

# TUTORIAL SHEET 9

## System Reliability

1. Consider a system of 5 components connected in series, each component having a constant failure rate of 0.1 per 1000 hours.
  - (i) Find an expression for the reliability of the system and use it to find the probability that the system is still working after 5000 hours.
  - (ii) How many of these components can be connected in series if there is to be at least a 25% chance that the system is still working after 5000 hours?
  - (iii) What is the expected lifetime of the original system with 5 components connected in series?
  
2. Consider a system of 4 components connected in parallel. These components have constant failure rates of 1, 3, 4 and 6 per year.
  - (i) Find the reliability function for the system and evaluate the probability that the system fails within one year.
  - (ii) The system is to be replaced by another system which consists of a number of identical components connected in parallel. These components all have a constant failure rate of 2 per year. How many would need to be connected in parallel so that the new system has at least as high a reliability at one year as the old one?  
  
How many would be needed if the new system is to have double the probability of still working after one year?
  
3. Part of the control system for a hydroelectric dam consists of 4 components. A and B are connected in parallel, C and D are connected in parallel and the A/B and C/D subsystems are connected in series. All components have the same constant failure rate  $\lambda$ .
  - (i) Show that the reliability function of this system is  $R_s(t) = 4e^{-2\lambda t} - 4e^{-3\lambda t} + e^{-4\lambda t}$ .
  - (ii) Show that the expected lifetime of the system is  $\frac{11}{12}$  of the expected lifetime of each individual component.
  - (iii) If  $\lambda = 0.5$  per 1000 hours, find the probability that the system will have a lifetime greater than the mean.

4. A navigational control device is made from five circuit boards A, B, C, D and E. The device has two circuits in parallel. The first of these consists of boards A, B and C connected in series. The second of these consists of another board A in series with boards D and E. However, D and E are connected in parallel.

During the operational life of the control device the circuit boards A to E have constant failure rates of 0.2, 0.3, 0.3, 0.4 and 0.4 per ten thousand hours of operation respectively. The device will fail as soon as no signal can pass through. The manufacturers claim an expected lifetime of 25,000 hours of operational life. Is this claim reasonable? (Find the reliability function first.)