}$$

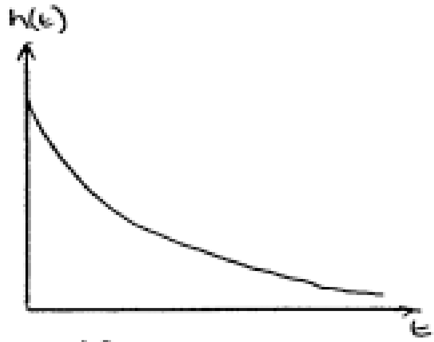
3.4.2 Time-dependant hazard function

For a Weibull lifetime distribution, the **hazard function** is given by

$$h(t) = \beta\alpha^{-\beta}t^{\beta-1}$$

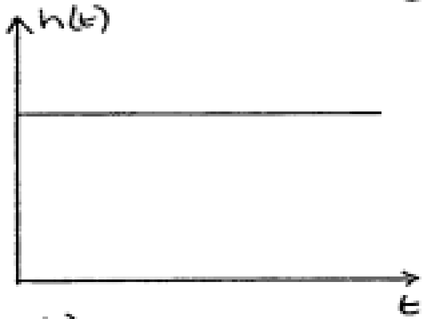
This will give various forms for the hazard function depending on the value of β, as follows:

(i) $\beta < 1$



i.e. ***'The older it is, the less likely it will fail.'***
This is called **NEGATIVE AGEING**.

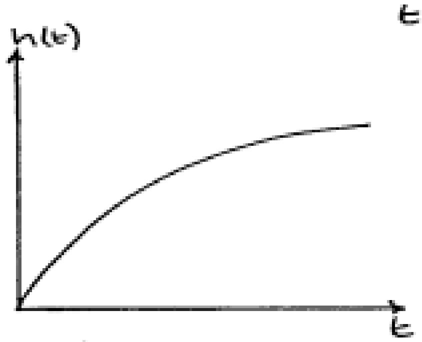
(ii) $\beta = 1$



$$\lambda(t) = \frac{1}{\alpha}$$

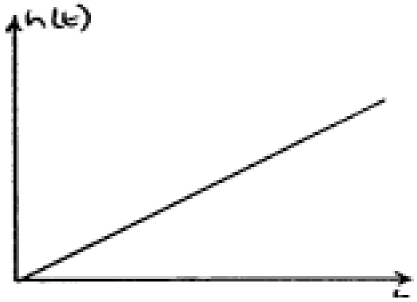
i.e ***age has no effect on how likely item is to fail.***
We say that the system has **'NO MEMORY'**.

(iii) $1 < \beta < 2$



i.e. ***'The older it is, the more likely it is to fail'.***
This is called **'POSITIVE AGEING'**.

(iv) $\beta = 2$



$\lambda(t) = 2\alpha t$
i.e. ***'the older it is, the more likely it is to fail'***

(Special case: Lifetime then has a Rayleigh distribution)

(v) $\beta > 2$

i.e. '*The older it is, the more likely it is to fail*'.

(vi) $\beta \doteq 3.4$

(Special case: Lifetime then has a normal distribution)

Example

The lifetime of a component (in thousands of hours) is found to have a Weibull distribution with α = 0.5 and β = 2. Find the probability that the component will fail before 2000 hours of operation.

Here $R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = e^{-\left(\frac{t}{0.5}\right)^{2}} = e^{-4t^2}$

Then $R(2) = e^{-4\times 4} = e^{-16}$

$= 1.13\times 10^{-7}$

Required probability $= 1 - 1.13\times 10^{-7}$

≈ 1 (i.e. almost certainly.)

3.4.3 Summary

The standard reliability results for the Weibull distribution are then:

- $F(t) = 1 - e^{-\left(t/\alpha\right)^{\beta}}$
- $R(t) = e^{-\left(t/\alpha\right)^{\beta}}$
- $h(t) = \beta\alpha^{-\beta}t^{\beta-1}$
- $H(t) = \left(\frac{t}{\alpha}\right)^{\beta}$
- $\text{MTBF} = \alpha\,\Gamma\left(1 + 1/\beta\right)$

where Γ denotes the gamma function. Tables of gamma functions are available to assist in calculating the mean time between failures.

Note that when $\beta = 1$, the Weibull distribution becomes the exponential distribution, with $\lambda = 1/\alpha$.

<u>Example</u>

Find the MTBF for the component on the previous example.

The lifetimes (in thousands of hours) have a Weibull distribution with $\alpha = 0.5$ and $\beta = 2$.

Then $MTBF = \alpha\Gamma\left(1+\frac{1}{\beta}\right) = 0.5\Gamma\left(1+\frac{1}{2}\right)$

$$= \frac{1}{2}\times\Gamma\left(\frac{3}{2}\right) = \frac{1}{2}\times 0.886227$$ (from tables of the gamma function.)

= 0.443 (thousands of hours)

i.e. ***443 hours.***

3.4.4 Bathtub Curve

For the "Bathtub Curve" shown earlier, we note that the different periods of the item's lifetime can be modelled by the following distributions

Section (A) (initial 'burn in'):
– Weibull distribution, $\beta < 1$

Section (B) (Constant failure rate):
– Exponential distribution

Section (C) ('wear out'):
– Weibull distribution, $\beta > 2$.

In practice, lifetime distributions other than the Bathtub curve may be obtained, in which case other combinations of distributions can be used to model the item's lifetime at the different periods throughout its life, as appropriate.

3.5 LIFETIME FOLLOWING NORMAL OR LOGNORMAL DISTRIBUTIONS

Although the normal distribution is the most commonly used distribution in statistics, it is rarely used as a lifetime distribution. The lognormal distribution is used more frequently, in which case *log(lifetime)* is taken to have a normal distribution. The associated functions of these distributions are more complex than those for the exponential and Weibull distributions although Minitab for example can deal with them just as easily.

3.6 USING THE FUNCTIONS: AN APPLICATION IN MINITAB

Having fitted a standard probability distribution to a set of reliability data, the next step is to use the associated reliability characteristics (hazard function, reliability function, ….) together with the 'best fit' parameters for the fitted distribution, to make predictions, and answer questions of interest related to the data.

This process will be demonstrated by means of an example.

Example

A relay is critical in the operation of a piece of communications equipment. Sixteen relays are life-tested and the following actuations to failure recorded:

Failure i	Actuations t_i
1	3.8
2	6.6
3	8.2
4	9.5
5	11.0
6	11.9
7	14.7
8	17.1
9	19.2
10	21.9
11	23.5
12	24.5
13	27.9
14	29.9
15	33.0
16	37.2

(Actuations are in 100,000 (i.e. multiply by 10^5))

Do the data suggest that the lifetime of this relay follow a Weibull distribution?

If so:

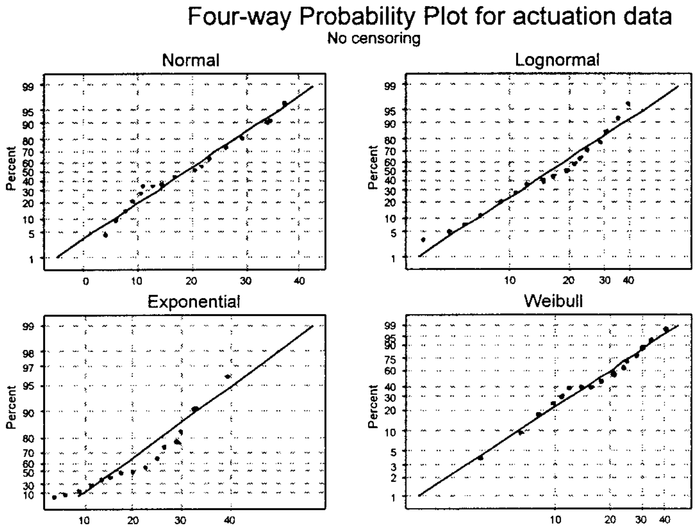
(i) Estimate the Weibull parameters α and β.

(ii) Estimate the expected lifetime of this relay.

(iii) Find the lifetime distribution and failure rate. What do these plots suggest?

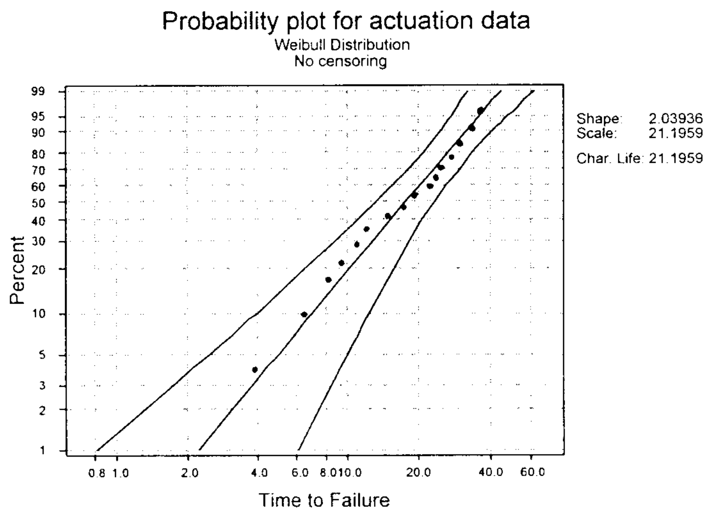
(iv) Write down the reliability function and calculate the probability that one of these relays will still be operating after a million actuations.

Solution



Points on the above ID-plot are approximately linear for the Weibull distribution indicating that it is reasonable to assume that the lifetime of this relay does follow a Weibull distribution.

(i) From STAT>REL/SURV>PROB PLOT:



estimate of the shape parameter β is 2.03936

estimate of the scale parameter α is 21.19591×10^5 *(note: 10^5 are the units of the data.)*

and fitted Weibull distribution has pdf

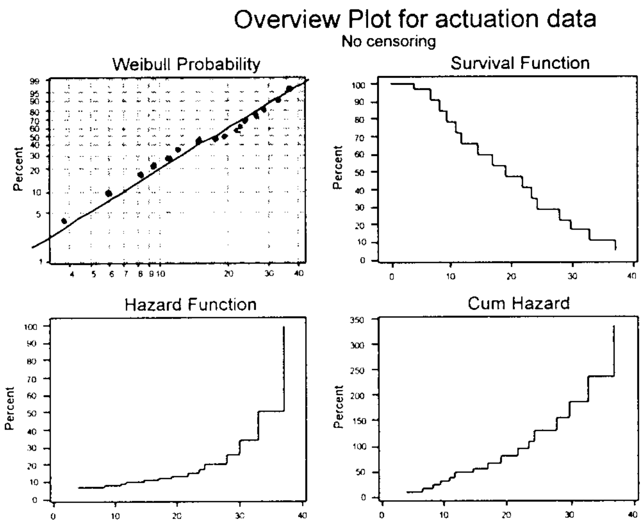
$$f(t) = (2.04)(21.19)^{-2.04} t^{1.04} \exp\left[-\left(\frac{t}{21.19}\right)^{2.04}\right]$$

i.e. $$f(t) = (0.004)t^{1.04} \exp\left[-\left(\frac{t}{21.19}\right)^{2.04}\right]$$

(ii) Then, for a Weibull distribution

$$\begin{aligned} MTBF &= \alpha\Gamma\left(1+\frac{1}{\beta}\right) \\ &= (21.19).\Gamma\left(1+\frac{1}{2.04}\right) \\ &= 21.19.\Gamma(1.49) \\ &= 21.19(0.886) = 18.77x10^5\, actuations \end{aligned}$$

(ii) STAT>REL>overview plot



The hazard function increases more and more steeply with time, so the relay is ageing with time. This is ***positive ageing***.

Note that the shape of the hazard function h(t) above suggests β > 2 (by comparison with plots on pages 120 and 121). Compare this with the estimate of β of 2.04.)

(v) For a Weibull distribution:

$$R(t) = e^{-(t/\alpha)^{\beta}}$$

In this case

$$R(t) = \exp\left[-\left(\frac{t}{21.19}\right)^{2.04}\right]$$

Note that 1 million actuations = 10×10^5 i.e. 10 'time units', so we require the reliability at t = 10:

$$R(10) = \exp\left[-\left(\frac{10}{21.19}\right)^{2.04}\right] = 0.806$$

i.e. There is an **81%** chance that the relay will still be operating after 1 million actuations, based on the sample data given.

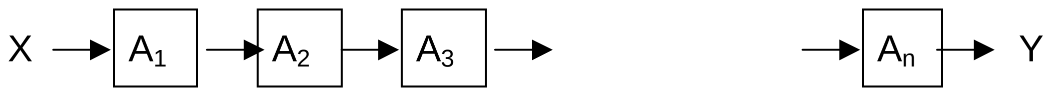
3.7 SYSTEM RELIABILITY

Suppose that we have to calculate the Reliability of a system made up of several components. The total reliability can be calculated by calculating the reliability of each individual component, and combining these individual reliabilities. The way in which they are combined depends on the way in which the components are connected. That is, whether they are connected:

- in series
- in parallel
- combination of series and parallel.

3.7.1 Series System

Consider a system of n components connected in series so that the system will only work (i.e. a signal will pass from X to Y) if *all of the components work.*

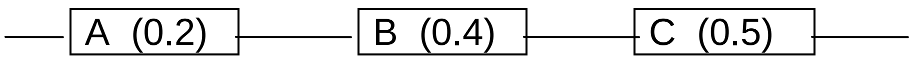


If the components fail independent of each other it is easy to show that if $R_1, R_2, \dots, R_n$ are the reliabilities of the individual components, then the reliability of the *system* is given by:

$$R_{system} = R_1 \times R_2 \times \dots \times R_n$$

Example

Consider a system of 3 components connected in series, each component having a constant failure rate. (In other words, the components have *exponential lifetimes*.) These rates for components A, B and C are 0.2, 0.4 and 0.5 per 10,000 hours respectively. Thus we have



For constant failure rate λ, reliability

$$R(t) = e^{-\lambda t}.$$

Thus for component A,

$$R_A = e^{-0.2t}$$

Similarly, $R_B = e^{-0.4t}$ and $R_C = e^{-0.5t}$.

Hence the reliability of the system is:

$$R(t) = e^{-0.2t} \times e^{-0.4t} \times e^{-0.5t}$$

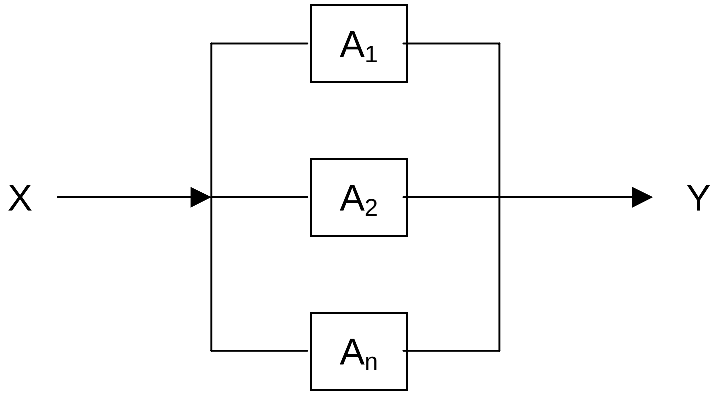
$$= e^{-0.2t - 0.4t - 0.5t}$$

$$= e^{-1.1t}$$

Then, for example, the probability that the system is still working after 20,000 hours = R(2) = $e^{-2.2}$ = <u>0.1108</u>.

3.7.2 **<u>Parallel System</u>**

If n components are connected in parallel



so that the system works (signal from X to Y) as long *as at least one of the components works*. The reliability of the system (again assuming independent failures) is then:

$$R_{system} = 1 - P(\text{all fail}) = 1 - [P(A_1 \text{ fails}) \times P(A_2 \text{ fails}) \times \ldots \times P(A_n \text{ fails})]$$

$$= 1 - (1 - R_1)(1 - R_2) \ldots (1 - R_n)$$

$$R_{system} = 1 - \prod_{i=1}^{n} (1 - R_i)$$

<u>Example</u>

Component A with a constant failure rate of 1.5 per 1000 hrs and component B with a constant failure rate of 2 per 1000 hrs are connected in parallel. Find the overall reliability of this system. (Note that components A and B have exponential lifetimes.)

<u>Solution</u>

For the individual components, assuming a constant failure rate

$$R_A(t) = e^{-1.5t}$$

$$R_B(t) = e^{-2t}$$

Then if these components are connected in parallel,

$$R_{system}(t) = 1 – [(1 – R_A(t)) \times (1 – R_B(t))]$$

$$= 1 – [(1 – e^{–1.5t}) (1 – e^{–2t})]$$

$$= 1 – [1 – e^{–1.5t} – e^{–2t} + e^{–1.5t} e^{–2t}]$$

$$= e^{–1.5t} + e^{–2t} – e^{–1.5t-2t}$$

$$= e^{–1.5t} + e^{–2t} – e^{–3.5t}$$

Then, for example, the probability that the system is still working after 1000 hours is

$$R(1) = e^{–1.5(1)} + e^{–2(1)} – e^{–3.5(1)}$$

$$= 0.3283$$

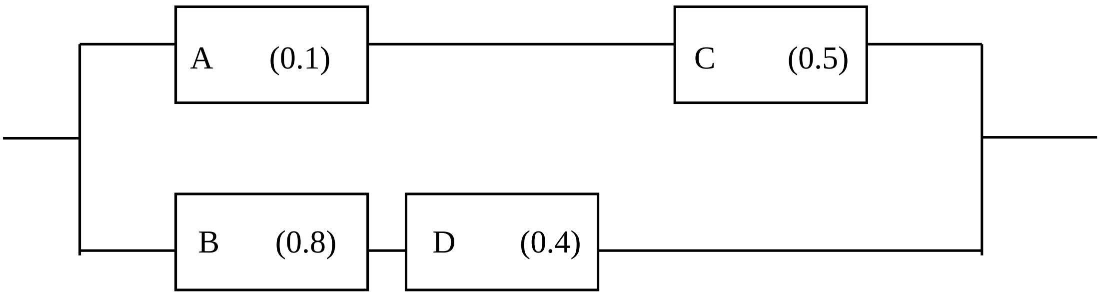
ie. there is a **33%** chance that such a system will still be working after 1000 hours.

3.7.3 Mixed System

To evaluate reliability of a system comprising both parallel and series sections, divide the system into 'series only' and 'parallel only' subsystems. Find the reliability of each subsystem as above then combine in a suitable manner.

Example

Find the reliability of the system shown, where the number in brackets indicate the constant failure rates per year for each of the components.



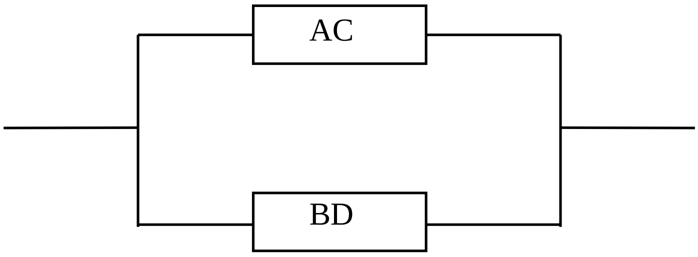
Solution

Consider the two branches (AC) and (BD) first. These are both series 'subsystems'. Then

Reliability of top branch = $R_{top}(t) = R_A(t) \times R_C(t) = e^{–0.1t} \times e^{–0.5t} = e^{–0.6t}$

Reliability of bottom branch = $R_{bottom}(t) = R_B(t) \times R_D(t) = e^{–0.8t} \times e^{–0.4t} = e^{–1.2t}$

We can then consider these two subsystems (AC) and (BD) as part of a larger system, namely



and for this parallel system

$$R_{system}(t) = 1 – [(1 – R_{AC}(t)) \times (1 – R_{BD}(t))]$$

$$= 1 – [(1 – e^{–0.6t}) \times (1 – e^{–1.2t})]$$

$$= 1 – [1 – e^{–0.6t} – e^{–1.2t} + e^{–0.6t} e^{–1.2t}]$$

$$= e^{–0.6t} + e^{–1.2t} – e^{–1.8t}$$

Then, if for example we wish to find the probability that the system will still be operational after 6 months, R(0.5), we have

$$R(0.5) = e^{–0.6(0.5)} + e^{–1.2(0.5)} – e^{–1.8(0.5)}$$

$$= 0.741 + 0.549 – 0.407$$

$$= 0.883$$

(i.e. there is an 88% chance that the mixed system under consideration will still be operational after 6 months.)

Example (continued)

What is the expected lifetime of this system?

Solution (continued)

MTBF or E(T) = $\int_0^\infty R(t)dt = \int_0^\infty (e^{-0.6t} + e^{-1.2t} - e^{-1.8t})dt$

$$= \left[\frac{e^{-0.6t}}{-0.6} + \frac{e^{-1.2t}}{-1.2} - \frac{e^{-1.8t}}{-1.8} \right]_0^\infty = (0) - \left(\frac{1}{-0.6} + \frac{1}{-1.2} - \frac{1}{-1.8} \right)$$

$$= \frac{1}{0.6} + \frac{1}{1.2} - \frac{1}{1.8} = 1.944 \text{ units}$$

Thus the expected lifetime is **1.94 years**.

NB *The above examples all have constant failure rates (or hazards) – i.e. exponential lifetimes. The same approach can be used in other situations (e.g. Weibull lifetimes.)*