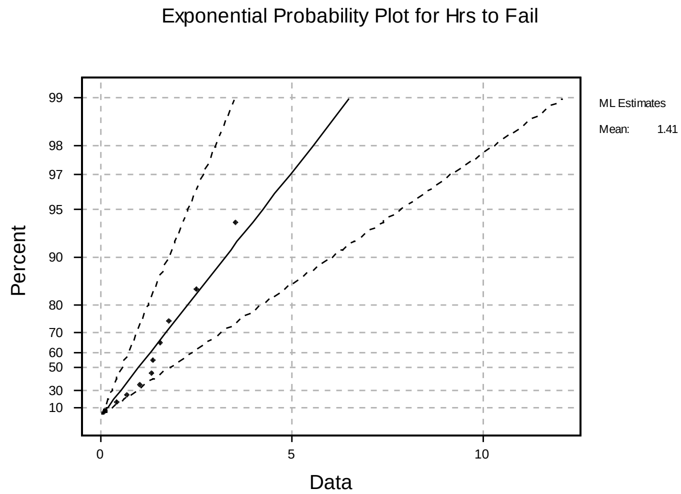
Outline Solutions to Tutorial Sheet 8

1. The exponential probability plot is :



The plot gives the estimate of the mean as 1.41. Thus, since the mean is $\frac{1}{\lambda}$, the estimate of λ is $\frac{1}{1.41} = 0.709$

(i) $p(X < 1) = F(1) = 1 - e^{-0.709 \times 1} = 1 - 0.4921 = 0.5079$

(ii) $p(X > 2) = R(2) = e^{-0.709 \times 2} = 0.2422$

(iii) $p(1.5 < X < 2.5) = F(2.5) - F(1.5)$ or

$R(1.5) - R(2.5) = e^{-0.709 \times 1.5} - e^{-0.709 \times 2.5} = 0.3452 - 0.1699 = 0.1753$

The hazard function is $h(t) = \lambda = 0.709$ (i.e. constant) and the MTBF is 1.41 hours

2. The Weibull plot shows a good fit. Estimates of α (scale) and β (shape) are given as 0.735 and 2.61 respectively.
For this value of β, (>2) we expect a hazard function shape

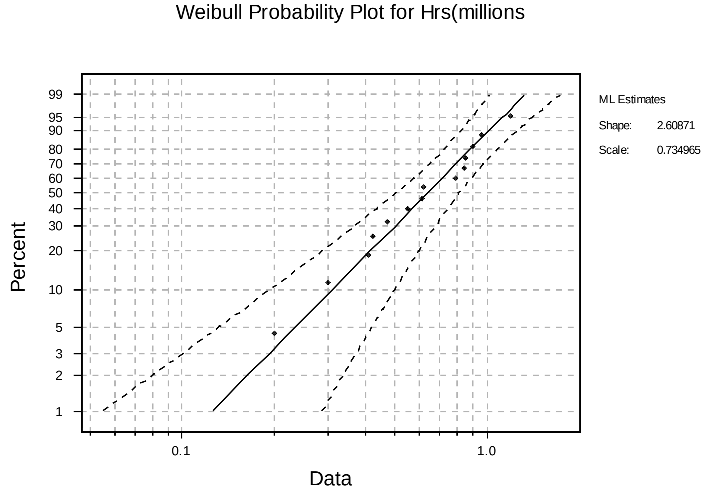
This shows positive ageing. Failure rates are *increasing* at an *increasing* rate.

$$R(t) = e^{-\left(\frac{t}{\alpha}\right)^{\beta}} = e^{-\left(\frac{t}{0.735}\right)^{2.61}} \quad \text{(in units of 1 million hours)}$$

$$\text{Then } R(0.75) = e^{-\left(\frac{0.75}{0.735}\right)^{2.61}} = e^{-1.054} = 0.3485$$

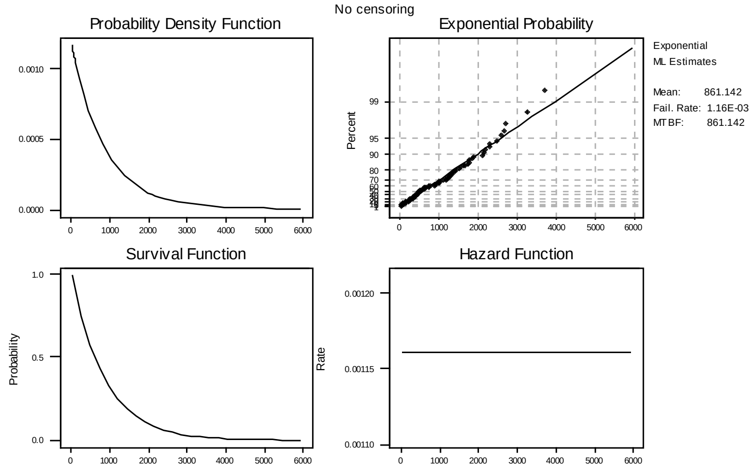
$$MTBF = \alpha\Gamma\left(1+\frac{1}{\beta}\right) = 0.735\Gamma\left(1+\frac{1}{2.61}\right) = 0.735\Gamma(1.38) = 0.888537$$ from tables of the gamma function. Thus the MTBF is approximately 889,000 hours.

Weibull Probability Plot for Hrs(millions



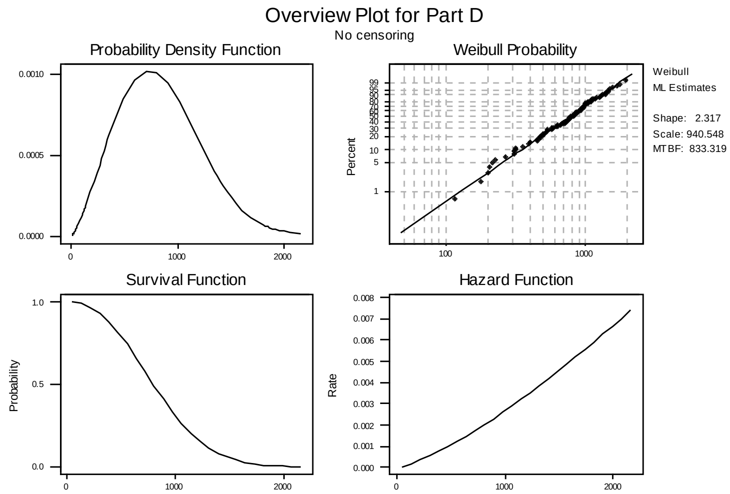
3. The *Distribution Overview Plot* for the *Part B*'s exponential lifetimes is:

Overview Plot for Part B
No censoring



In particular, notice the reverse-J shape of the pdf and the constant hazard function (constant failure rate). Also note the rapid decline of the survival function as so many components fail early.

For the Weibull lifetimes of *Part D*, the plot is:



Note the following.

(i) The distribution is positively skew

(ii) The parameter estimates for α and β. The fact that the shape parameter β is greater than 2 suggests that the hazard function should increase at an increasing rate (as the plot shows).

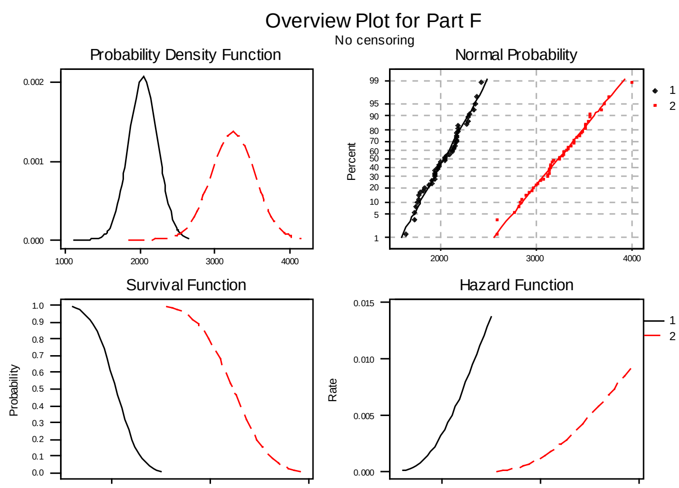
(iii) Compared to the exponential survival function above, more components are surviving in the early stages.

Calculating the MTBF from the estimates of α and β gives:

$$\text{MTBF} = \alpha\Gamma\left(1+\frac{1}{\beta}\right) = 940.548\times\Gamma\left(1+\frac{1}{2.317}\right) = (940.548)\Gamma(1.43) = (940.548)(0.886) = 833.3\,\text{hours}$$

which is the same as the estimate on the plot.

Finally, for the two types of *Part F*, **normal distributions** provide good fits. The combined overview plot is then:



The pdf shows that Type 2 components have a higher average but more variable lifetime than Type 1. As a result of this, the survival plot shows that, at a given time, a higher proportion of Type 2 components will survive. The hazard functions (increasing at an increasing rate) show that, at a given time, Type 2 components have a lower risk of instantaneous failure. The fact that the hazard plot for Type 1 rises more steeply than that for Type 2 is a reflection of the fact that type 1 lifetimes are less variable – i.e. cover a smaller range.