

Outline Solutions to Tutorial Sheet 7

1. (i) $p(X = 5) = \frac{(2.5)^5 e^{-2.5}}{5!} = 0.067$
 (ii) $p(X \geq 2) = 1 - [p(X = 0) + p(X = 1)]$
 where $p(X = 0) = \frac{2.5^0 e^{-2.5}}{0!} = 0.082085$ and $p(X = 1) = \frac{2.5^1 e^{-2.5}}{1!} = 0.20521$
 Thus $p(X \geq 2) = 1 - 0.287 = 0.713$

2. The distribution is Poisson with $\lambda = 14/7 = 2$ per day. Then

- (i) $p(X = 0) = 0.1353$
 (ii) $p(X = 3) = 0.1804$

Other probabilities that will be needed to calculate parts (iii) to (vii) are:

$$p(X = 1) = 0.2707 \quad p(X = 2) = 0.2707 \quad p(X = 4) = 0.0902$$

- (iii) $p(X \leq 2) = p(X = 0) + p(X = 1) + p(X = 2) = 0.6767$
 (iv) $p(X > 1) = 1 - [p(X = 0) + p(X = 1)] = 1 - 0.4060 = 0.5940$
 (v) $p(X \geq 3) = 1 - p(X < 3) = 1 - [p(X = 0) + p(X = 1) + p(X = 2)] = 1 - 0.6767 = 0.3233$
 (vi) $p(X < 4) = p(X = 0) + p(X = 1) + p(X = 2) + p(X = 3) = 0.8571$
 (vii) $p(2 \leq X \leq 4) = p(X = 2) + p(X = 3) + p(X = 4) = 0.5413$

3. The number answered correctly has a binomial distribution with $n = 25$ and $p = 0.25$. We require

$$p(X < 5) = p(X = 0) + p(X = 1) + p(X = 2) + p(X = 3) + p(X = 4)$$

$$= \frac{25!}{0! 25!} 0.25^0 0.75^{25} + \frac{25!}{1! 24!} 0.25^1 0.75^{24} + \frac{25!}{2! 23!} 0.25^2 0.75^{23} + \frac{25!}{3! 22!} 0.25^3 0.75^{22} + \frac{25!}{4! 21!} 0.25^4 0.75^{21} = 0.2137$$

4. Under *Calc* → *Probability Distributions* → *Binomial*, use $n = 50$ and $p = 0.20$. Then enter 14 as *Input Constant* to get the *cumulative probability* $p(X \leq 14) = 0.9393$

$$\text{Thus, } p(X \geq 15) = 1 - 0.9393 = 0.0607$$

5. A good design misses only 1% of welds. The number of missed welds out of 100 then has a binomial distribution with $n = 100$ and $p = 0.01$. It will be rejected if it misses 3 or more of the 100.

$$\text{Thus } p(X \geq 3) = 1 - [p(X = 0) + p(X = 1) + p(X = 2)] = 1 - [0.3660 + 0.3697 + 0.1849] = 0.0794$$

A poor design misses 5% of welds. The number of missed welds out of 100 then has a binomial distribution with $n = 100$ and $p = 0.05$. It will be accepted if it misses 2 or less of the 100.

$$\text{Then } p(X \leq 2) = p(X = 0) + p(X = 1) + p(X = 2) = 0.00592 + 0.03116 + 0.08118 = 0.1183$$

6. We expect $\lambda = 0.05 \times 10 = 0.5$ in 10 square feet

- (i) $p(X = 0) = e^{-0.5} = 0.6065$ (ii) $0.6065^{10} = 0.006738$