Outline Solutions to Tutorial Sheet 4

1. (i) $\Pr(X > 0.08) = \Pr(Z > \frac{0.08-0.06}{0.008}) = \Pr(Z > 2.5) = 0.00621.$

(ii)

$$\begin{aligned}\Pr(X < 0.055) &= \Pr(Z < \frac{0.055-0.06}{0.008}) \\ &= \Pr(Z < -0.625) \\ &= \Pr(Z > 0.625) \\ &= \frac{0.2676+0.2643}{2} \\ &= 0.2660.\end{aligned}$$

(iii)

$$\begin{aligned}\Pr(0.05 < X < 0.07) &= \Pr(\frac{0.05-0.06}{0.008} < Z < \frac{0.07-0.06}{0.008}) \\ &= \Pr(-1.25 < Z < 1.25) \\ &= 1 - 2\times\Pr(Z > 1.25) \\ &= 0.7795.\end{aligned}$$

(iv)

$$\begin{aligned}&\Pr(X < 0.045 \text{ or } X > 0.065) \\ =\ &\Pr(X < 0.045) + \Pr(X > 0.065) \\ =\ &\Pr(Z < \frac{0.045-0.06}{0.008}) + \Pr(Z > \frac{0.065-0.06}{0.008}) \\ =\ &\Pr(Z < -1.875) + \Pr(Z > 0.625) \\ =\ &0.0304 + 0.2660 \\ =\ &0.2964.\end{aligned}$$

2. (i)

$$\begin{aligned}\Pr(X < 6520) &= \Pr(Z < \frac{6520-6000}{100}) \\ &= \Pr(Z < 5.2) \\ &= 1 - \Pr(Z > 5.2) \\ &\approx 1 - 0 \\ &= 1.\end{aligned}$$

(ii)

$$\begin{aligned}\Pr(5800 < X < 5900) &= \Pr(\frac{5800-6000}{100} < Z < \frac{5900-6000}{100}) \\ &= \Pr(-2 < Z < -1) \\ &= 0.1587 - 0.02275 \\ &= 0.13585\end{aligned}$$

(iii) Required strength is 1.6449 SD's **below** the mean.
i.e. $6000 - 1.6449\times100 = 5836$ Kg cm^{-2}

(iv) Required strength is 1.2816 SD's **above** the mean.
i.e. $6000 + 1.2816\times100 = 6128$ Kg cm^{-2}

3. (i) Required length is 2.3263 SD's **above** the mean.
i.e. $90 + 2.3263\times0.1 = 90.23$mm

(ii) Required length is 1.6449 SD's **below** the mean
i.e. $90 - 1.6449\times0.1 = 89.84$mm

(iii) First, find the probability that one particular case is between 89.7 and 90.3 millimetres:

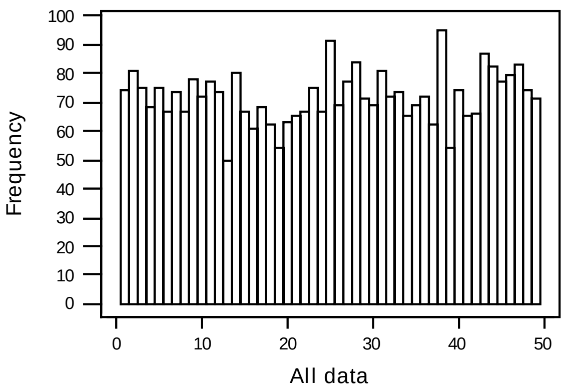
$$\begin{aligned}
\Pr(89.7 < X < 90.3) &= \Pr\left(\frac{89.7-90}{0.1} < Z < \frac{90.3-90}{0.1}\right) \\
&= \Pr(-3 < Z < 3) \\
&= 1 - 2\times\Pr(Z > 3) \\
&= 0.9973.
\end{aligned}$$

Then, by independence the probability that all 10 cases are between 89.7 and 90.3 millimetres is:

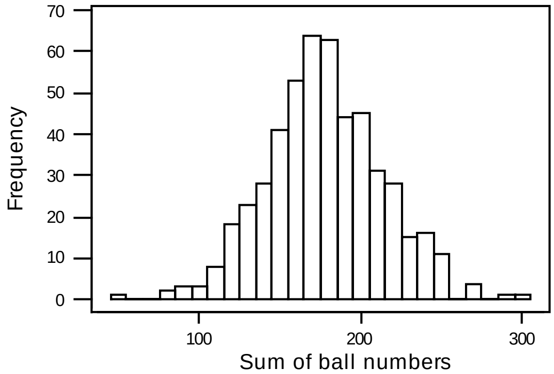
$$\begin{aligned}
&\Pr(89.7 < X_1 < 90.3 \text{ and } \ldots \text{ and } 89.7 < X_{10} < 90.3) \\
= &\Pr(89.7 < X_1 < 90.3)\times \cdots \times\Pr(89.7 < X_{10} < 90.3) \\
= &(0.9973)^{10} \\
= &0.9733.
\end{aligned}$$

(iv) $10\times\Pr(89.7 < X < 90.3) = 10\times0.9973 = 9.973.$

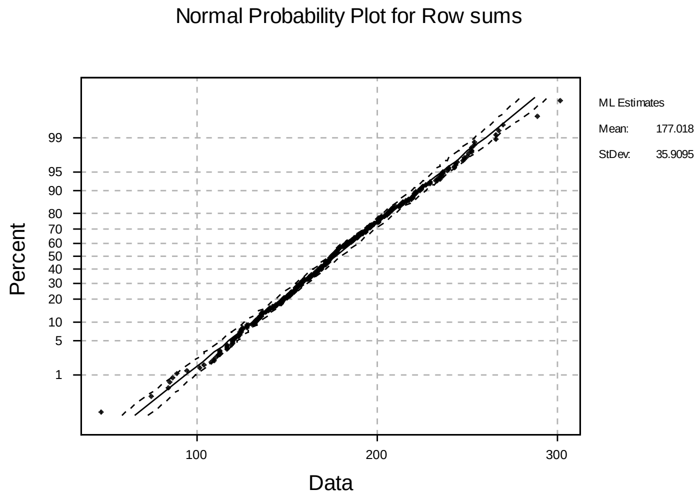
4. (i) The histogram is approximately uniform.



(ii)

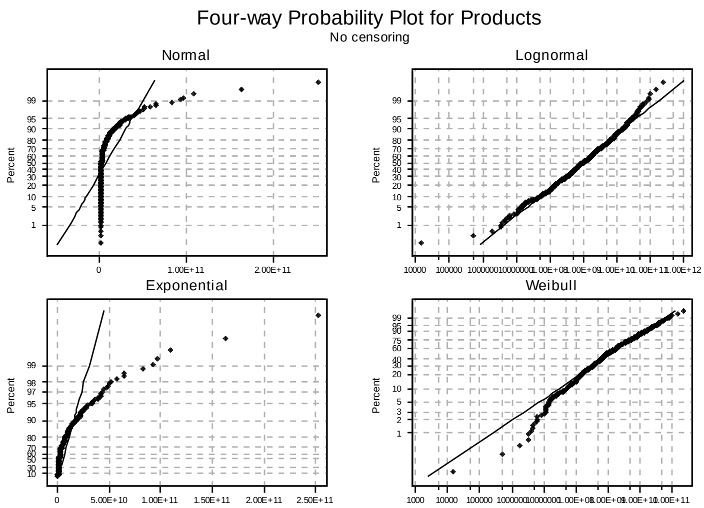


The ID plot shows the normal is the best fit of the four.

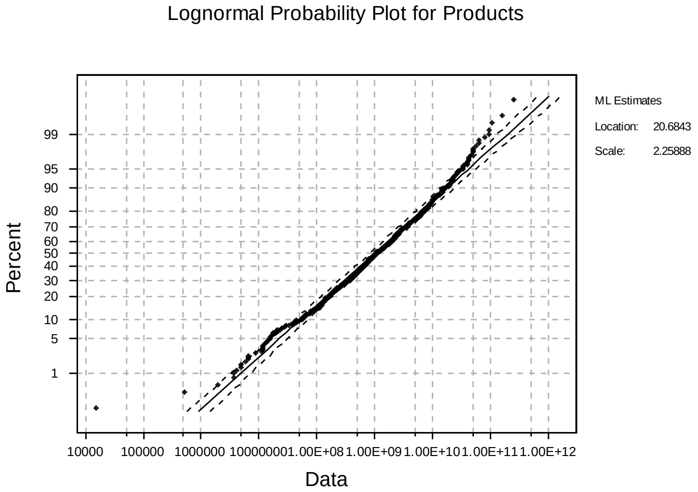


There are three points at the ends outside the error limits but the sample is very small (only seven numbers are being added). The fit would be even better if more numbers were summed.

(iii)



The lognormal distribution looks the best of the four but even that is not very good.



Again, this is probably due to the small sample.