TUTORIAL SHEET 6

Using Calculus

1. Gap width is an important property of a magnetic recording head. In coded units, if the width has probability distribution function (pdf):

$$f(x) = kx \qquad 0 < x < 2$$

   (i) Find the value of k and sketch the pdf.
   (ii) Find the mean gap width, E(X).
   (iii) Determine the cumulative distribution function and hence find the median gap width.
   (iv) Relate the relationship between the mean and the median to the shape of the distribution.

2. Find the mean size of contamination particles if their size (in microns) can be modelled by the pdf:

$$f(x) = \frac{2}{x^3} \qquad x > 1$$

   A filter, which can trap particles of at least 1.1 microns in size, is used to reduce the amount of contamination. Find the percentage of particles which will pass through the filter.

3. Scrap tubing is left over from fabricating a compressor. Each piece has a length (in inches) represented by the probability density function

$$f(x) = \begin{cases} x & 0 \le x \le 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & \textit{otherwise.} \end{cases}$$

   Determine the following:
   (i) $p(X \ge 0.5)$;
   (ii) $p(X < 1.5)$;
   (iii) $p(0.5 < X \le 1.5)$;
   (iv) $F(x)$, the cumulative distribution function;
   (v) The mean, $E(X)$, by integration and by deduction from a sketch of the pdf.

4. The cumulative distribution function of wave heights (in metres) at a coastal site is $F(x) = 1 - \exp(-x^2/16)$.

   (i) What is the probability of getting waves higher than 4 metres?
   (ii) If a sea wall is to be built to protect from flooding, how high should the wall be so that the probability of getting waves higher than the wall is no more than 2%.