Outline Solutions to Tutorial Sheet 10

1. 95% CI for μ : $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$ = $5.06 \pm 1.96\frac{0.06}{\sqrt{25}}$ = **5.06 ± 0.024** pounds

   or **5.036 → 5.084** pounds

2. Sample is small, σ is unknown so estimate with s and use $\bar{x} \pm t_{n-1}\frac{s}{\sqrt{n}}$.

   Sample mean and standard deviation are $\bar{x} = 25.5$ kg and s = 3.240kg.

   For 90% confidence (tail area of 0.05) and ν = 9 degrees of freedom, tabulated t-value is 1.833.

   Then 90% CI for μ : $25.5 \pm 1.833\frac{3.240}{\sqrt{10}}$ = **25.5 ± 1.88 kg** or **23.62kg → 27.38 kg**

   This assumes lead sulphate amounts are normally distributed.

3. Sample is small, σ is unknown so estimate with s and use $\bar{x} \pm t_{n-1}\frac{s}{\sqrt{n}}$.

   Sample mean and standard deviation are $\bar{x} = 231.67$ Hz and s = 1.531Hz.

   For 99% confidence (tail area of 0.005) and ν = 4 degrees of freedom, tabulated t-value is 4.604.

   Then 99% CI for μ : $231.67 \pm 4.604\frac{1.531}{\sqrt{5}}$ = **231.67 ± 3.152 Hz** or **228.518Hz → 234.822 Hz**

4. Answers should be the same (except for round-off error).

5. $\hat{p} = \frac{7}{50} = 0.14$. Then the 95% CI for p is

   $$0.14 \pm 1.96\sqrt{\frac{(0.14)(1-0.14)}{50}}$$

   i.e. **0.14 ± 0.096** or **0.044 → 0.236**

   There are 1 million boards in the shipment, so 4.4% to 23.6% of these is 44,000 to 236,000. Thus we are 95% confident that the range 44,000 to 236,000 contains the number of boards excessively warped.

   This is far too wide to be of any practical use. The sample is too small. Many more than 50 boards would be needed to get a reasonably precise estimate. (See question 8.)

6. Using a 1-sample t interval gives confidence limits **646 to 1255 hours**.

The histogram is positively skew (i.e. breakdown times do *not* appear to be normally distributed. However, this doesn't matter since the sample size is 'large' (≥ 30), so the interval is still valid.

7. Minitab output, using 1-sample t intervals, is:

	N	MEAN	STDEV	SE MEAN	95.0 PERCENT C.I.
Supp1	100	599.548	0.619	0.062	(**599.425, 599.671**)
Supp2	100	600.230	1.874	0.187	(**599.858, 600.602**)

These intervals do *not* overlap, which suggests that there really is a difference in mean lengths from the two suppliers.

8. Using $\hat{p} = 0.14$ as a planning value, we require n so that $1.96\sqrt{\dfrac{(0.14)(1-0.14)}{n}} = 0.03$

This can be solved to give n = 513.9. So **at least 514** boards are required in the sample.

If no estimate of p is available, use the pessimistic estimate $\hat{p} = 0.50$. Then replacing 0.14 with 0.50 in the above calculation gives n = 1067.1 So **at least 1068** boards would be required.

9. From the 95% CI for μ : $\bar{x} \pm 1.96\dfrac{\sigma}{\sqrt{n}}$ using σ = 8, we need to find n so that $1.96 \times \dfrac{8}{\sqrt{n}} = 2$.

Solving this gives n = 61.5. So the operator would have to perform the task **at least 62** times.

For 99% confidence, replace 1.96 in the above with 2.58. Then n = 106.5 and **at least 107** observations would be needed.

10. Minitab output, using 1-sample t intervals, is then:

Variable	N	Mean	StDev	SE Mean	95.0 % CI
Part B	100	861.1	805.9	80.6	(701.2, 1021.0)
Part E	100	276.4	341.1	34.1	(208.7, 344.0)

Note that the interval for component B includes the claimed value of 1000 hours. Thus 1000 hours *is* a plausible value for the mean lifetime and the manufacturer's claim is reasonable.

On the other hand, the interval for component E *does not* include the claimed value of 350 hours. Thus 350 hours is *not* a plausible value for the mean lifetime and the manufacturer's claim is *not* reasonable. The true (population) mean lifetime appears to be less than 350 hours.