Outline Solutions to Tutorial Sheet 5

1. Estimates are 5.06924 for the location parameter and 1.02544 for the scale parameter.

   (i) $\log_e(50) = 3.9120$.

   Standardizing gives z = $\dfrac{3.9120 - 5.06924}{1.02544} = -1.13$

   Then, from normal tables, $p(Z < -1.13) = 0.129$

   (ii) Corresponding value of Z is –1.6449 so $\log_e(X)$ should be 1.6449 standard deviations below the mean i.e. $\log_e(X) = 5.06924 - (1.6449)(1.02544) = 3.3825$

   Then $X = e^{3.3825} = 29.44$ hours.

2. The sample mean is $\overline{X} = 861.14$ hours, so the estimate of λ is $\dfrac{1}{\overline{X}} = 0.001161$ per hour

   (i) $p(X > 2500) = e^{-0.001161 \times 2500} = e^{-2.9025} = 0.055$

   (ii) $p(1000 < X < 2000) = p(X > 1000) - p(X > 2000) = e^{-0.001161 \times 1000} - e^{-0.001161 \times 2000}$
   $= 0.3131 - 0.0981 = 0.215$

   (iii) Working backwards, we require the lifetime (t, say) so that $p(X > t) = 0.05$

   i.e. $e^{-0.001161t} = 0.05$.

   Solving this gives $-0.001161t = \log_e(0.05) = -2.9957$

   Thus the required lifetime is $\dfrac{-2.9957}{-0.001161} = 2580$ hours.

   (iv) The median lifetime is that exceeded by 50% of the values. Repeating part (iii) with 0.5 in place of 0.05 gives the median lifetime as $\dfrac{\log_e(0.5)}{-0.001161} = \dfrac{-0.6931}{-0.001161} = 597$ hours.

   The sample median is 544 hours – similar to the calculated value in part (iv).

3. (i) The shape is slightly positively skew and suggests a value for β of about 2.

(ii) From the plot, the estimates are for the scale, α = 940.548 and for the shape, β = 2.31695.

(iii) $$p(X > 400) = \exp\left\{-\left(\frac{400}{940.548}\right)^{2.31695}\right\} = e^{-0.13793} = 0.871$$

$$p(X > 1400) = \exp\left\{-\left(\frac{1400}{940.548}\right)^{2.31695}\right\} = e^{-2.5133} = 0.081$$

(iv) Should get the same values as in (iii). (Don't forget to subtract Minitab's answers from 1.)

4. (i) 14 faults per week is a rate of λ = 1 per 12 hours.

Then $p(X < 12) = 1 - e^{-\left(\frac{1}{12}\right)(12)} = 1 - e^{-1} = 0.632$

(ii) $p(X > 48 hours) = e^{-4} = 0.018$

5. $$p(X > 6000) = \exp\left\{-\left(\frac{6000}{5000}\right)^{0.5}\right\} = e^{-1.2} = 0.301$$