TUTORIAL SHEET 3

Probability Rules and Plots

1. Explain briefly what is wrong with the following argument: "If the probability that a person's left eye is brown is 0.6 and the probability that his right eye is brown is 0.6, then the probability that both his eyes are brown is 0.6 * 0.6 = 0.36."

2. A company has two machines to produce a particular product. Machine A produces 45% of the product and Machine B produces 55%. The defective rate for Machine A is 8% and for Machine B it's 10%. Draw a tree diagram for this situation and evaluate all the joint probabilities. If a defective item is observed, what is the probability that it is from Machine A?

3. In a batch of 9 light bulbs 3 are defective. Three bulbs are drawn at random from the batch without replacement. Find the probability that
   (i) there are no defective bulbs;
   (ii) exactly one defective bulb;
   (iii) at least one defective bulb.

4. This is an example of a *diagnosis* problem. The scenario used here is medical but the same principle applies in any situation that involves imperfect testing.

   5% of a population are known to have some medical disorder. A testing procedure has been developed to screen for this disorder, but it is not infallible. If a person has this disorder, then the test will correctly give a positive result 95% of the time; if a person does *not* have the disorder, then the test will correctly give a negative result 90% of the time.

   A person is tested and has a positive result. What is the probability that the person has the disorder?

   A person is tested and has a negative result. What is the probability that the person does not have the disorder?

   Comment on your answers.

5. The Minitab worksheet *six_lifetimes.mtw* contains the lifetimes (in hours) of 100 items of each of 6 different machined parts. These are denoted Part A to Part F and are stored in columns 1-6.
   Use *Stat→Reliability/Survival→Distribution ID Plot* (and any other plots you think might be useful e.g. histograms) to identify an appropriate probability distribution to model the lifetimes of each part separately. Then use *Graph→Probability Plots…*to obtain the specific plot for your chosen distribution.

6. Consider the lifetimes of Part A. Use the probability plot to
   (i) estimate the proportion of items that have lifetimes less than 6000 hours
   (ii) estimate the lifetime that is exceeded by only 10% of items.

7. Consider the lifetimes of Part B. Use the probability plot to
   (i) estimate the percentage of items which will still be working after 3000 hours of operation. (This is called the **reliability** of the item at 3000 hours.)
   (ii) find the median lifetime.

   Also find the mean lifetime and estimate the proportion of items which will fail before the mean time to failure. Compare the mean and the median.

8. Consider the lifetimes of Part C. Use the probability plot to estimate the probability that an item will have a lifetime between 1000 and 2000 hours.

9. Consider the lifetimes of Part E. Use the probability plot to estimate the reliability of the item at 9000 hours. Find also the proportion of items in the sample of 100 items which lasted for at least 9000 hours and compare this with the previous estimate of the population reliability.